Structured Electronic Design
Amplifiers: small-signal dynamic behavior

Linear time-invariant dynamic systems

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Modeling with linear differential equations with constant coefficients

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Sum of a number of derivatives of the response = Sum of a number of derivatives of the excitation

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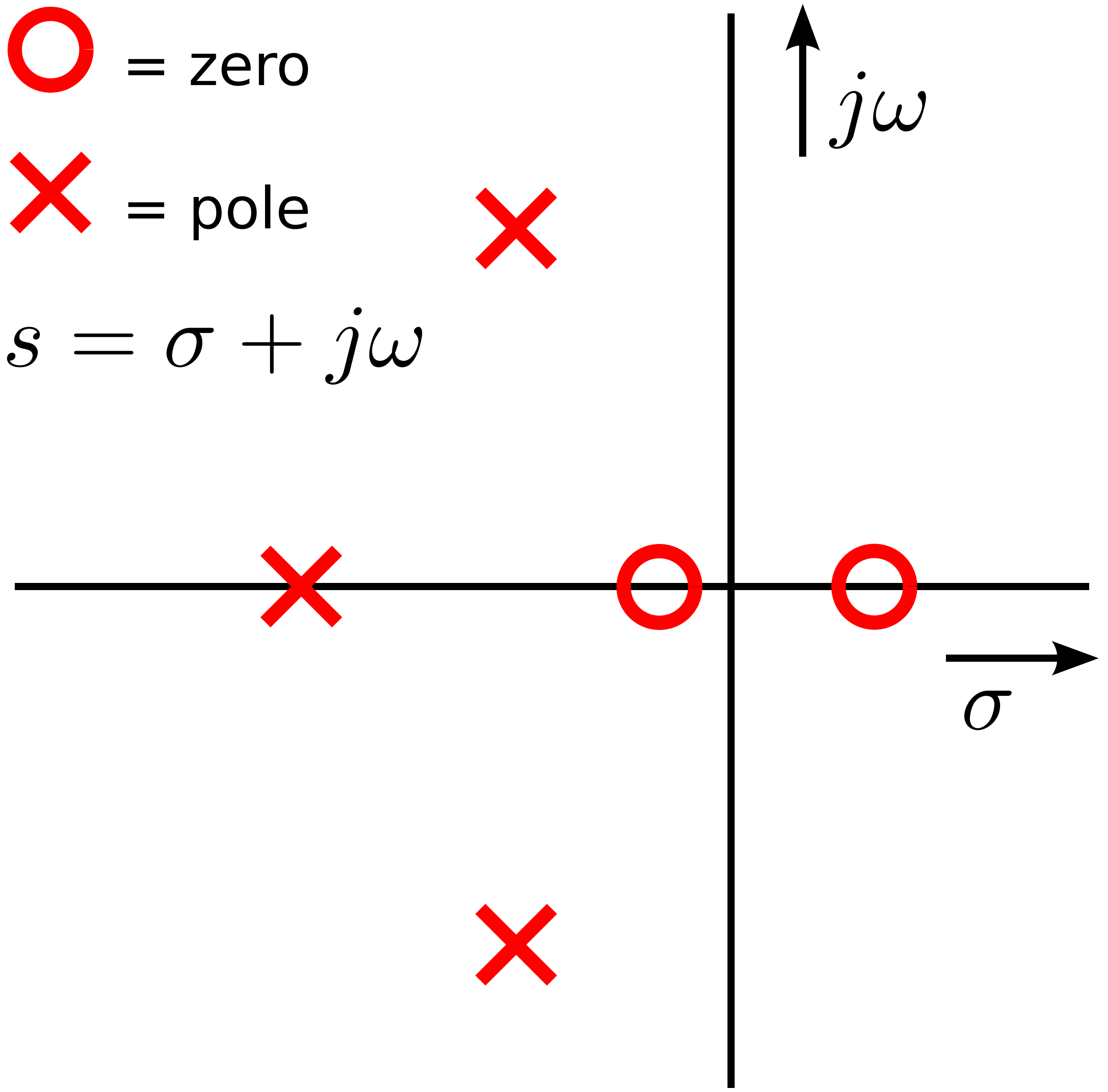

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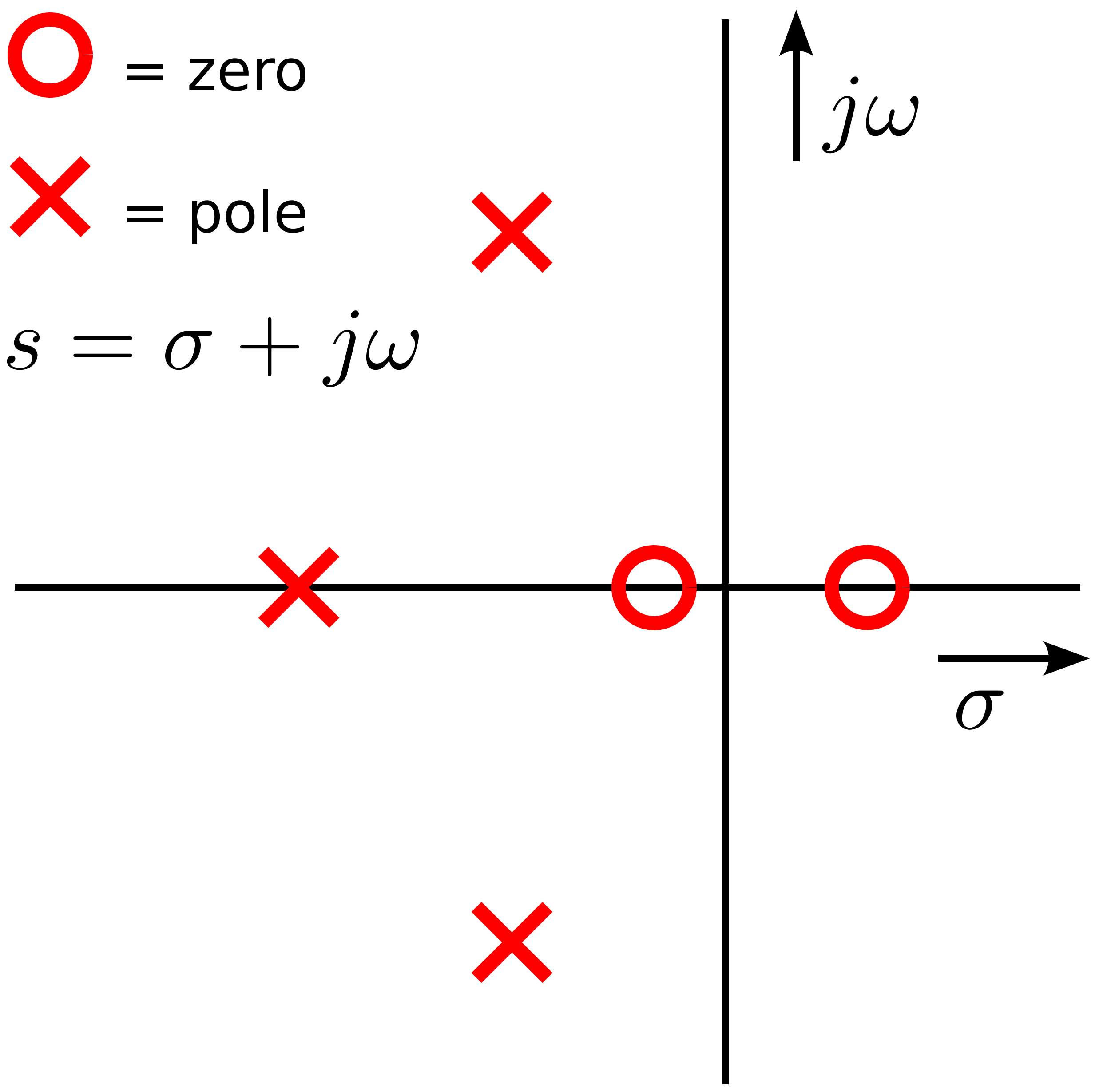

