Structured Electronic Design

Butterworth
Maximally Flat Magnitude
Frequency Responses

Anton J.M. Montagne

All-pole Butterworth (MFM) reponses

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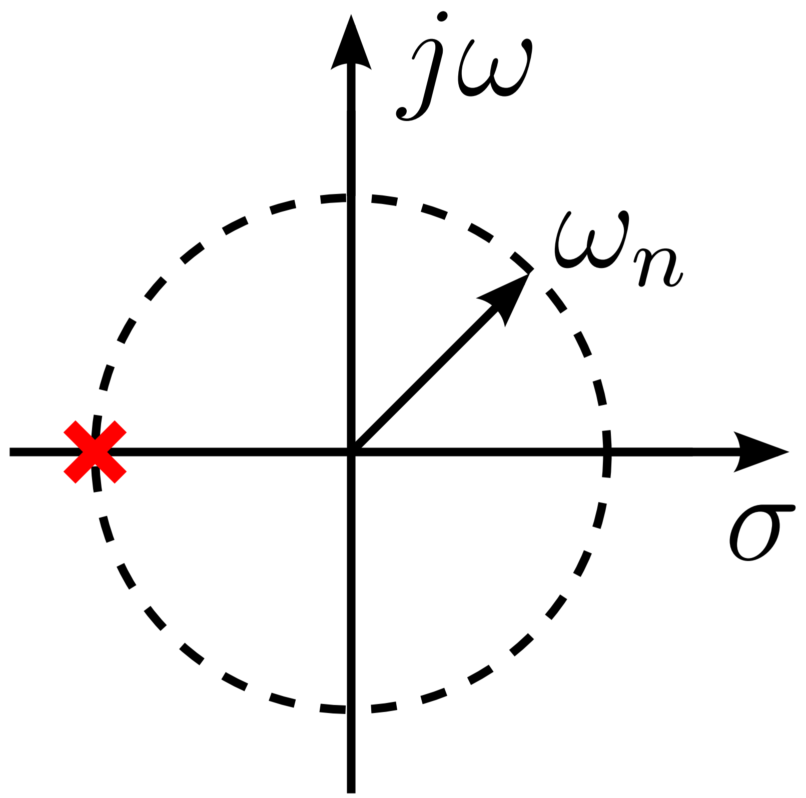

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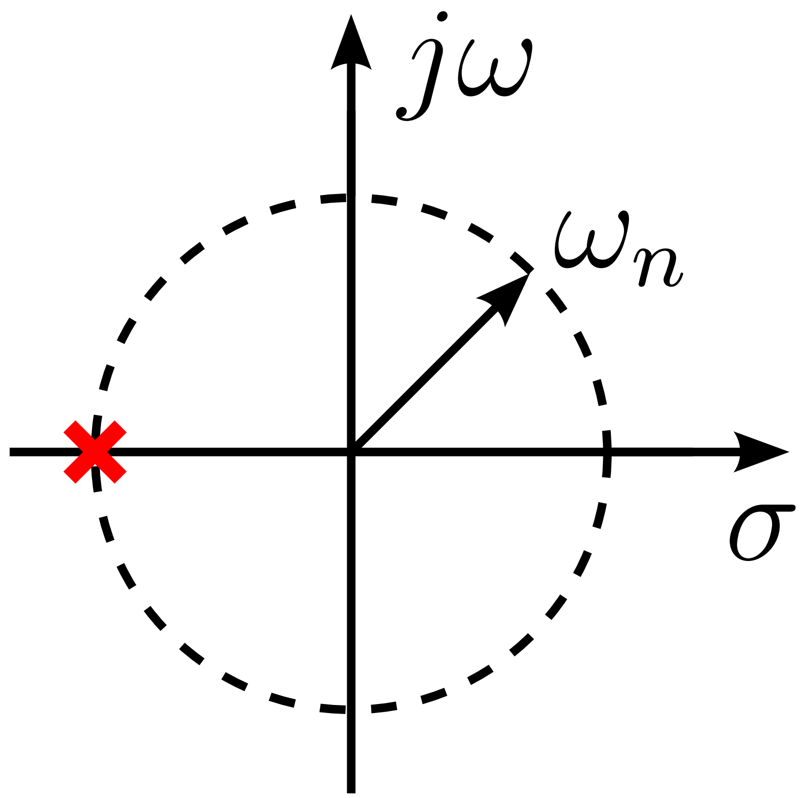

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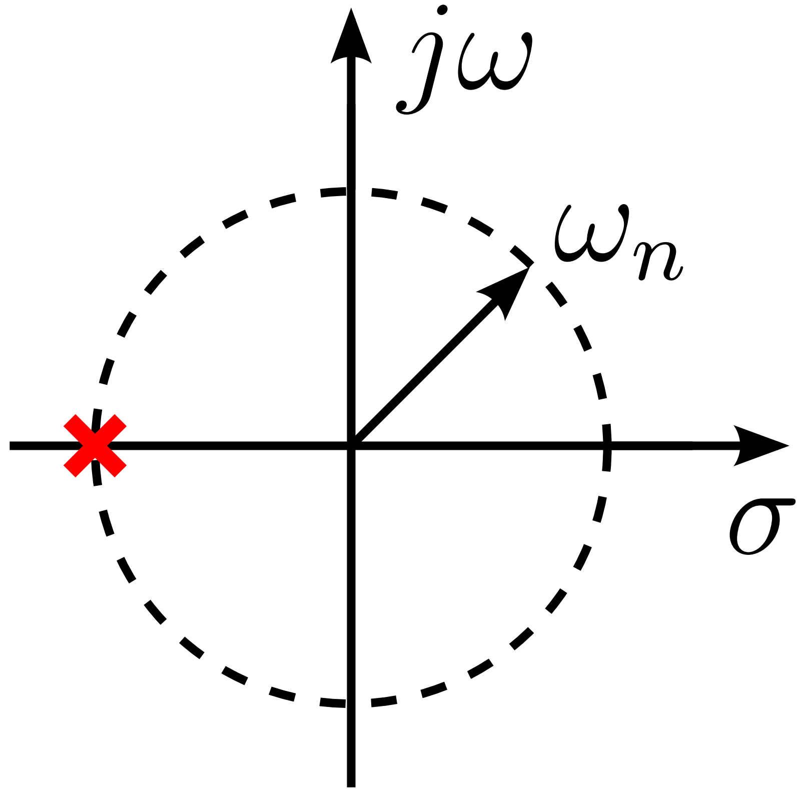

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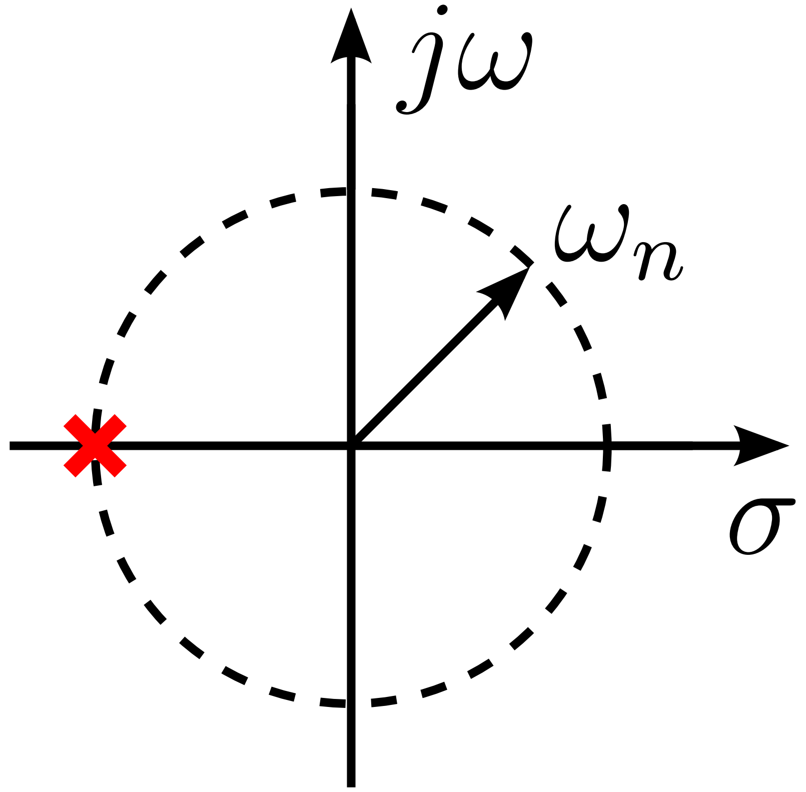

