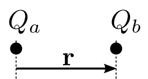
Electromagnetic Fields and Waves

Coulomb's Law
1785, Charles-Augustin de Coulomb

$\mathbf{F_{ab}} = \frac{1}{4\pi\epsilon_0}\frac{Q_aQ_b}{|\mathbf{r}|^2}\frac{\mathbf{r}}{|\mathbf{r}|}$



$\mathbf{F_{ab}}$: Electrical force exerted by Q_a on Q_b.
$\mathbf{r}$: Vector pointing from Q_a to Q_b.
Repulsive if both charges have the same sign.
Attractive if both charges have opposite sign.

Gauss' Law
1773, Johann Carl Friedrich Gauss

$\Phi_E = \frac{Q}{\epsilon_0}$ $\quad\Phi_E = \iint_S \mathbf{E}\cdot\mathbf{ds}$

Φ_E: The electric flux through a surface that encloses a volume.
Q: The total electric charge in that enclosed volume.

Ohms' Law
1827, Georg Ohm

$I = \frac{V}{R}$

I: The electric current flow in a conductor is proportional to the voltage V across it.
R: The electrical resistance.

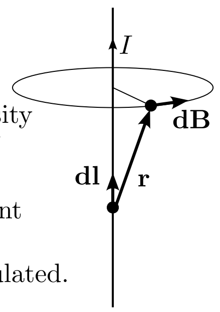
Ampère's Law
1827 André-Marie Ampère

$\oint \mathbf{H}\cdot\mathbf{dl} = I$

The integral of the magnetic field $\mathbf{H}$ over a closed path equals the current crossing a surface enclosed by this path.

Biot-Savart's Law
1820 Jean-Baptiste Biot and Félix Savart

$\mathbf{B} = \frac{\mu_o I}{4\pi}\oint \frac{\mathbf{dl}\times\frac{\mathbf{r}}{|\mathbf{r}|}}{|\mathbf{r}|^2}$



$\mathbf{dB}$: Contribution to the magnetic flux density $\mathbf{B}$ at some point in space due to a current I in a part $\mathbf{dl}$ of the contour.
$\mathbf{dl}$: Part of contour in direction of the current
$\mathbf{r}$: Vector that points from the point on the contour to the point at which $\mathbf{B}$ is calculated.

Faraday's Law
1834 Michael Faraday

$V = -\frac{d\Phi_M}{dt}$

V: Voltage induced in a loop that encloses a surface with a time-dependent magnetic flux.

Ampère-Maxwell's Law

Displacement current: $\epsilon_0\frac{\partial}{\partial t}\mathbf{E}$

Current flow resulting from a time-dependent electric field.

Maxwell's Laws
1861, James Clerk Maxwell

Gauss' law for magnetism:	$\oiint \mathbf{B}\cdot\mathbf{ds} = 0\,,$	$\nabla\cdot\mathbf{B} = 0$
Gauss' law:	$\oiint \mathbf{E}\cdot\mathbf{ds} = \iiint_V \frac{\rho}{\epsilon_0}dv\,,$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$
Faraday's law:	$\oint \mathbf{E}\cdot\mathbf{dl} = -\iint_S \frac{d}{dt}\mathbf{B}\cdot\mathbf{ds}\,,$	$\nabla\times\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$
Ampère-Maxwell's law:	$\oint \mathbf{B}\cdot\mathbf{dl} = \mu_0\left(\iint_S \mathbf{J}\cdot\mathbf{ds} + \epsilon_0\frac{d}{dt}\iint_S \mathbf{E}\cdot\mathbf{ds}\right)\,,$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

Static Electric Field

$\mathbf{E} = \frac{\mathbf{F_e}}{q}$

$\mathbf{E}$: The electric field vector, electrical force per unit of charge

$\oint \mathbf{E}\cdot\mathbf{dl} = 0 \qquad \nabla\times\mathbf{E} = 0$

The electric field is conservative:
The work does not depend on the path.

$\oiint \mathbf{E}\cdot\mathbf{ds} = \iiint_V \frac{\rho}{\epsilon_0}dv \qquad \nabla\cdot\mathbf{E} = \frac{\rho}{\epsilon_0} \qquad \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

Gauss' Law in integral and in differential form.

Static Electric Potential

$V_{1,2} = \int_{P_1}^{P_2}\mathbf{E}\cdot\mathbf{dl} \qquad -\nabla V = \mathbf{E}$

$V_{1,2}$: The potential difference between P_1 and P_2: The work required per unit of charge to move a charge from a point P_2 to a point P_1.

Self capacitance

$C_{a,a} = \frac{Q_a}{V_a}$

When charge is brought upon an isolated conductor, its potential energy changes

$C_{a,a}$: Capacitance: the ratio of the charge and the voltage of an isolated conductor

Mutual capacitance

$C_{a,b} = \frac{Q_a}{V_b}$

$C_{a,b}$: Mutual capacitance: the ratio of the charge Q_a induced on conductor a by a voltage change on conductor b.

Static Electric Current

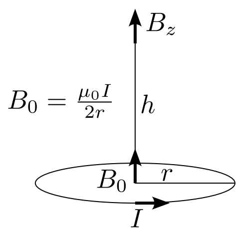
$I = \iint_S \mathbf{J}\cdot\mathbf{ds}$

$\mathbf{J}$: The current density through surface S.