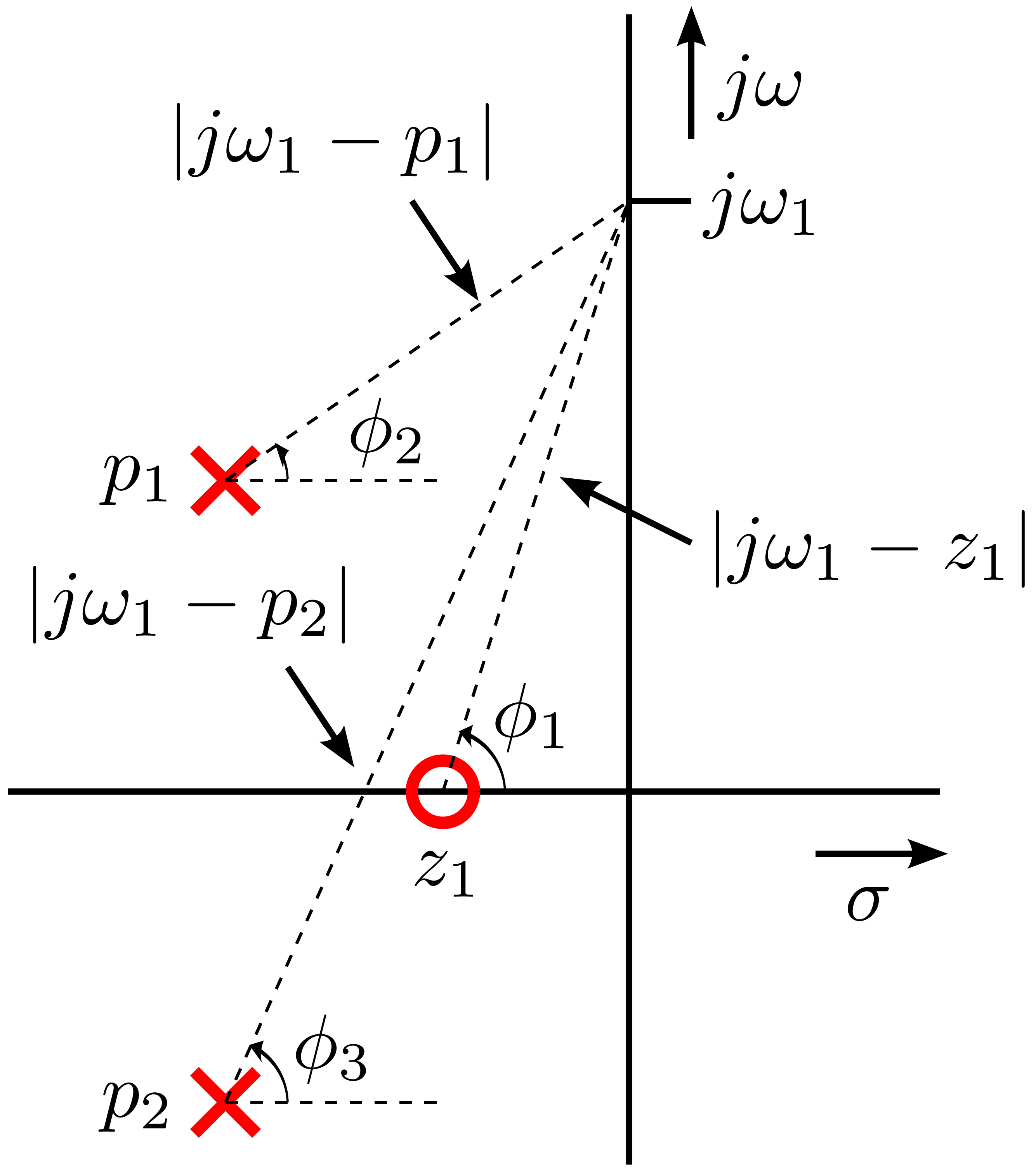

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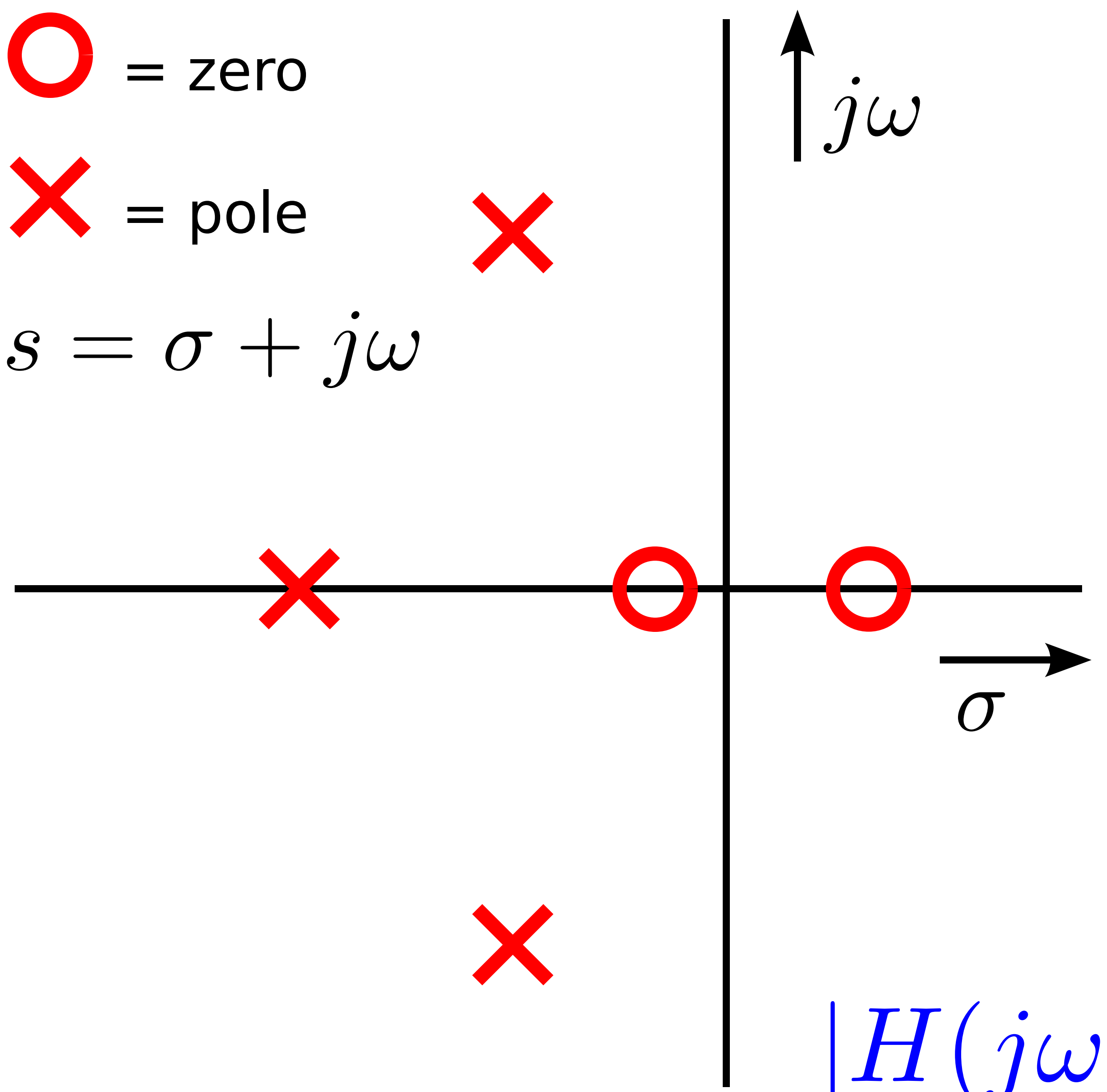
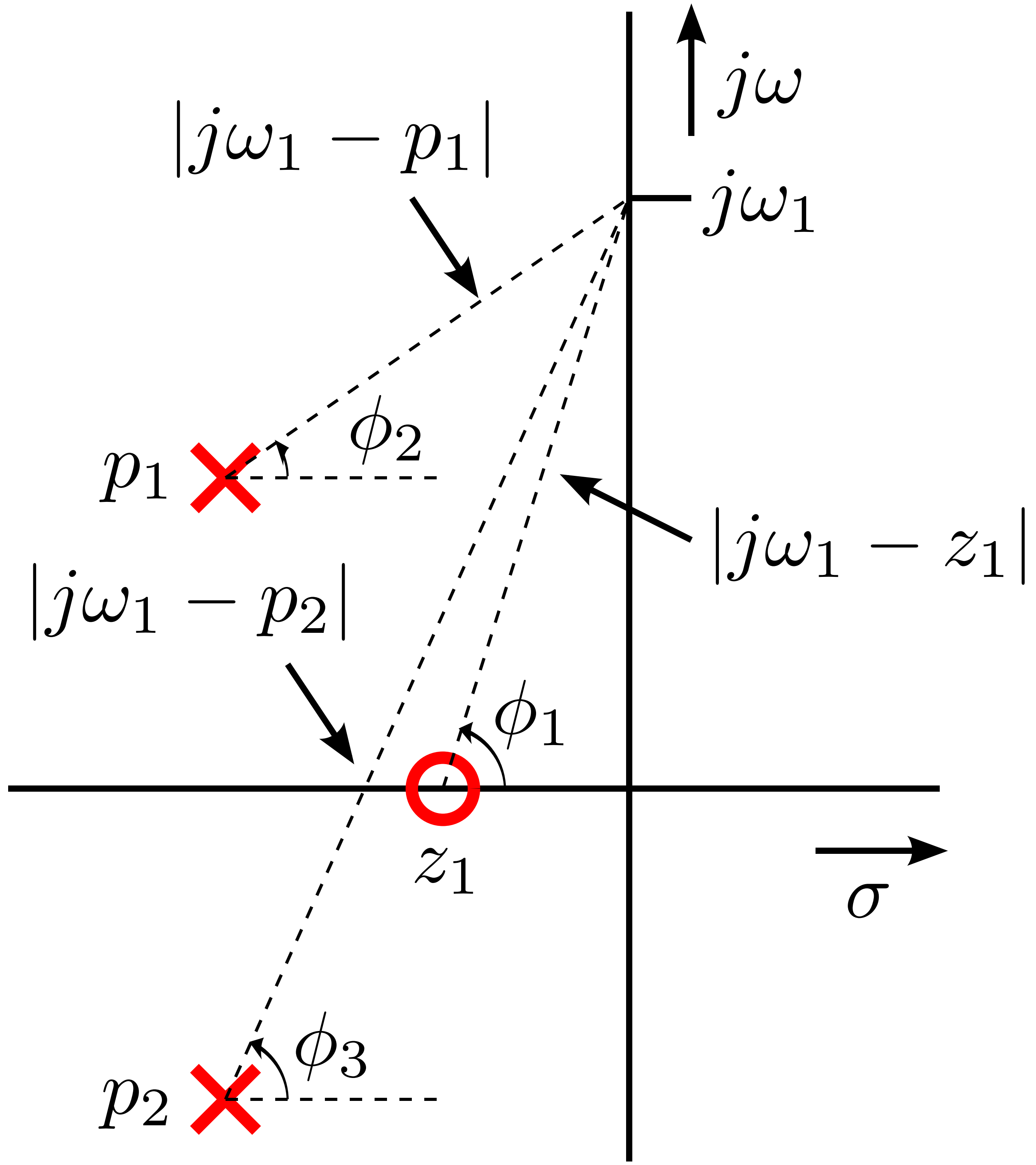


$s = \sigma + j\omega$

$$|H(j\omega)| = \frac{b_m}{a_n} \frac{\prod_{k=0}^{k=m} |j\omega - z_k|}{\prod_{i=0}^{i=n} |j\omega - p_i|}$$