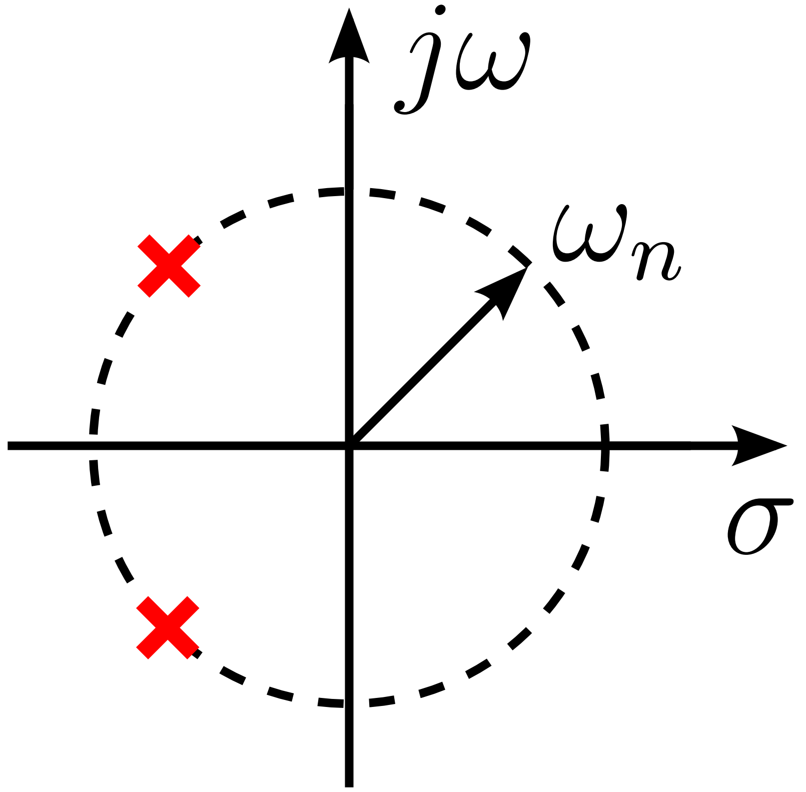

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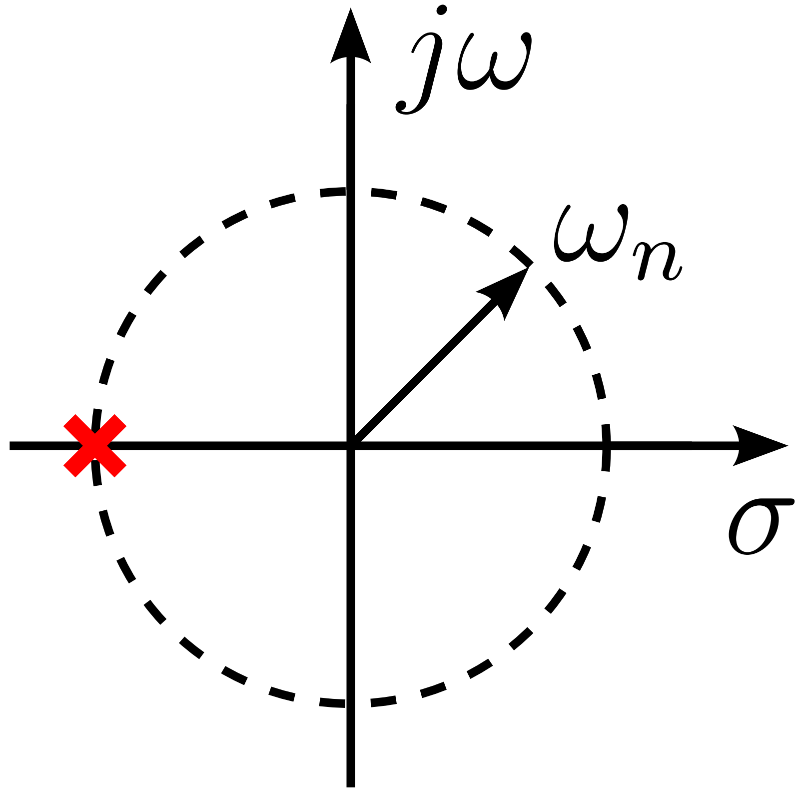

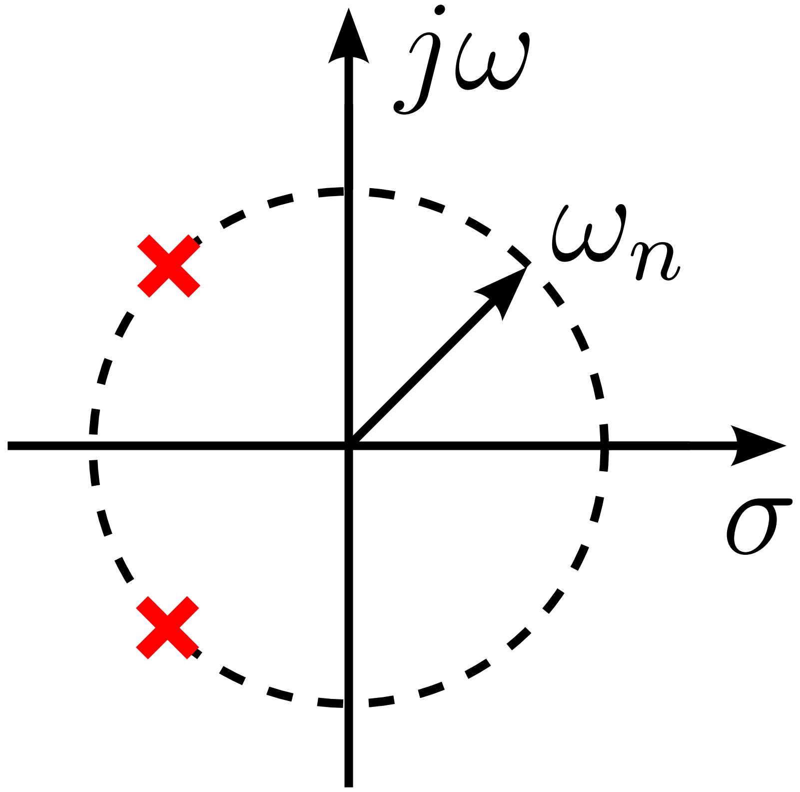

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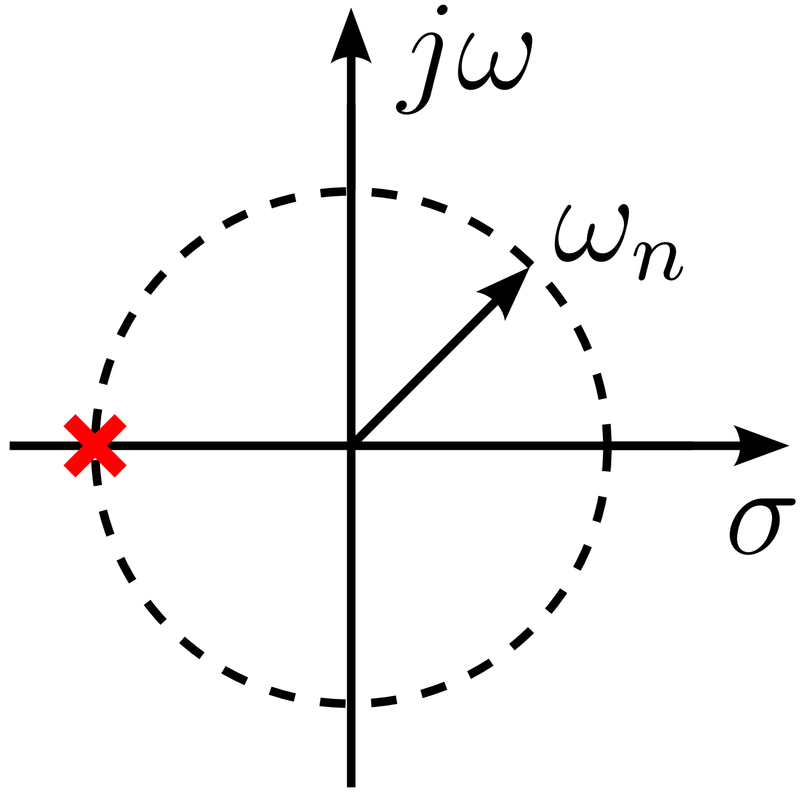