Static Magnetic Field

$\oint \mathbf{H}\cdot\mathbf{dl} = \iint_S \mathbf{J}\cdot\mathbf{ds}$
$\nabla\times\mathbf{H} = \mathbf{J}$

Ampère's law in integral and differential form.
Magnetic flux density: $\mathbf{B} = \mu_0\mathbf{H}$.
Magnetic flux $\Phi_M = \iint_S \mathbf{B}\cdot\mathbf{ds}$.

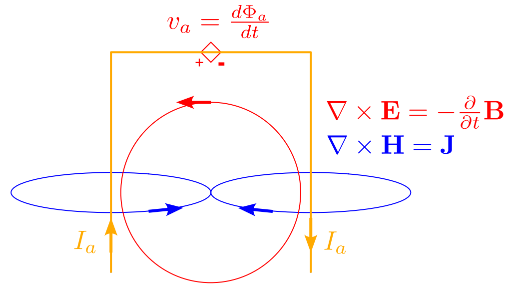


Circular current loop
$B_z = \frac{\mu_0 I r^2}{2(r^2+d^2)^{3/2}}$

B_z: magnetic flux density at distance h above the center of the loop.
I: loop current
r: loop radius
h: distance to loop

Static if all dimensions much smaller than wavelength.

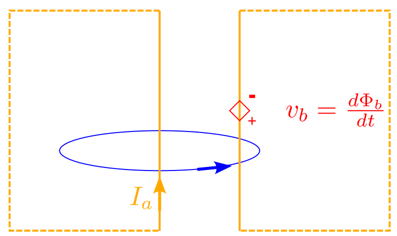
Self inductance



$L_{a,a} = -\frac{\Phi_a}{I_a}$

$L_{a,a}$: Self inductance: the flux Φ_a induced in a current loop by the loop current I_a.

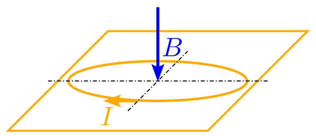
Mutual inductance



$L_{b,a} = -\frac{\Phi_b}{I_a}$

$L_{b,a}$: Mutual inductance: the flux Φ_b induced in a current loop by a current I_a in another loop.

Eddy current



Eddy currents are induced by alternating magnetic fields perpendicular to conductive planes.

Eddy currents are cause of

- damping in conductive shields
- skin effect
- proximity effect

Power losses P per unit of mass [W/kg]
(uniform magnetic field and material, no skin effect)
$P = \frac{(\pi B_p d f)^2}{6\rho D}$
B_p: Peak magnetic flux density [T]
d: Sheet thickness[m]
f: Frequency [Hz]
ρ: Sheet resistivity [Ωm]
D: Specific mass [kg/m^3]

Dynamics: interaction between E and B

$\oint \mathbf{E}\cdot\mathbf{dl} = -\iint_S \frac{\partial}{\partial t}\mathbf{B}\cdot\mathbf{ds} \qquad \nabla\times\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$

Faraday's law in integral and in differential form

$\oint \mathbf{B}\cdot\mathbf{dl} = \mu_0\left(\iint_S \mathbf{J}\cdot\mathbf{ds} + \epsilon_0\iint_S \frac{\partial}{\partial t}\mathbf{E}\cdot\mathbf{ds}\right)$
$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

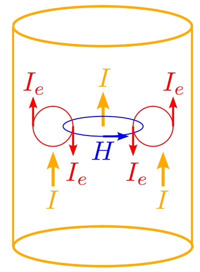
Ampère-Maxwell's law in integral and differential form.

Poynting Vector
1884, John Henry Poynting

$\mathbf{S} = \mathbf{E}\times\mathbf{H}$ [W/m^2]

$\mathbf{S}$: Poynting vector power flow of EM field.

Skin effect



Alternating currents I in a conductor cause an alternating magnetic field H inside the conductor. This gives rise to eddy currents I_e that move the current outwards to the surface.

Proximity effect

As skin effect but now with multiple adjacent conductors, mutually inducing eddy currents in each other.

If the currents in both conductors flows in the same direction, mutually induced eddy currents move the current conduction outwards.

If the currents in both conductors flows in opposite direction, mutually induced eddy currents move the current conduction inwards.

$\delta = \sqrt{\frac{\rho}{f\pi\mu_0}}$

δ: Skin depth [m]
ρ: Conductor resistivity [Ωm]
μ_0: permeability of vacuum $4\pi 10^{-7}$ [H/m].

$R_{AC} = R_{DC}\sqrt{\frac{f}{f_{skin}}}$

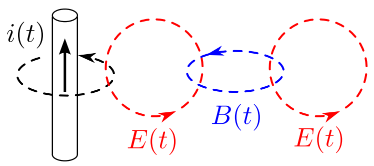
R_{AC}: AC resistance [Ω]
R_{DC}: DC resistance [Ω]
f_{skin}: Skin frequency [Hz].

$f_{skin} = \frac{4\rho}{\pi\mu_0 t^2}$

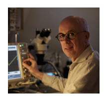
f_{skin}: Skin frequency [Hz]
ρ: Conductor resistivity [Ωm]
μ_0: permeability of vacuum $4\pi 10^{-7}$ [H/m]
t: trace thickness [m] (width larger than thickness).

Wave propagation

$\text{curl}\left(\vec{B}\right) = \mu_0\frac{i(t)}{A}$



$\text{curl}\left(\vec{E}\right) = -\frac{\partial\vec{B}}{\partial t}$
$\text{curl}\left(\vec{B}\right) = -\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}$



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