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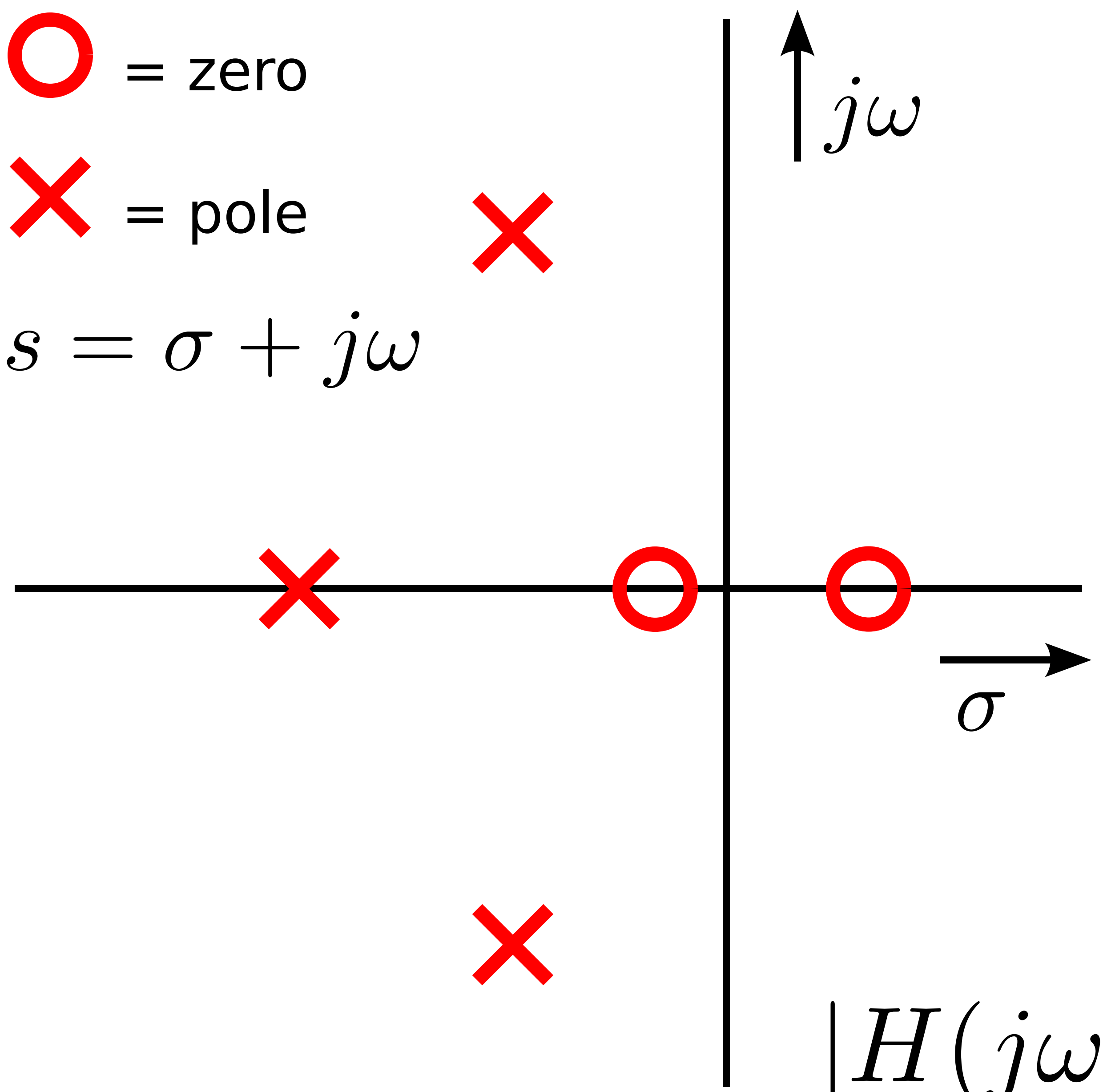

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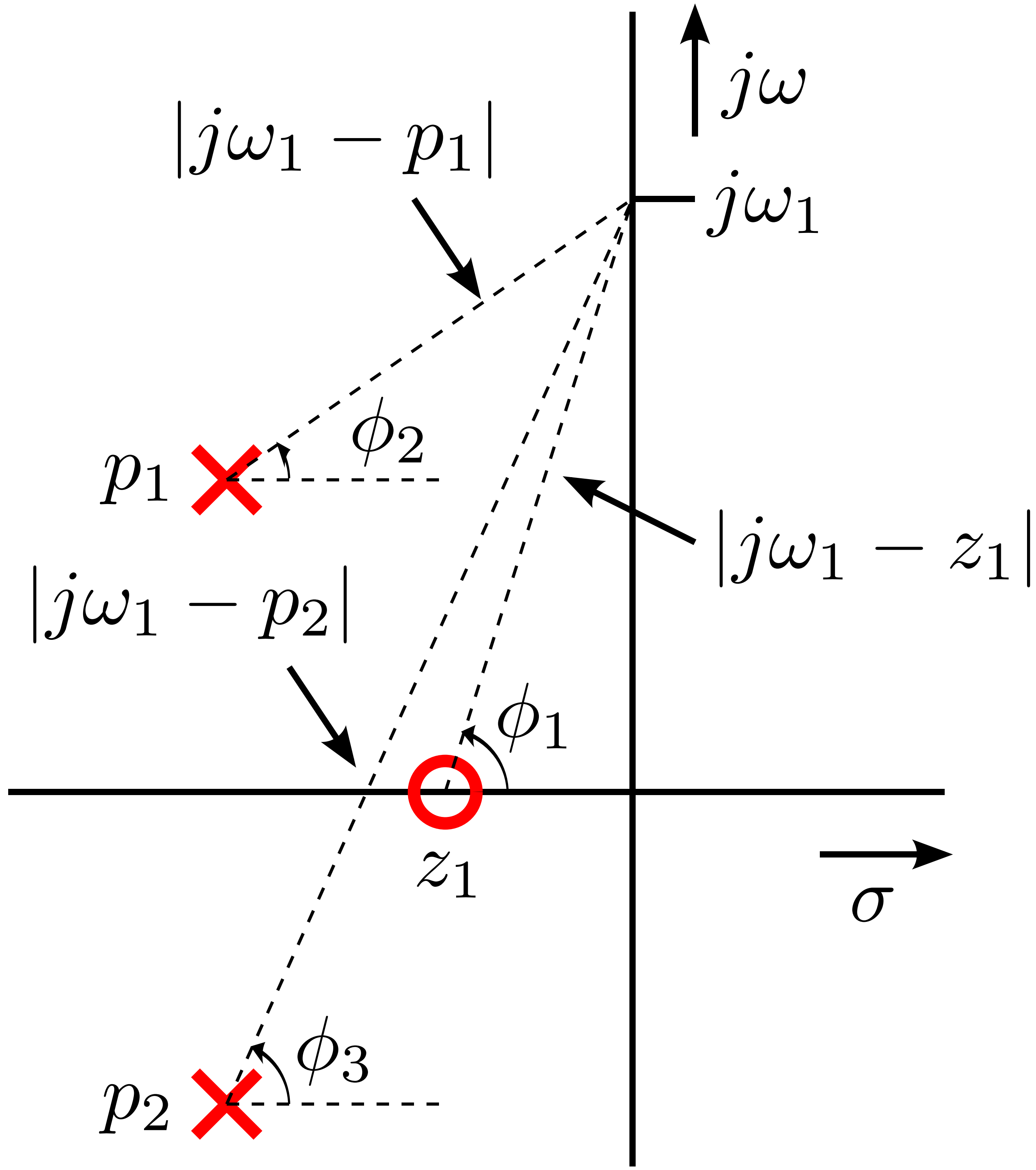

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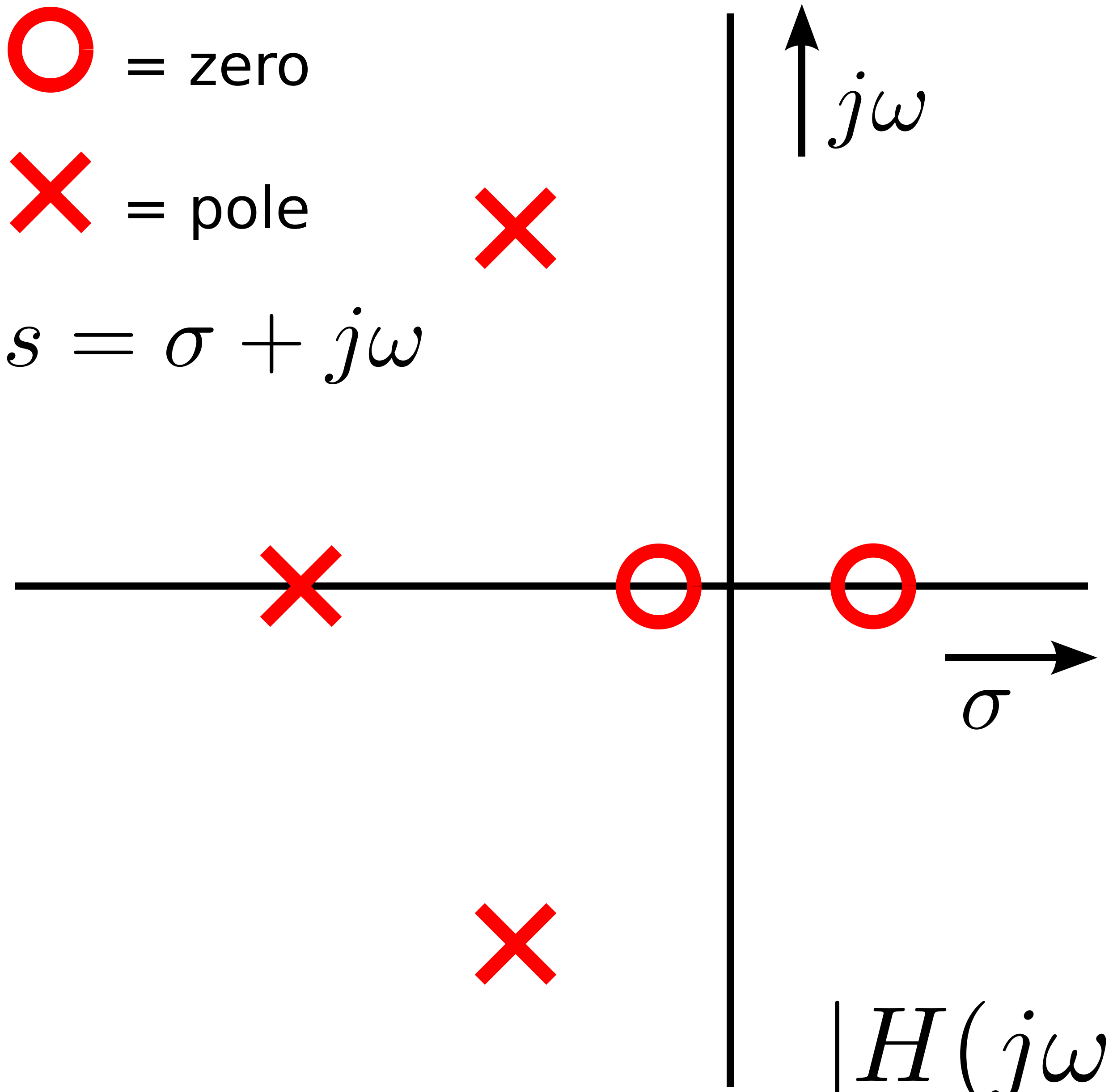