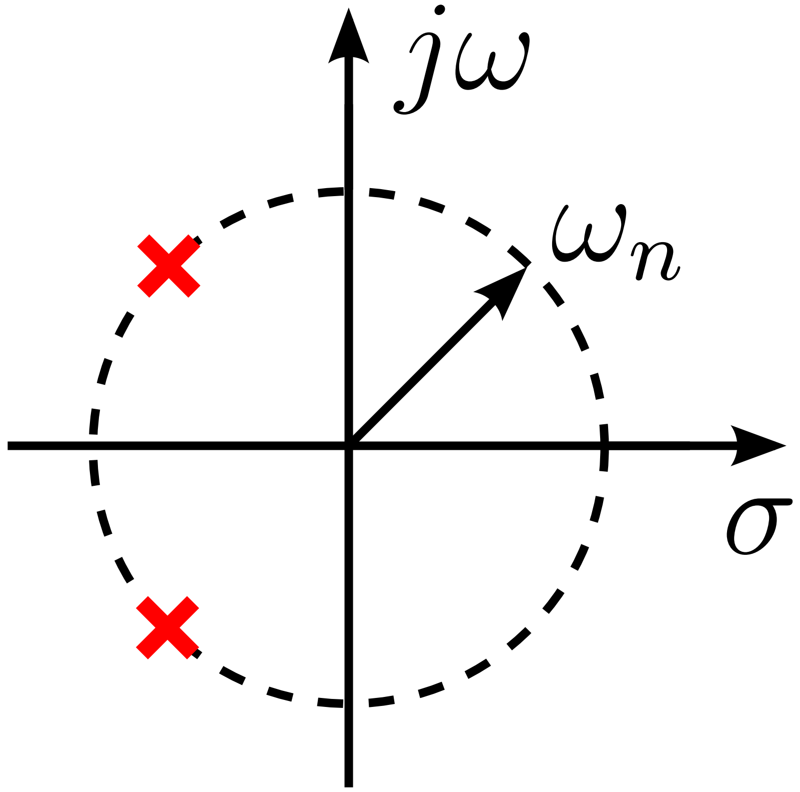

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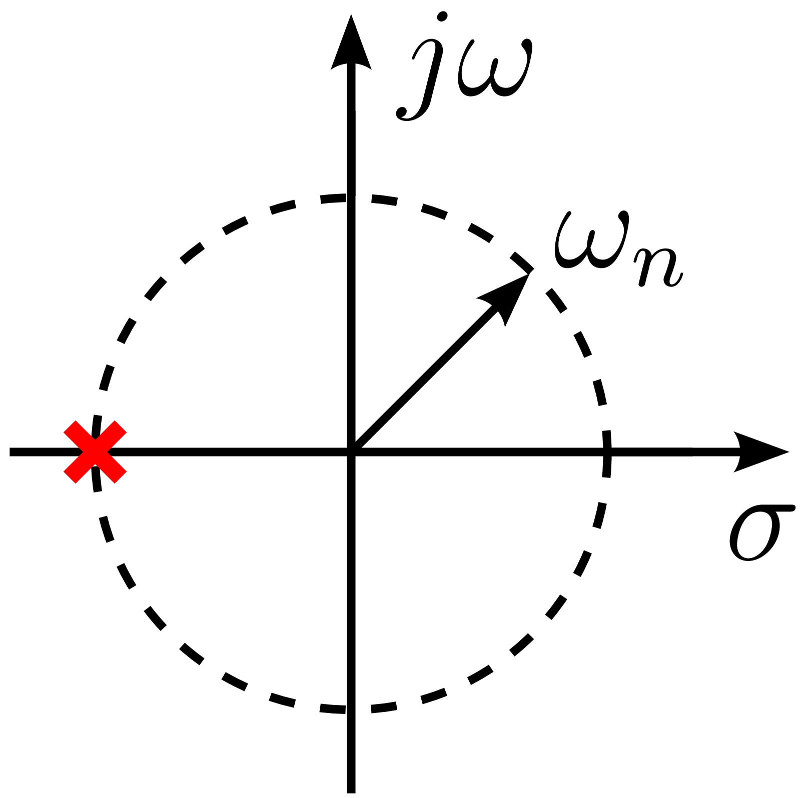

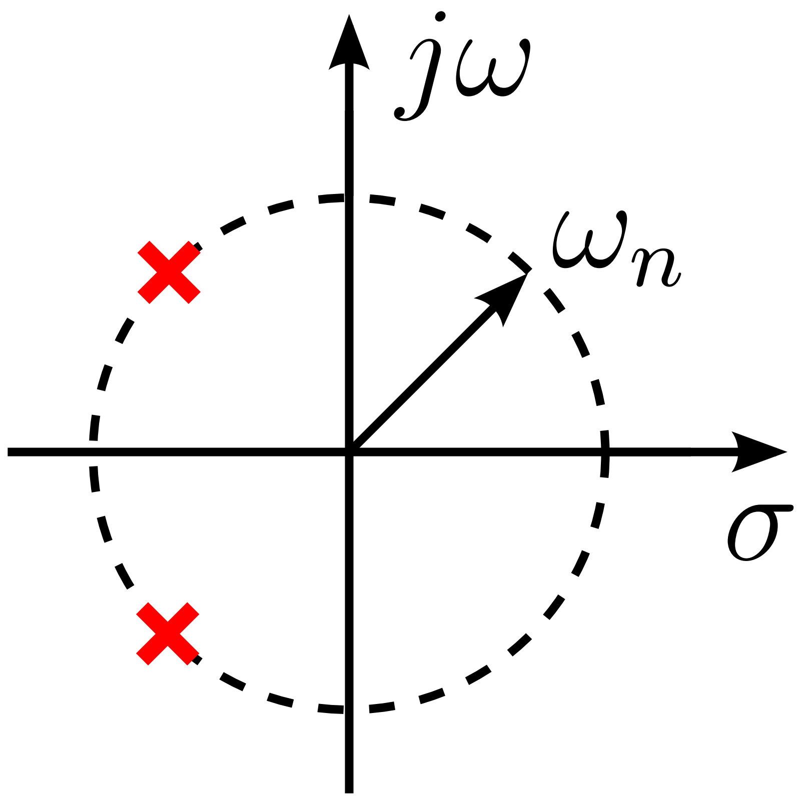

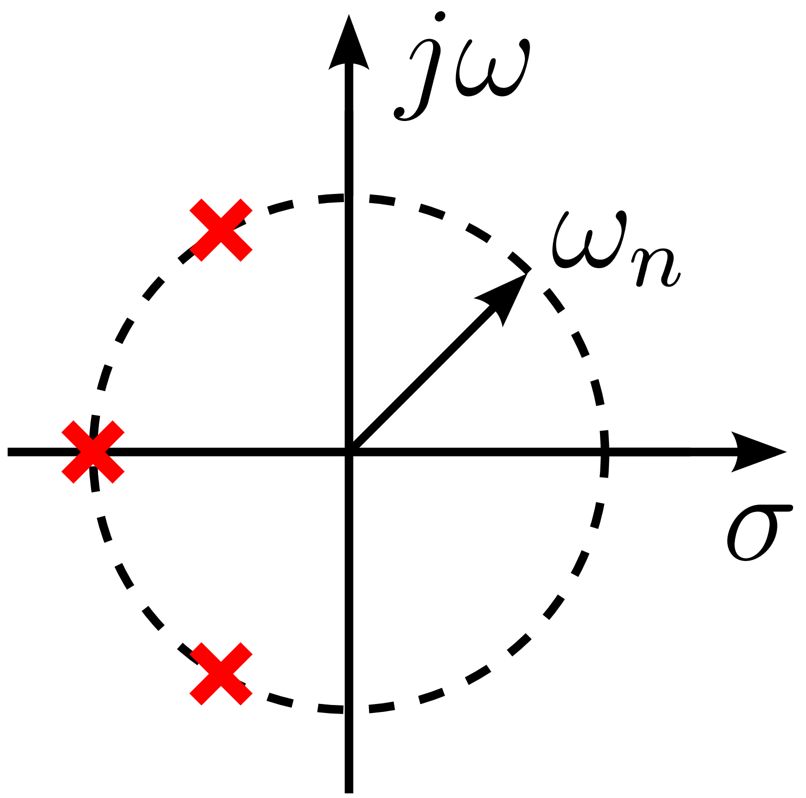

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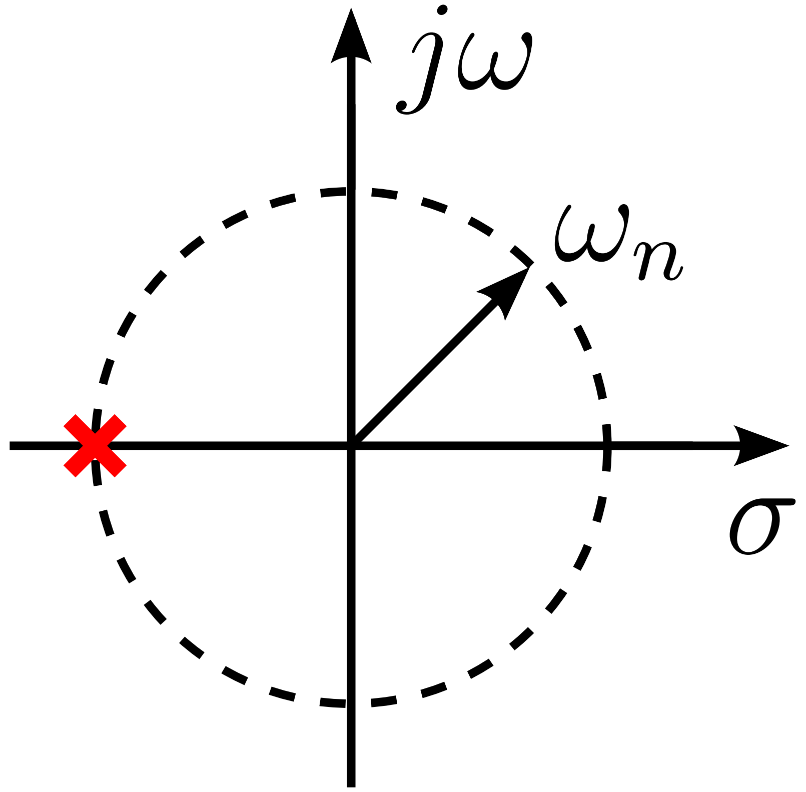