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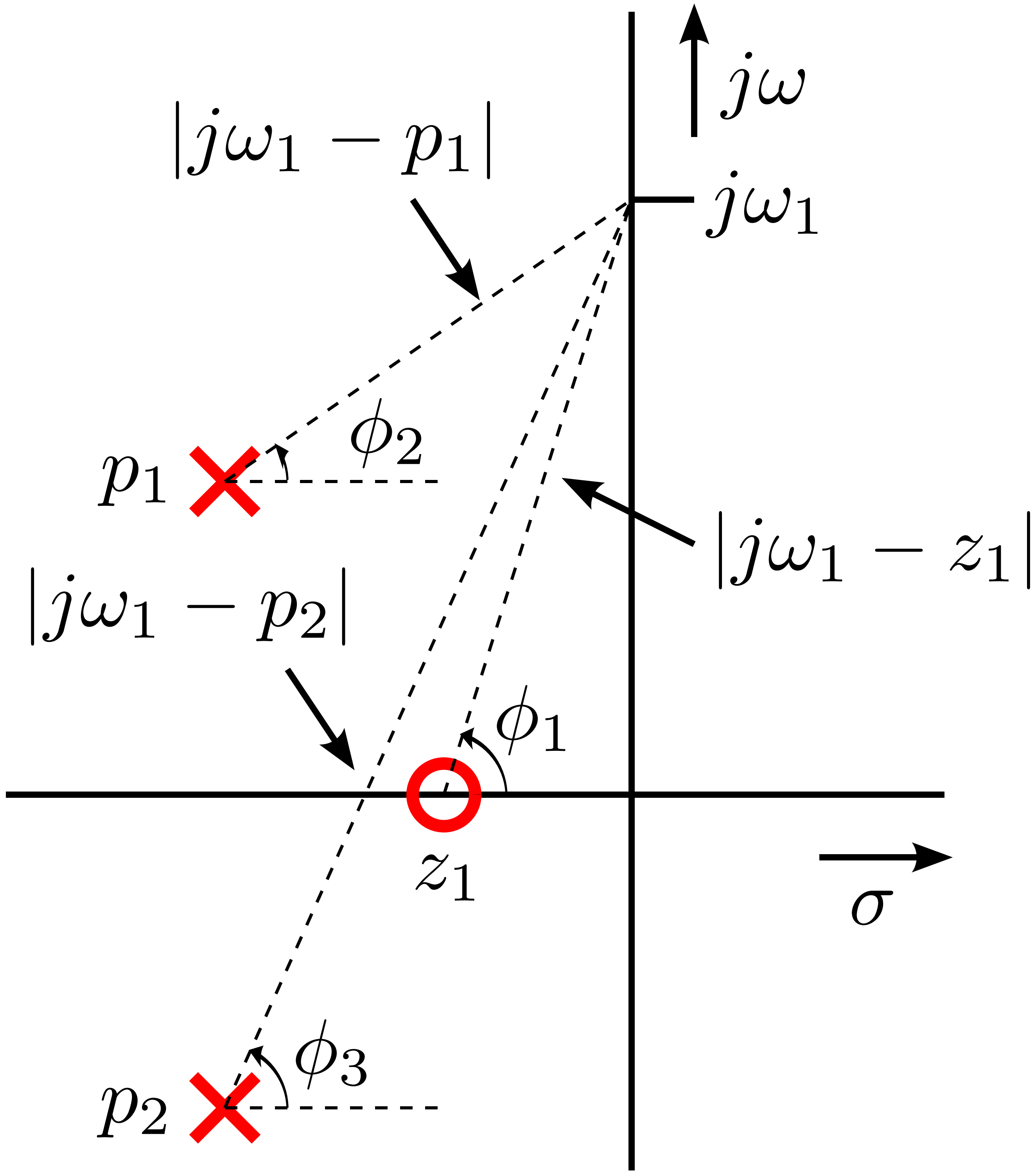

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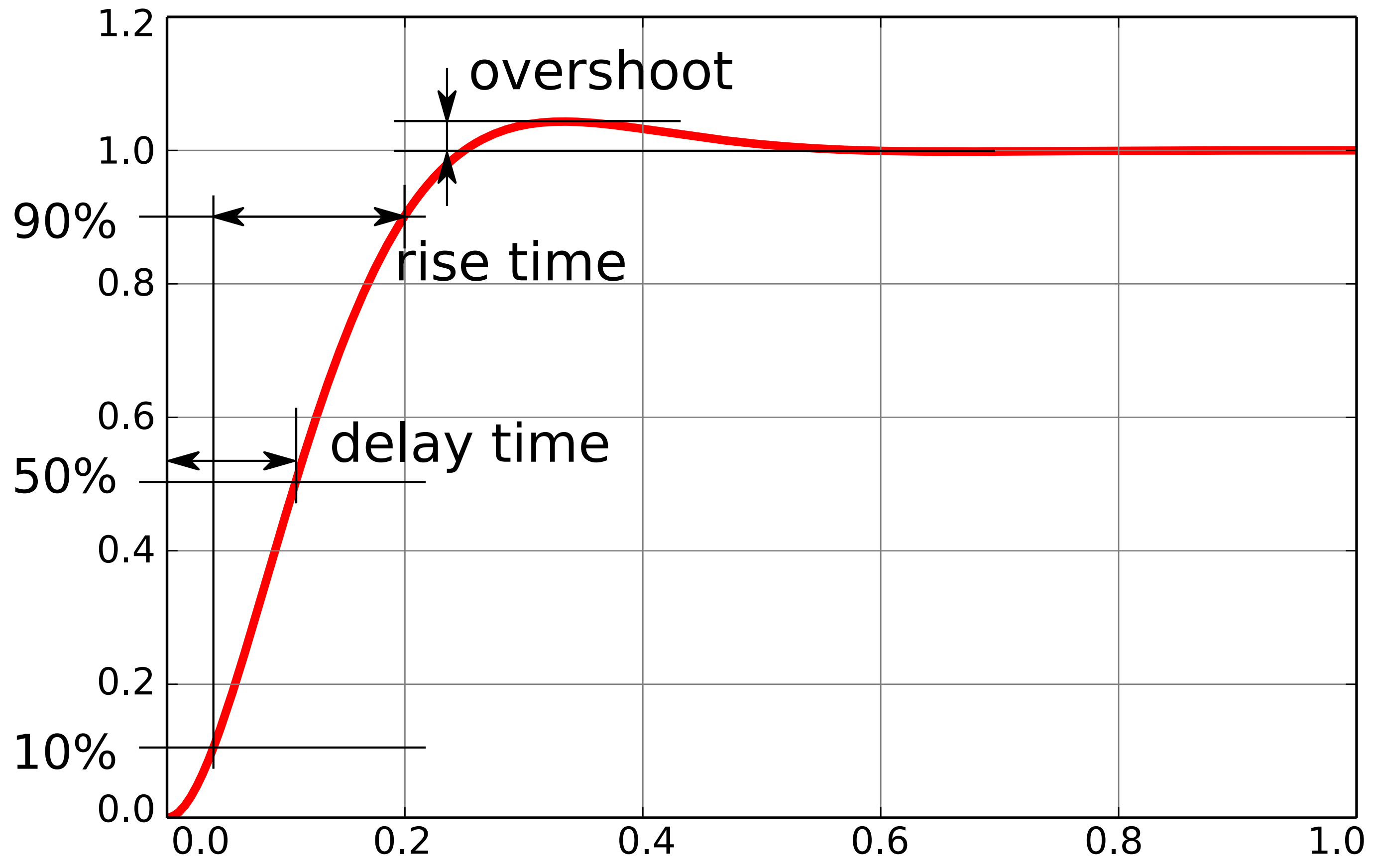

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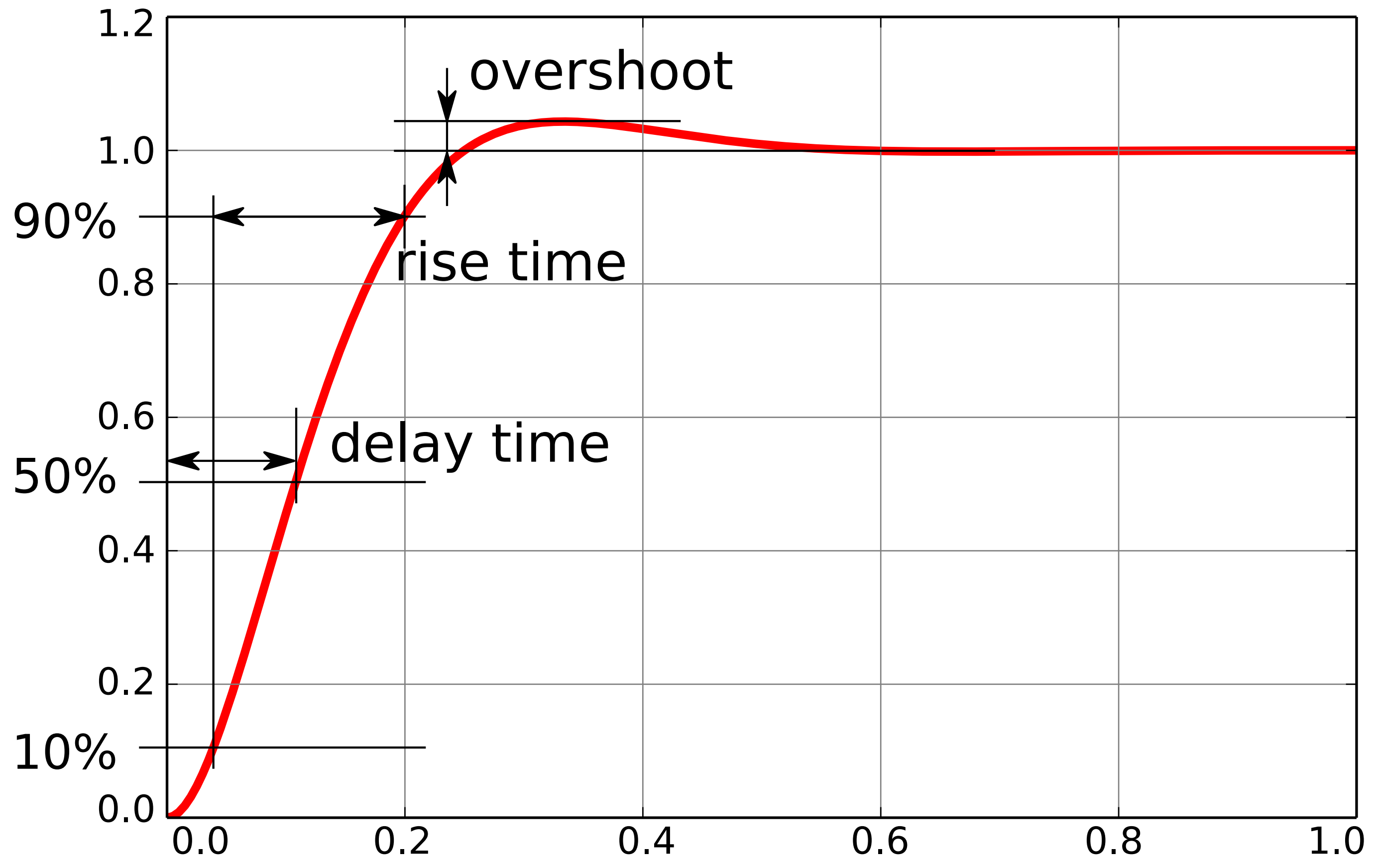

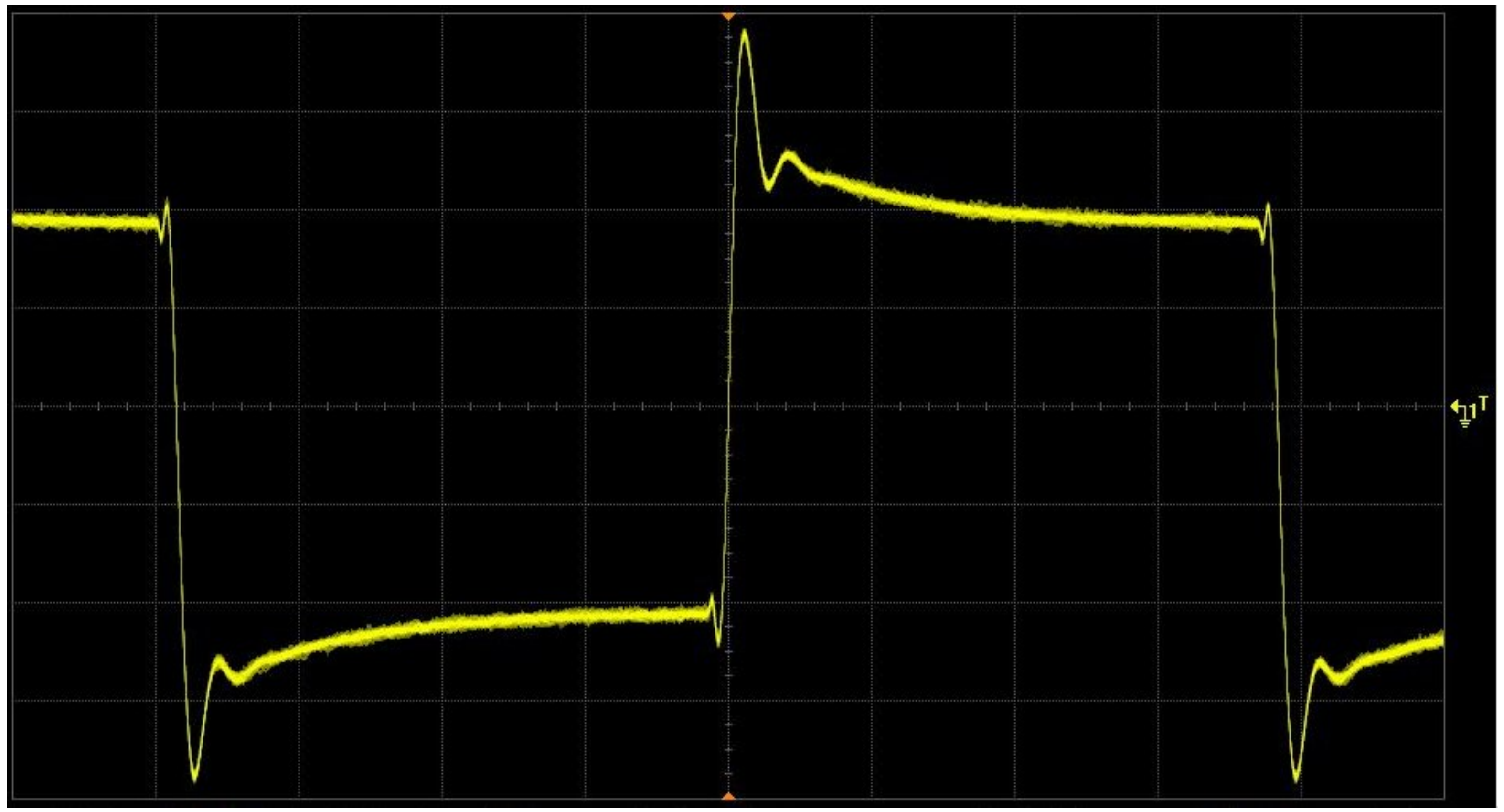

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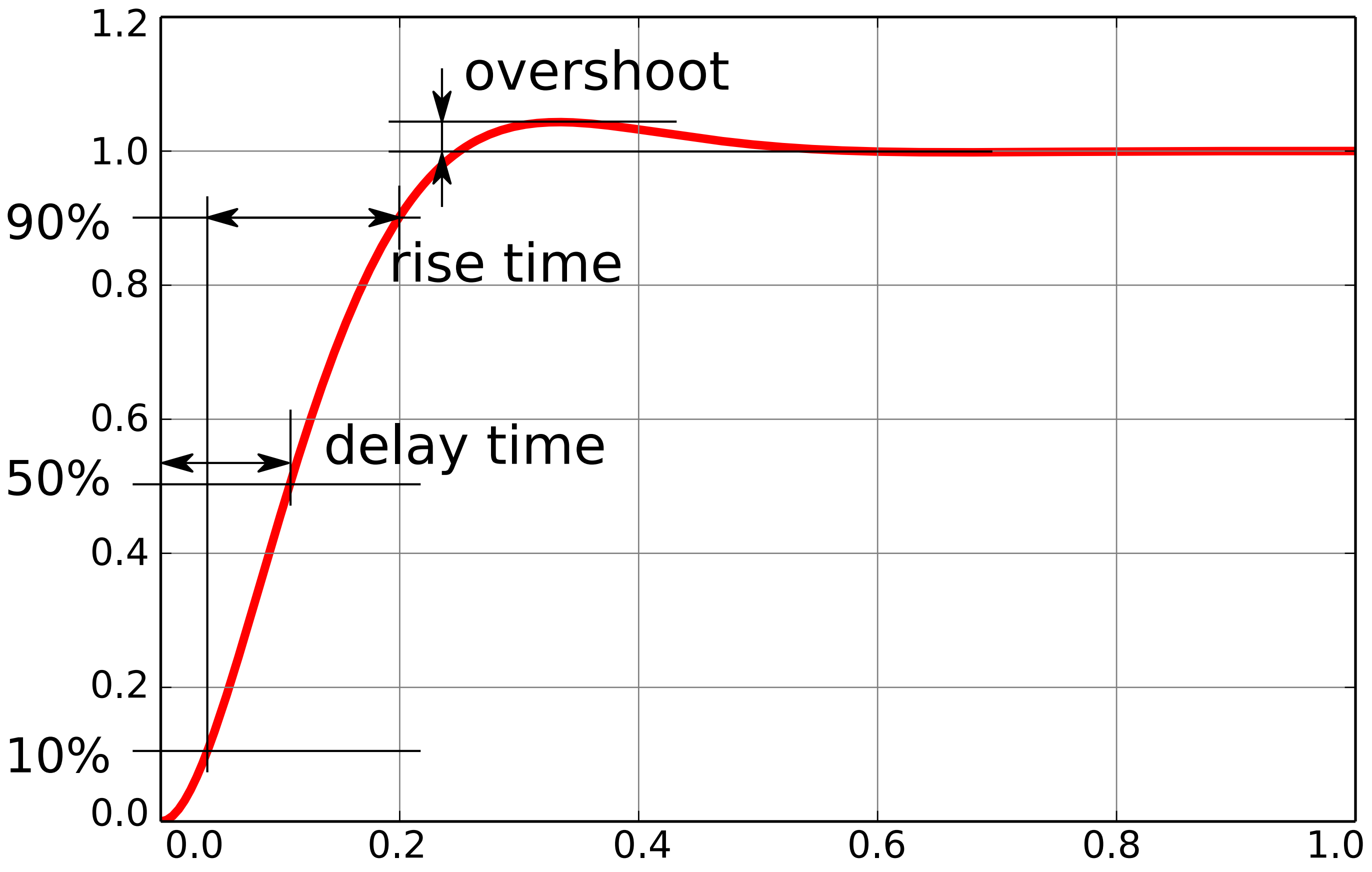