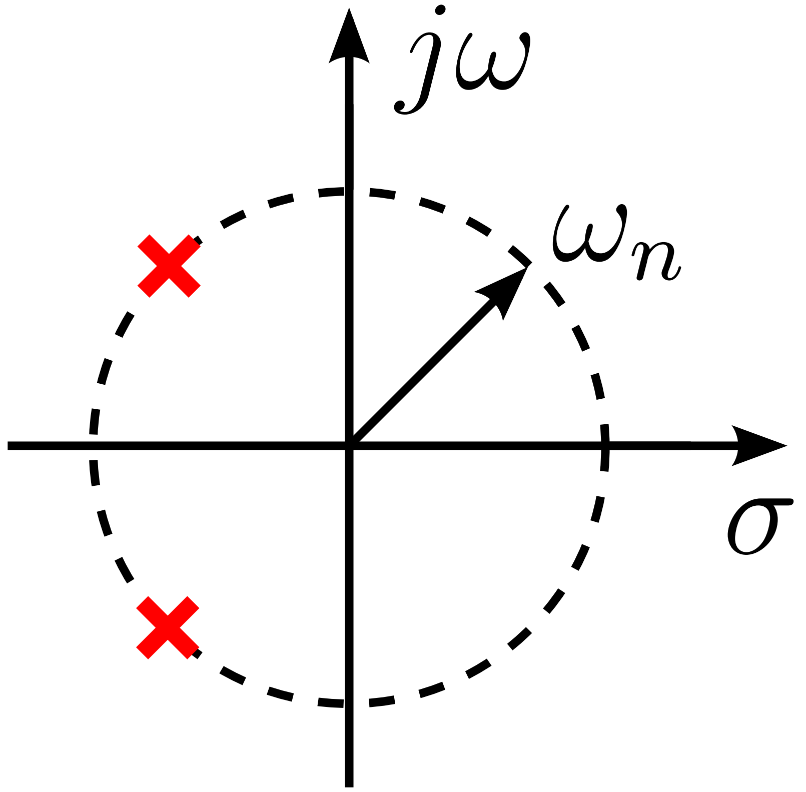

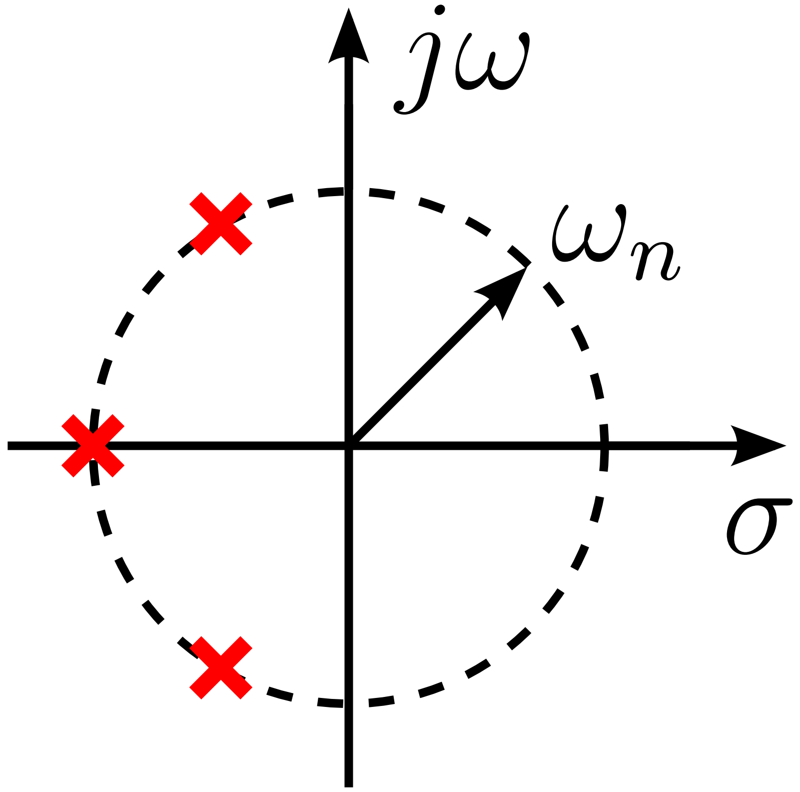

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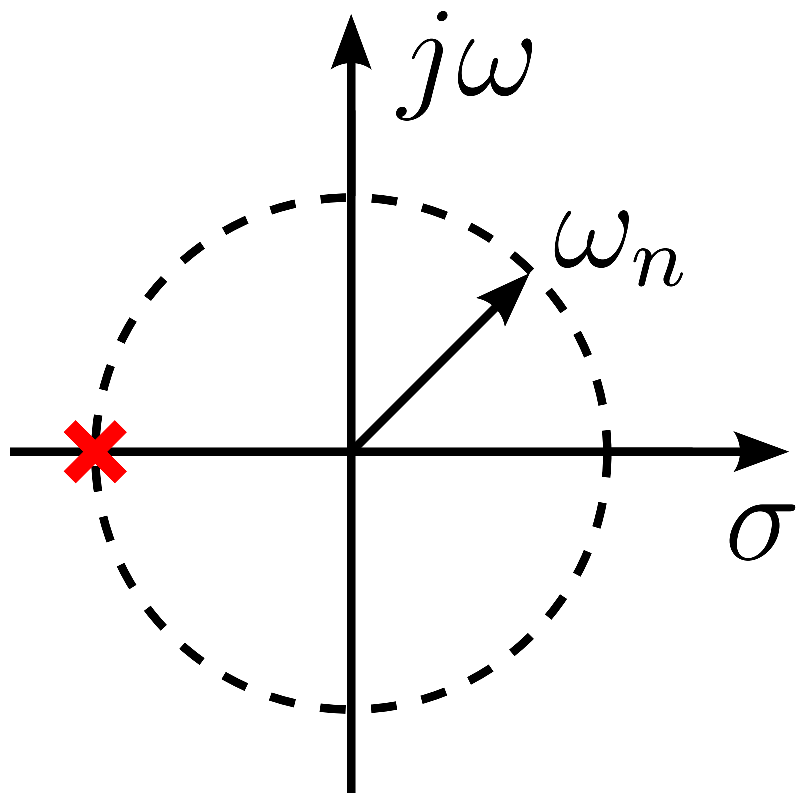