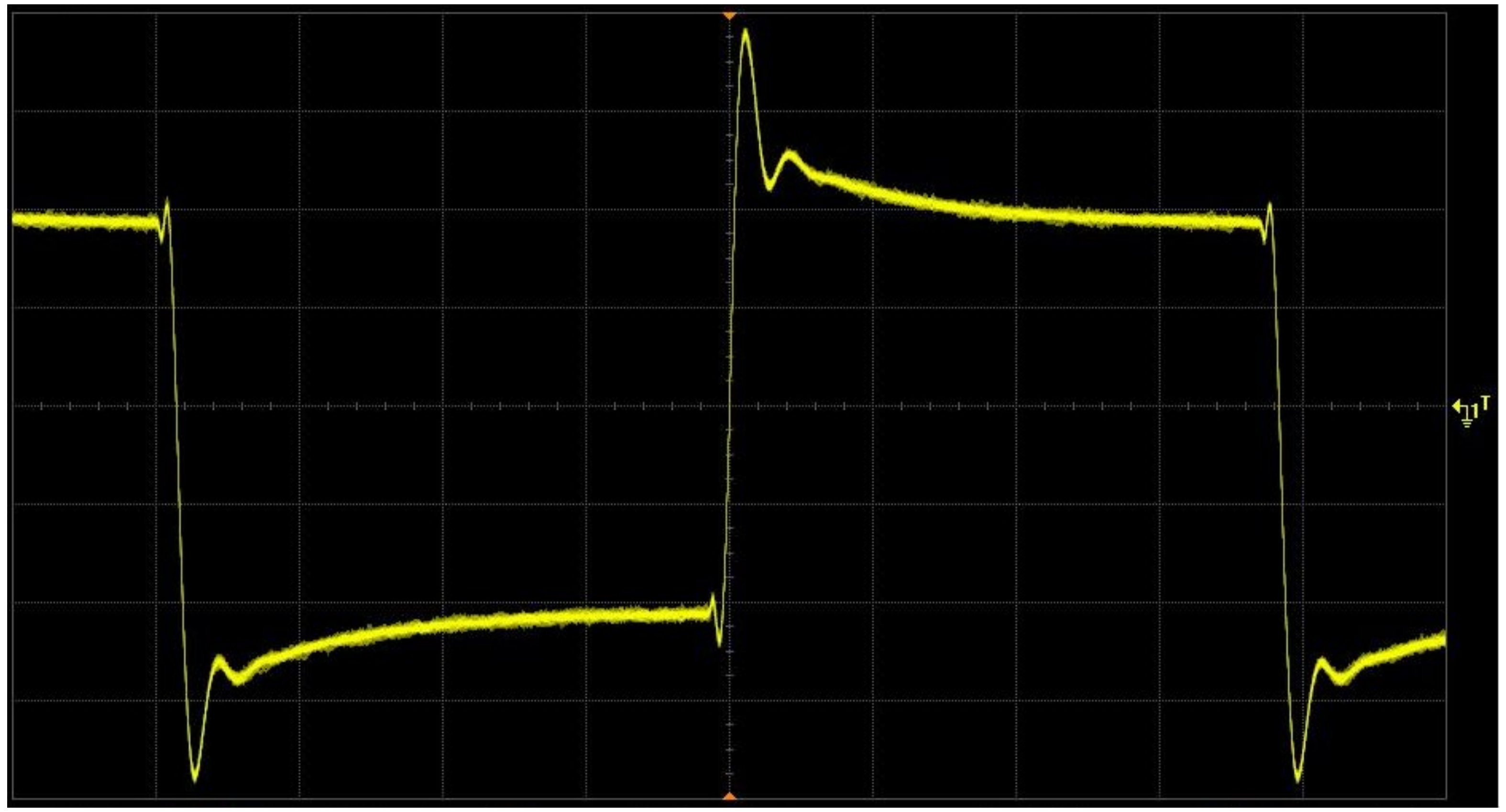

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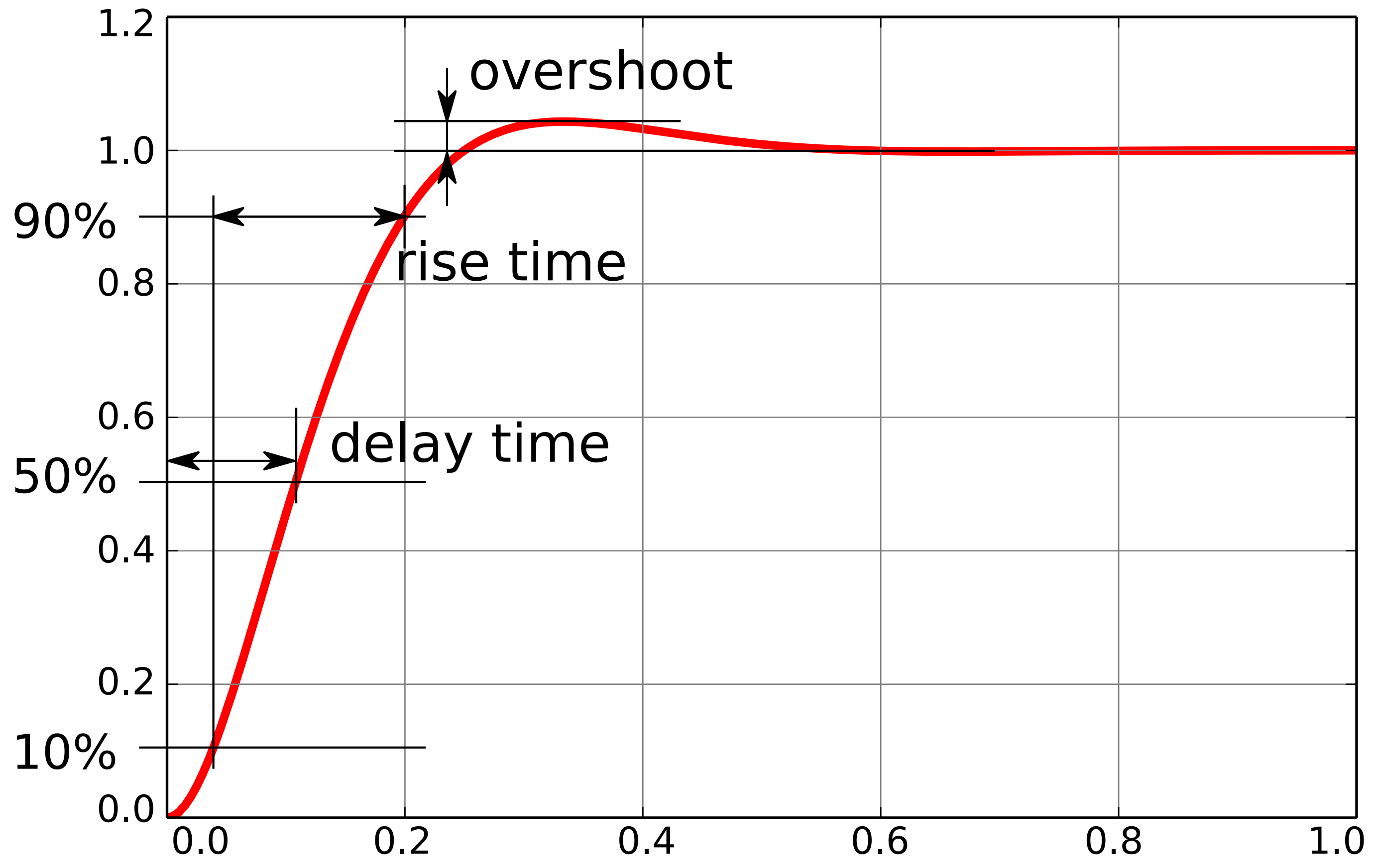

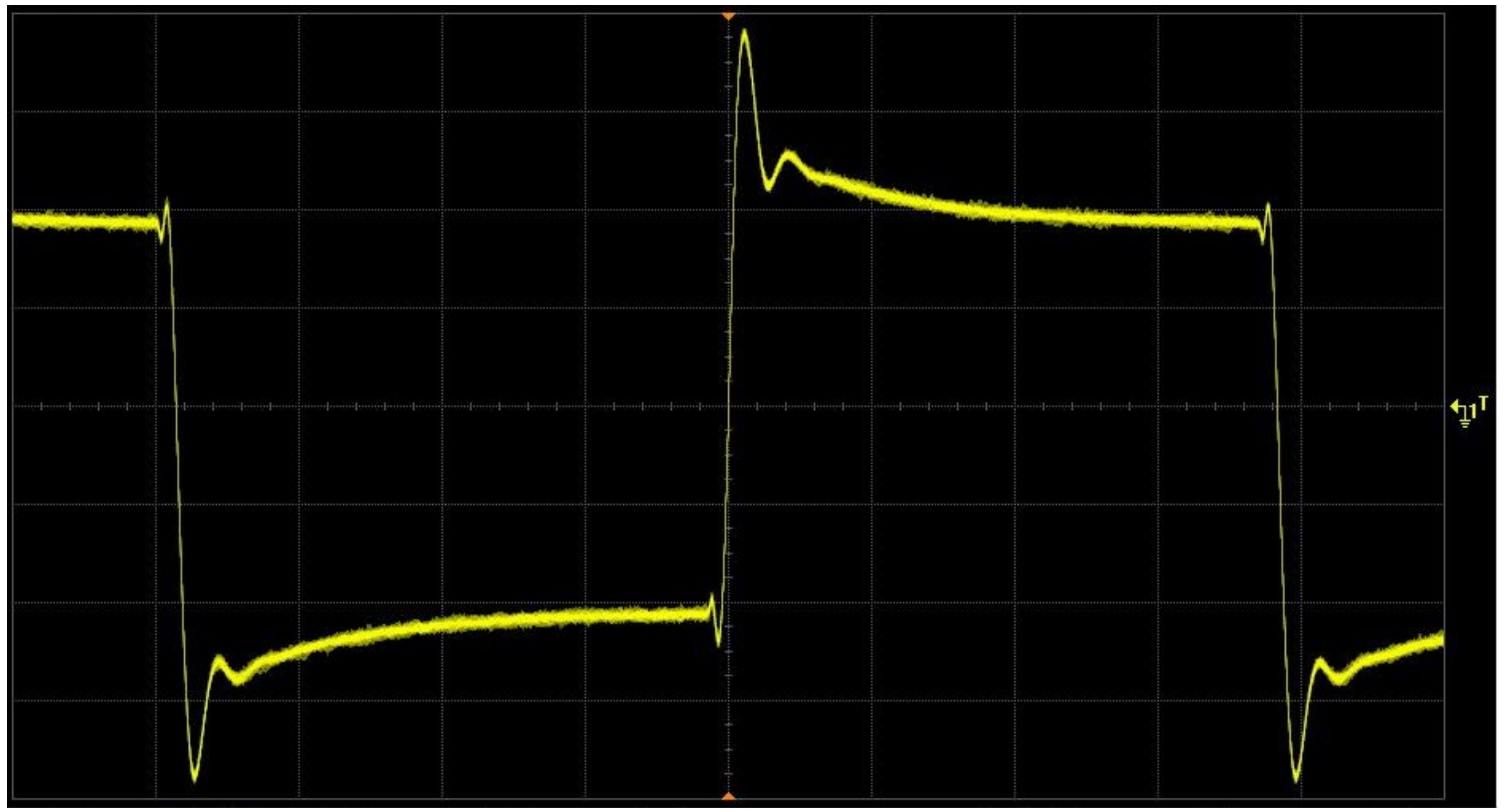

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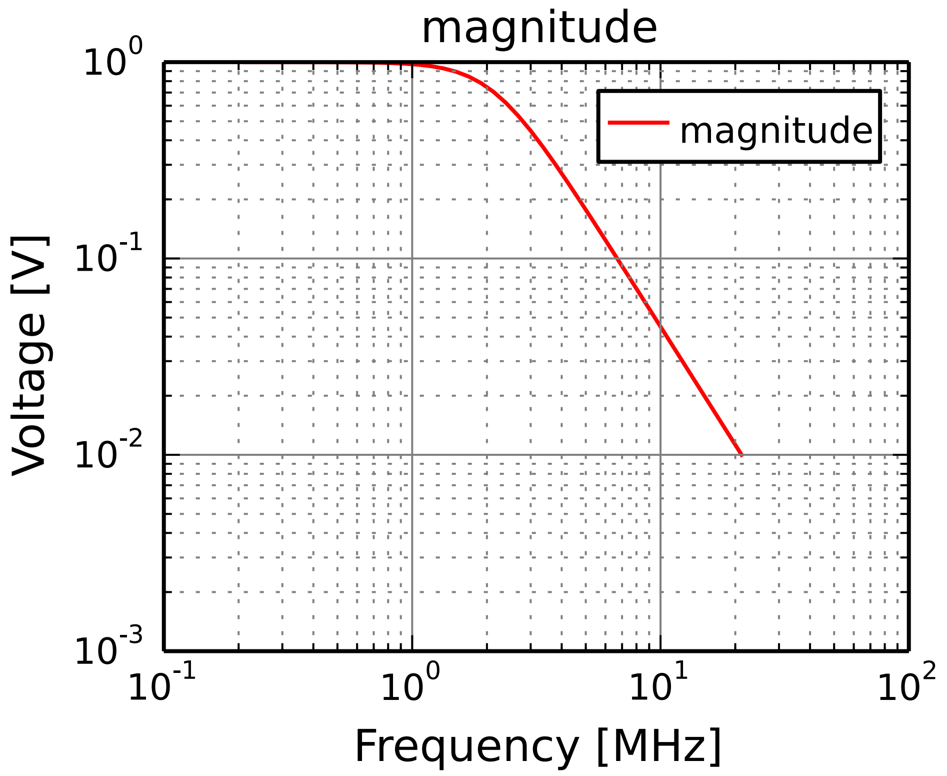

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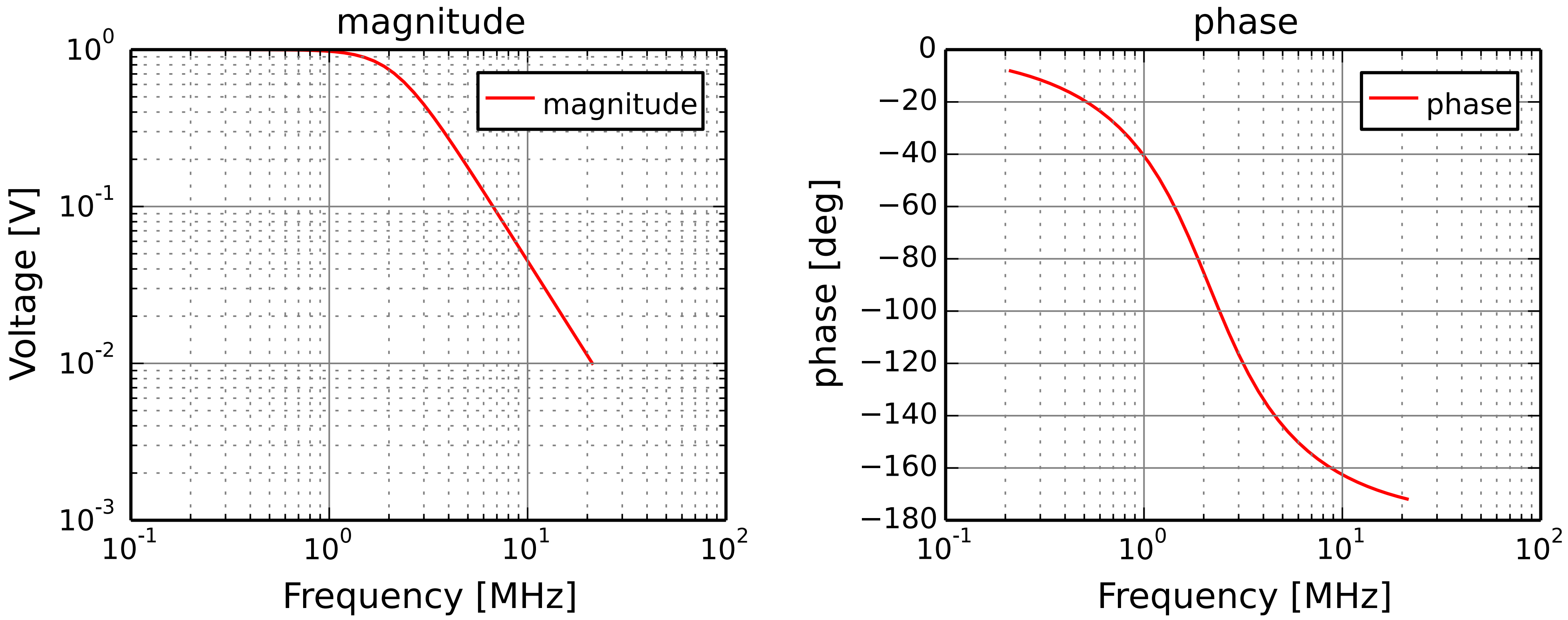

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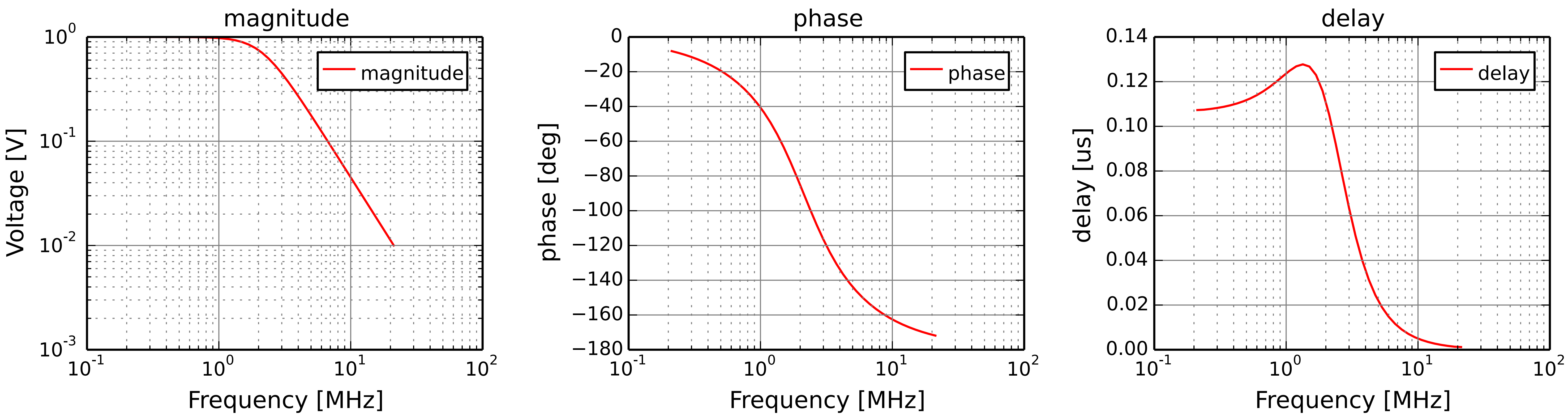

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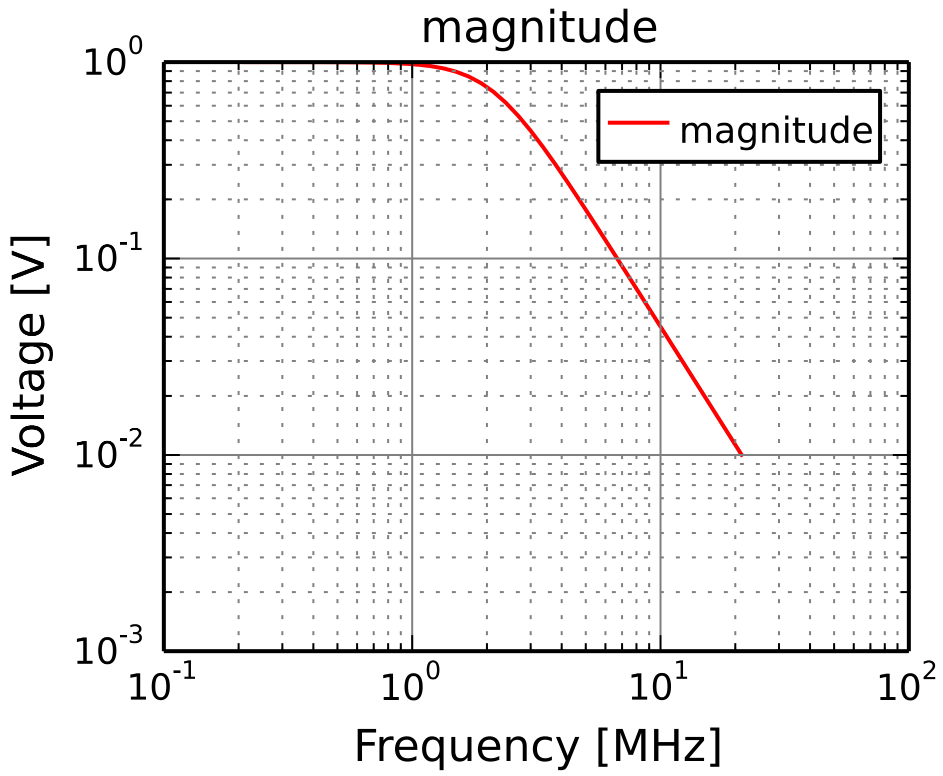