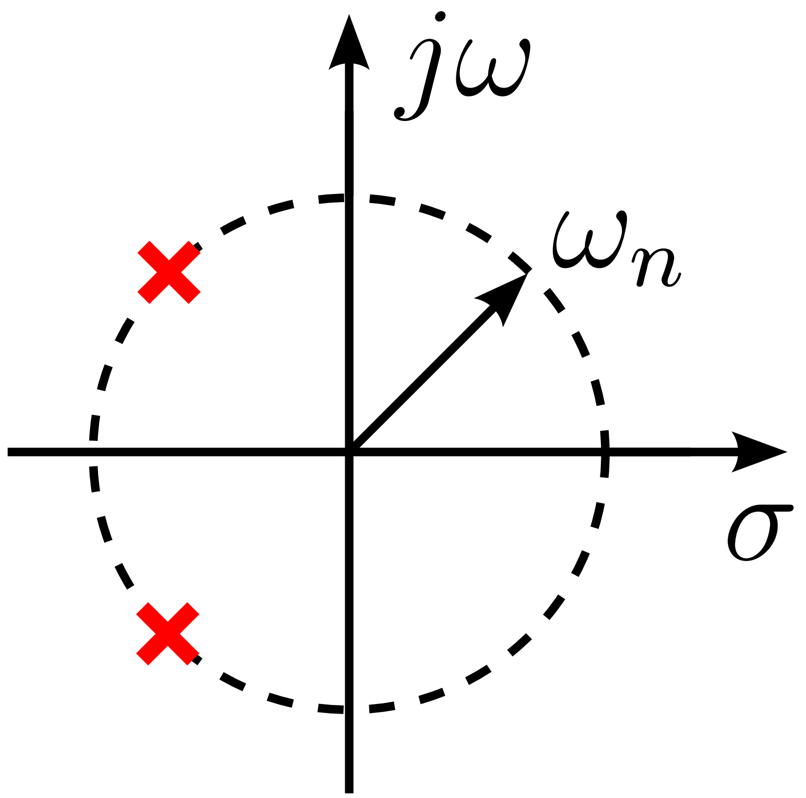

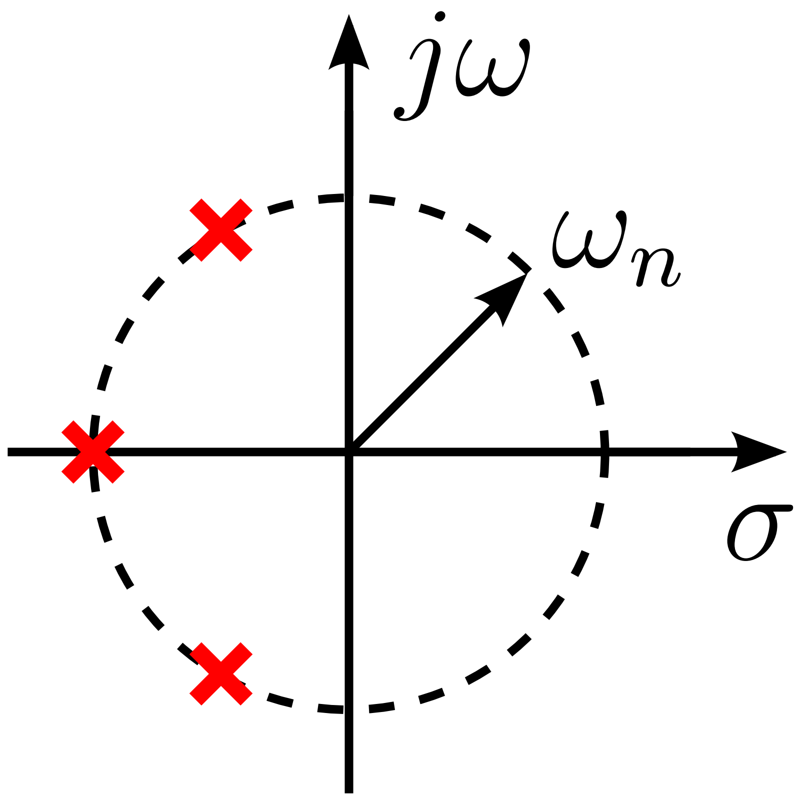

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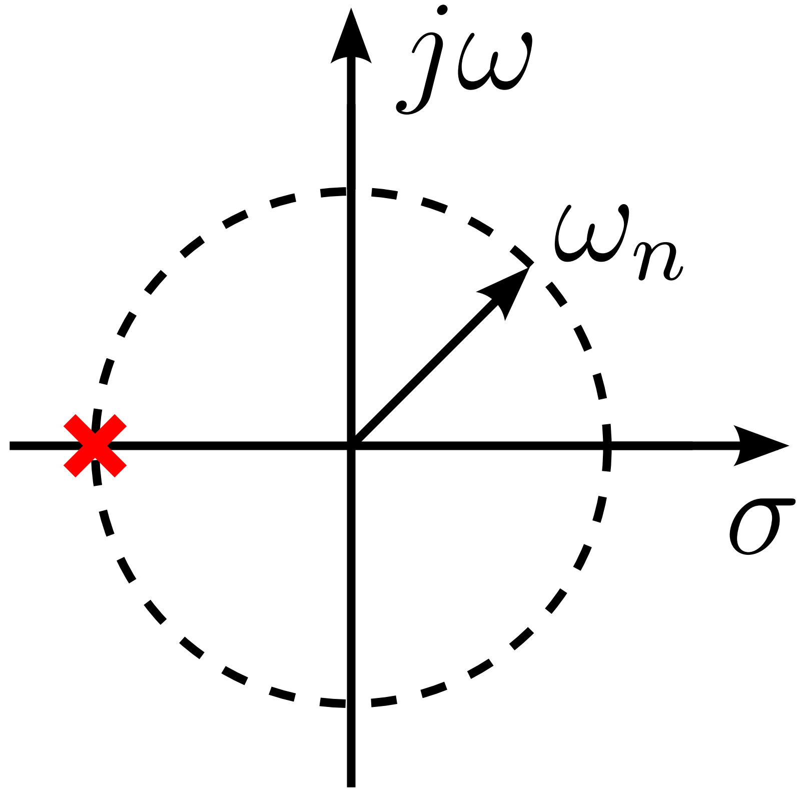

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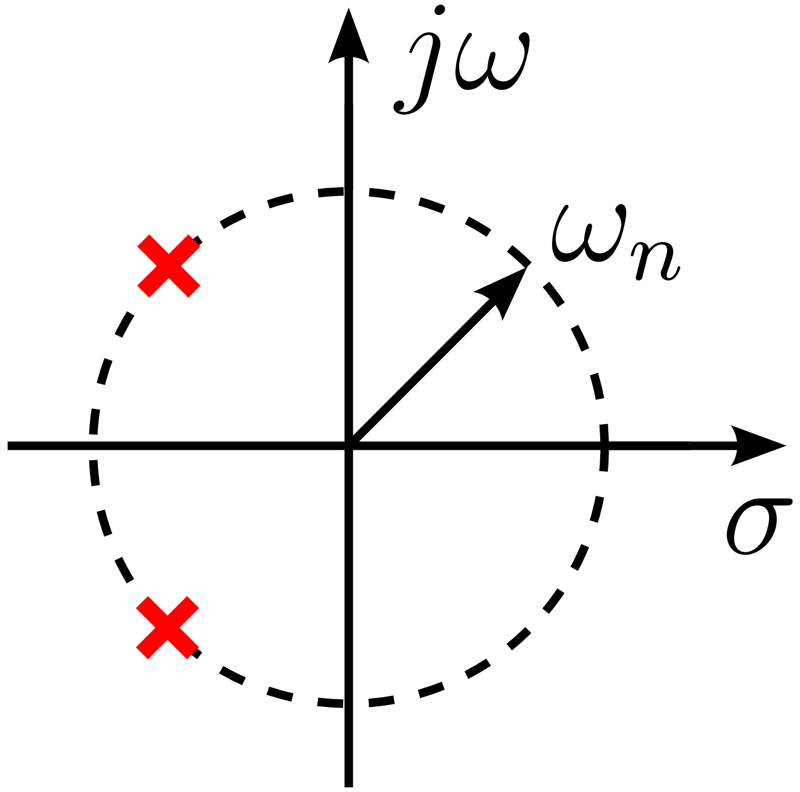

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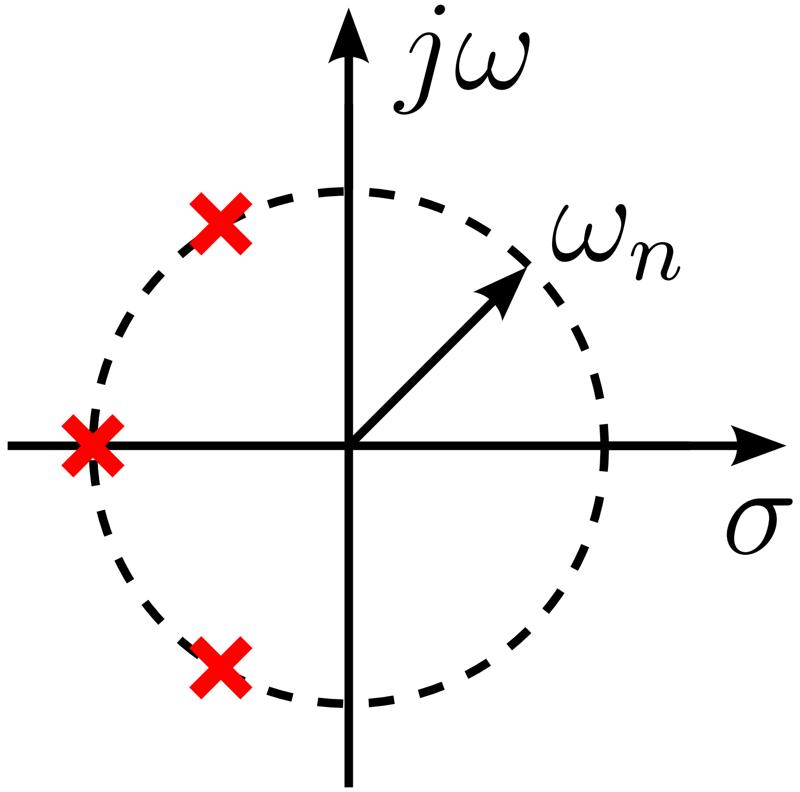

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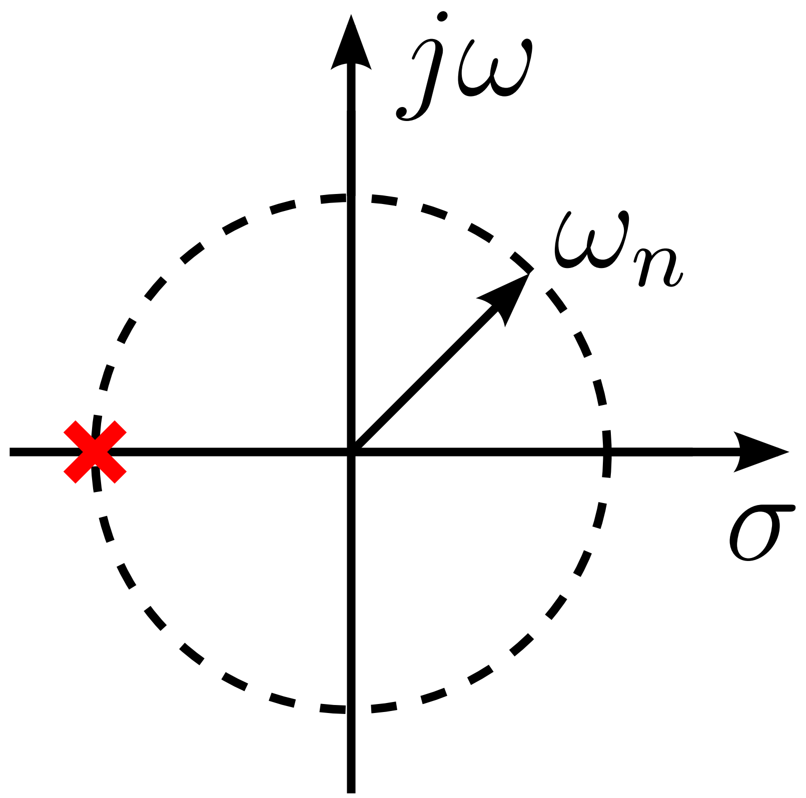