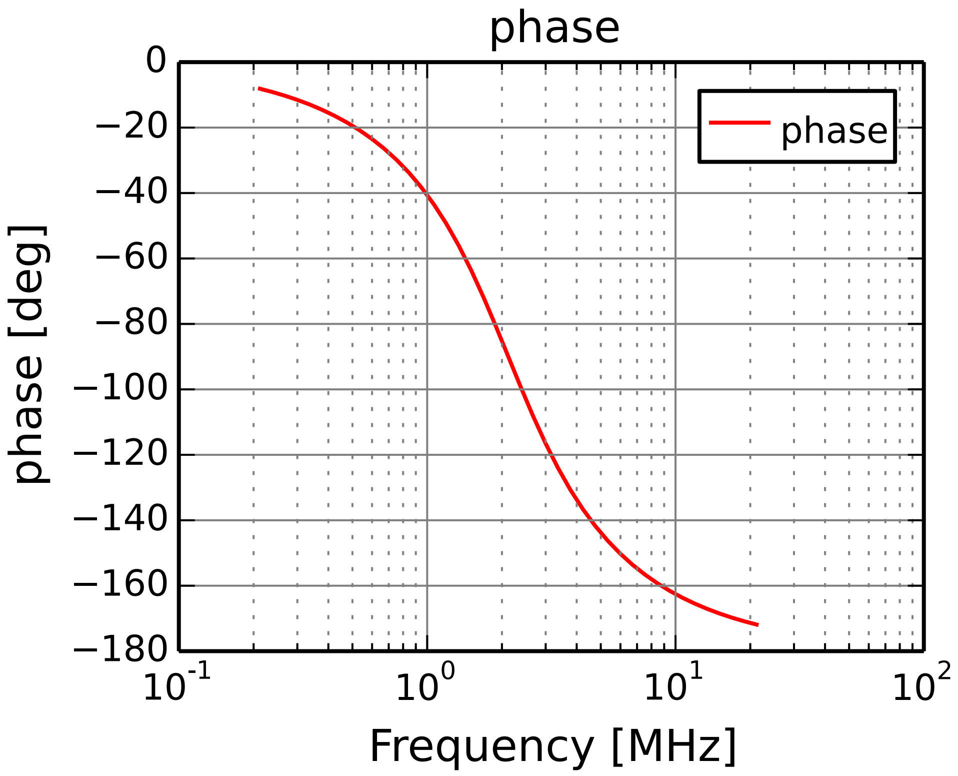

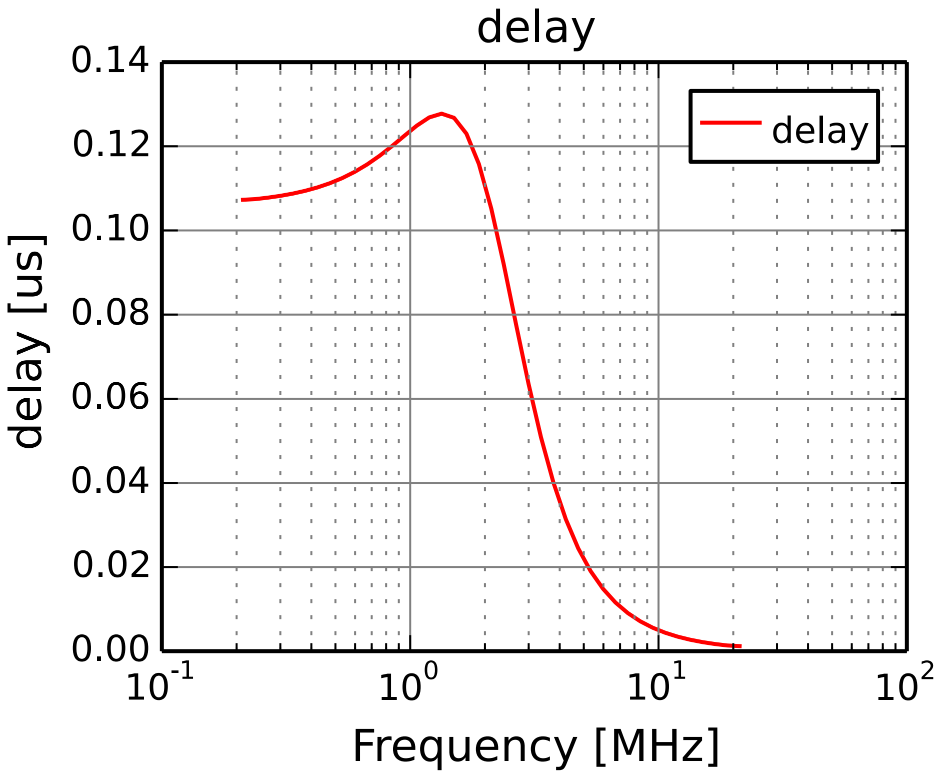

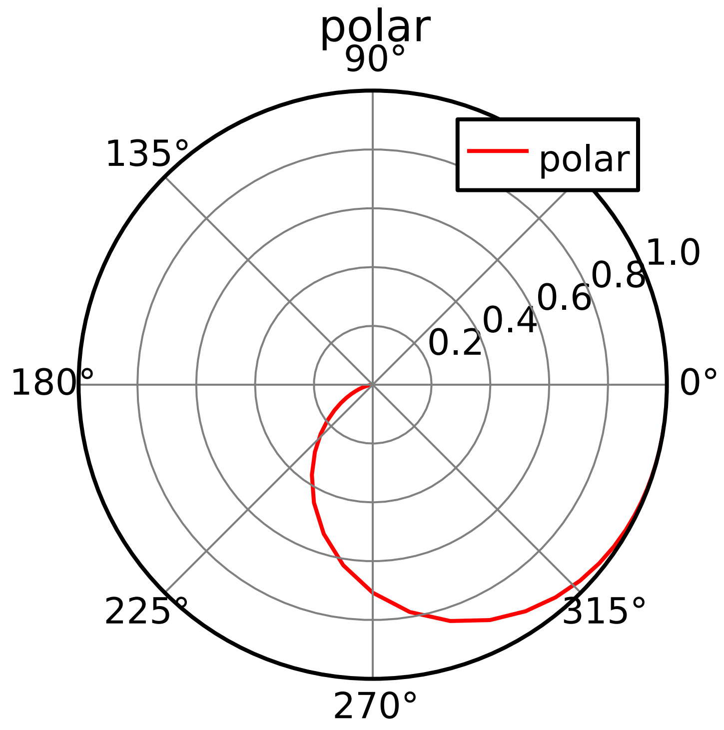

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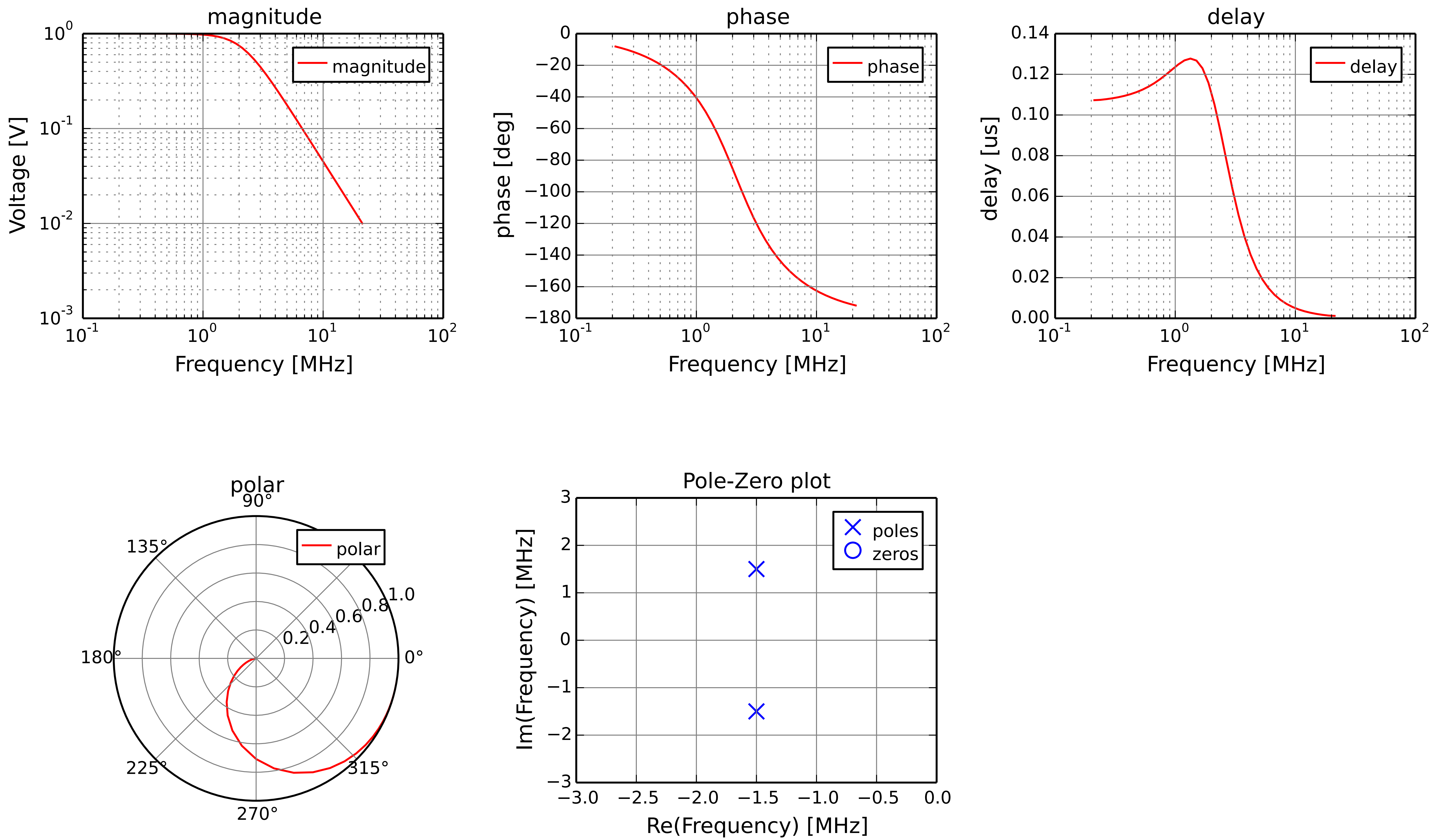

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