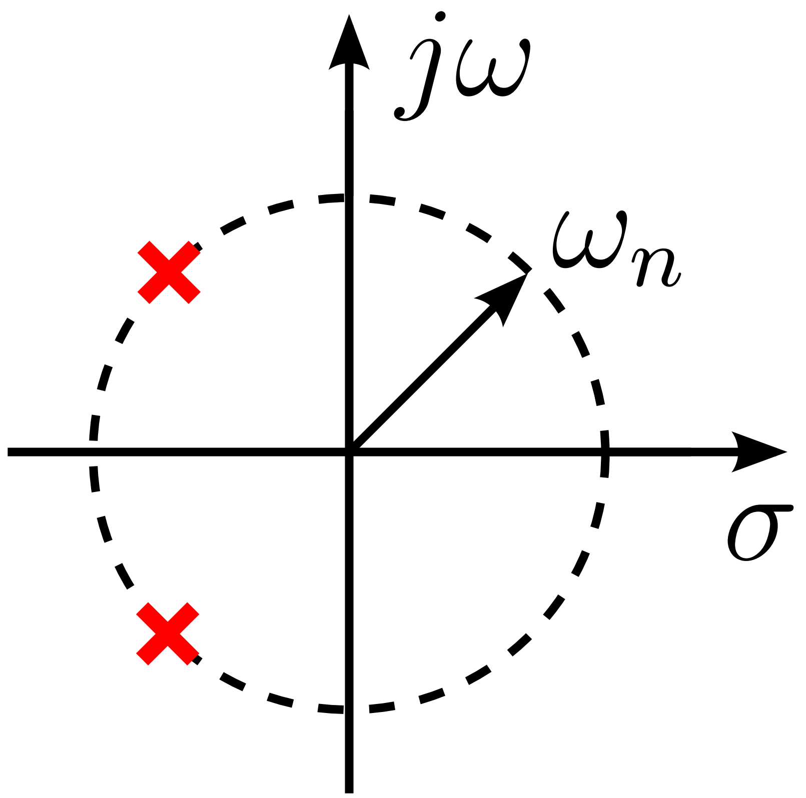

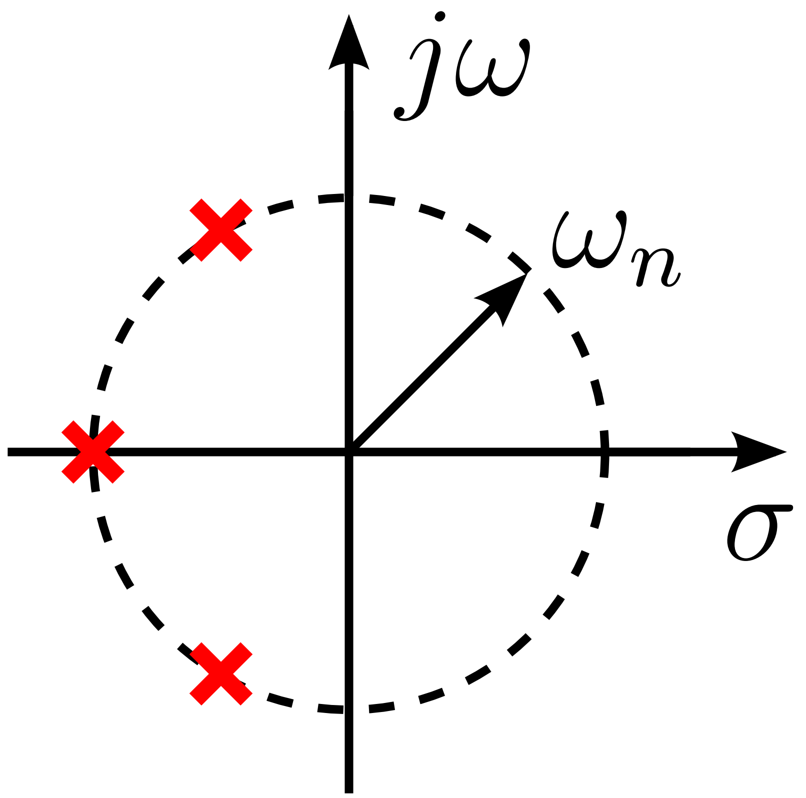

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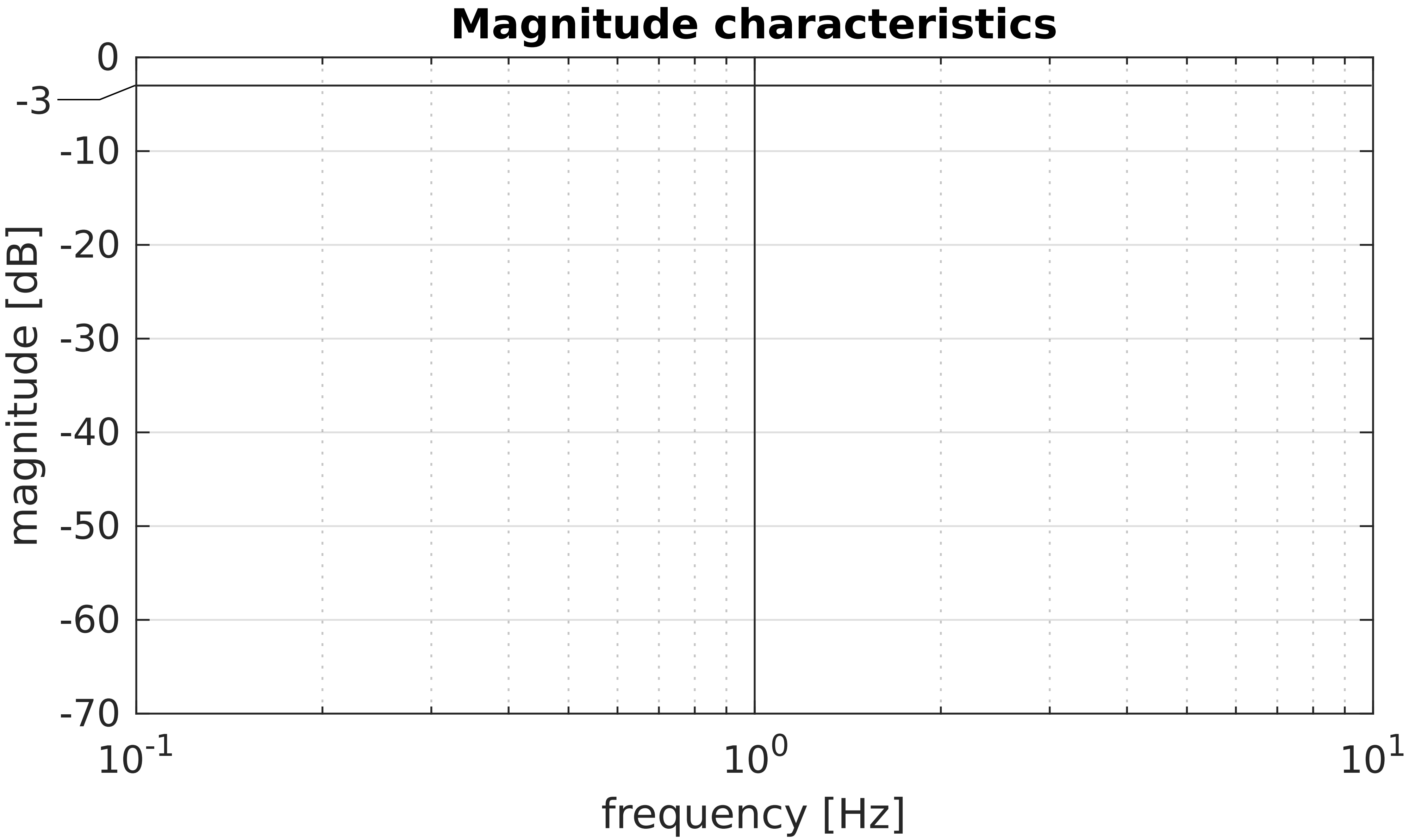

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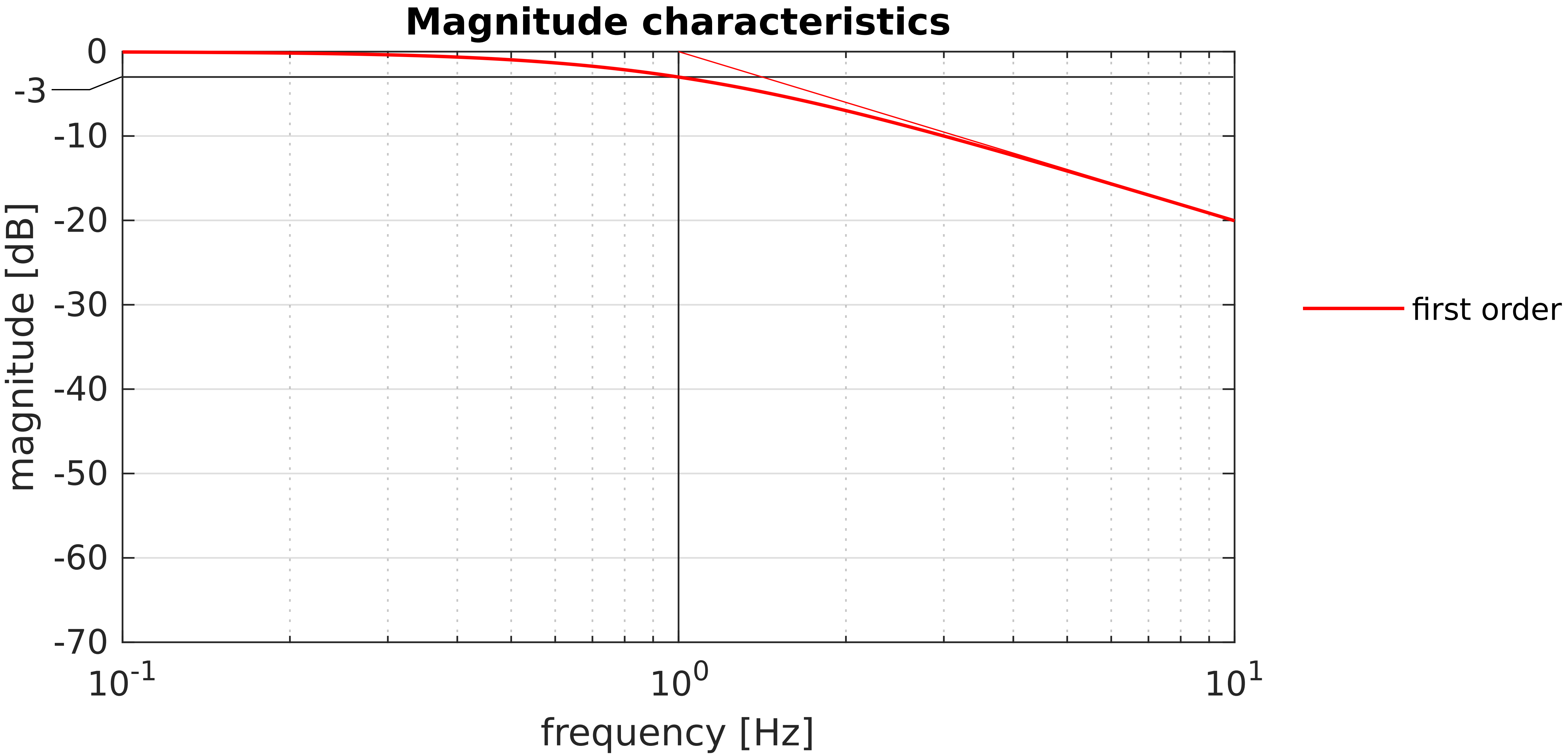

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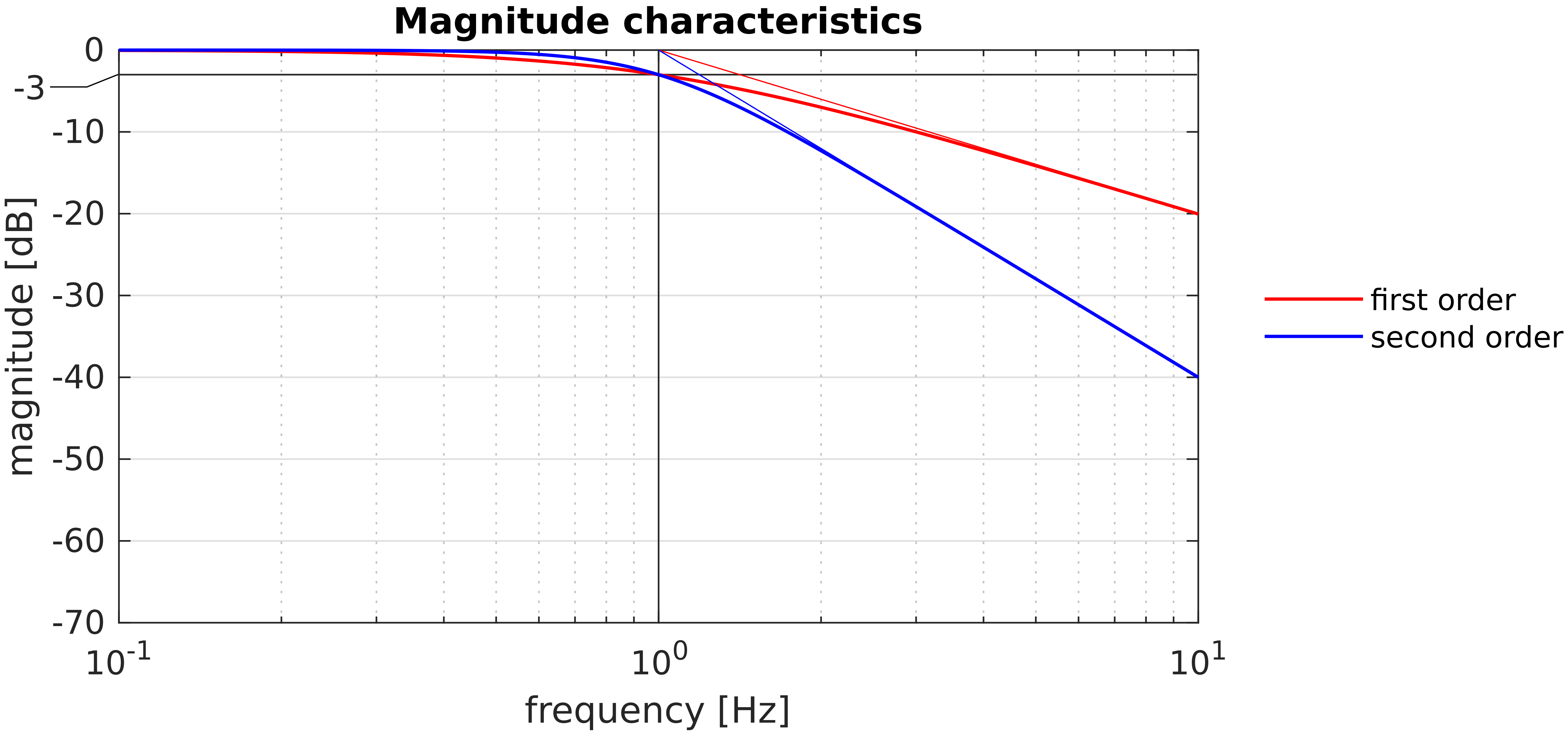

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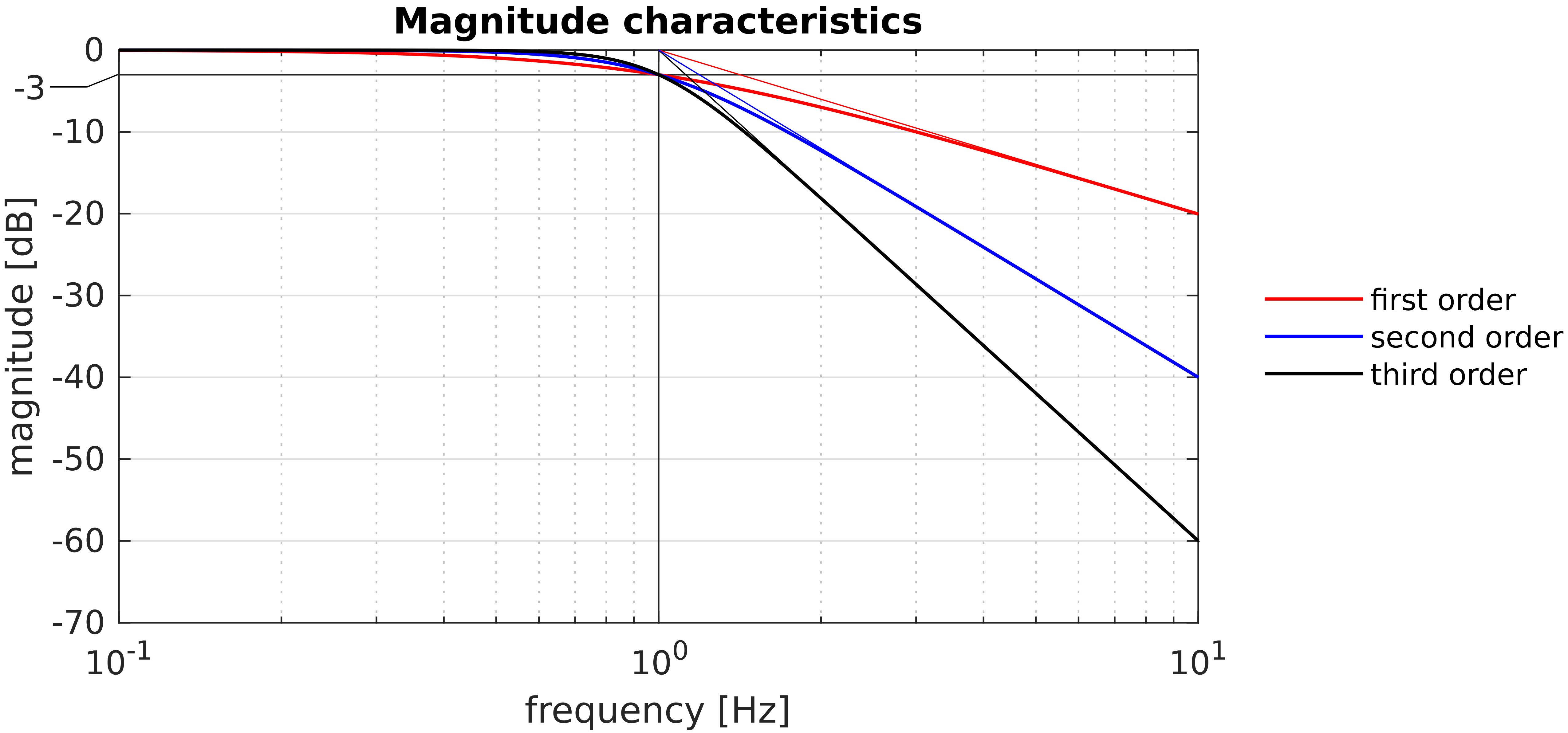

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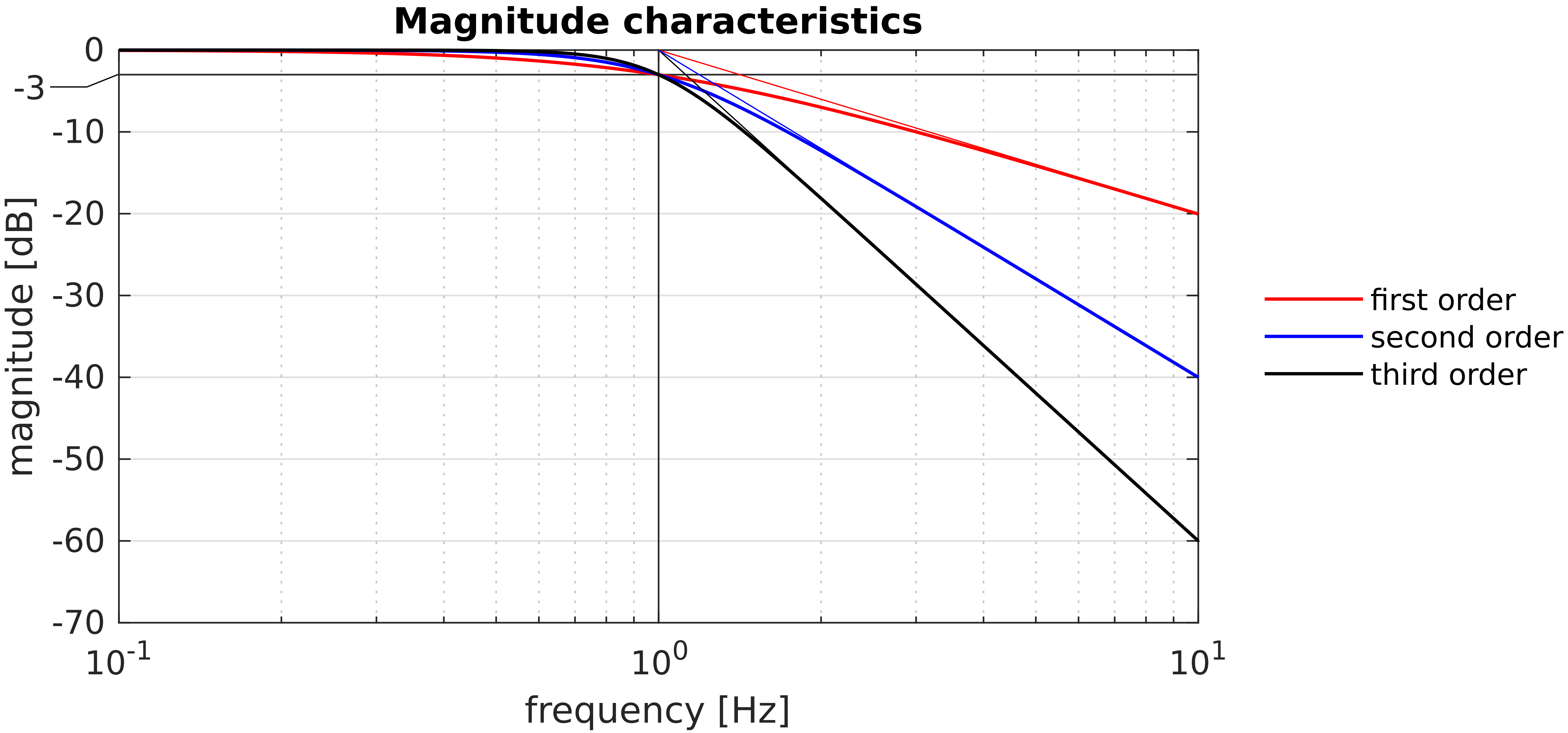

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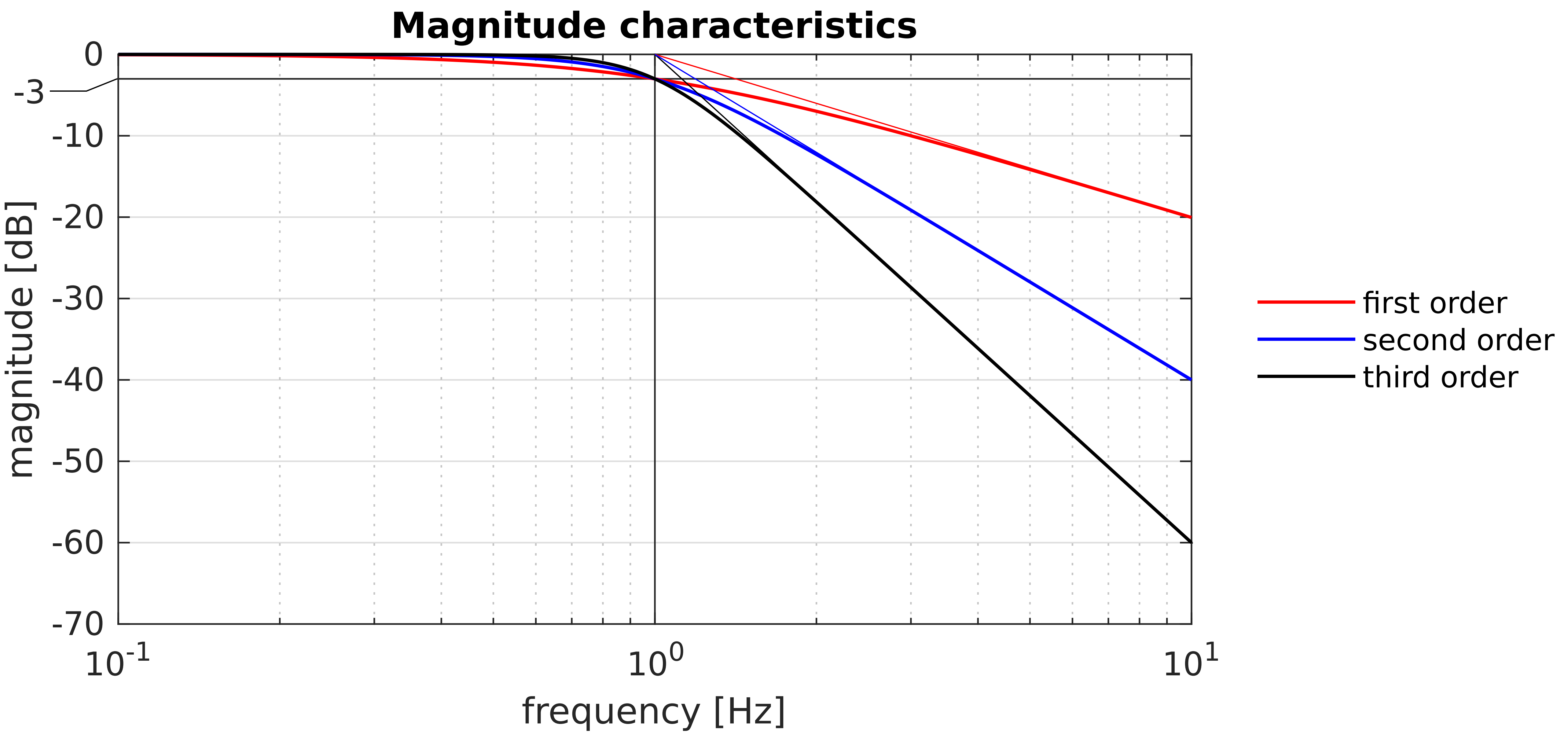

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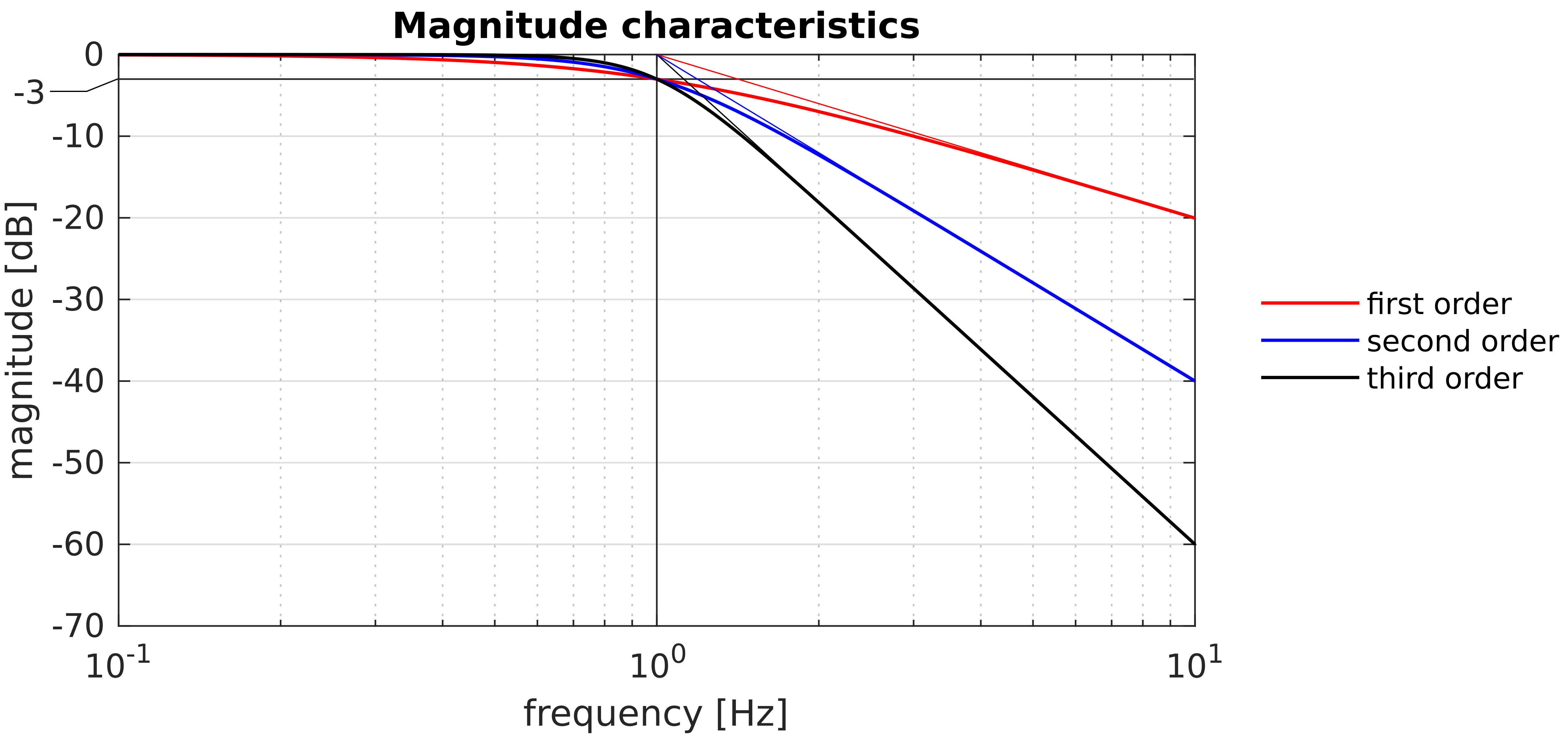

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