Structured Electronic Design

Frequency stability of feedback amplifiers

Anton J.M. Montagne

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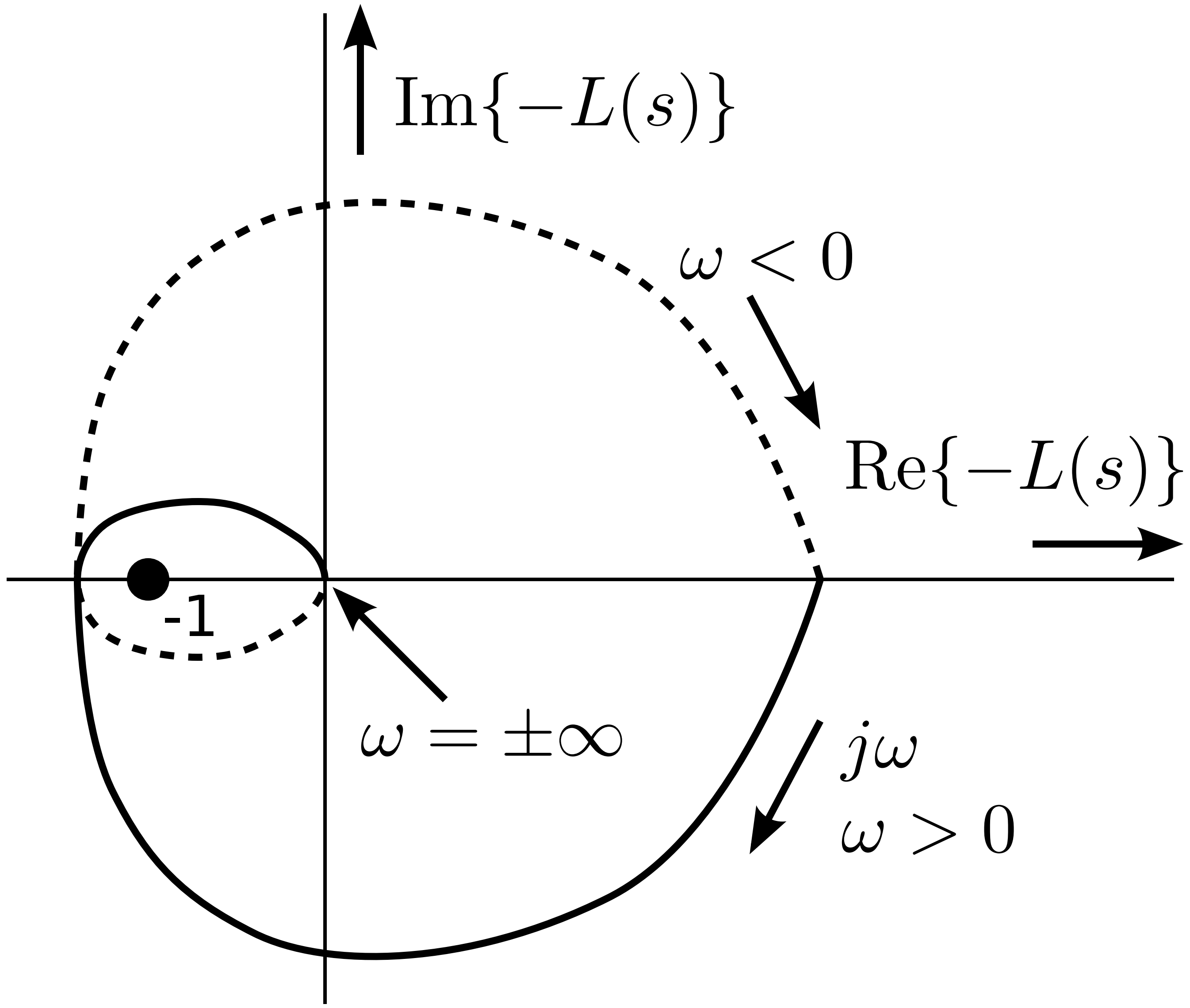

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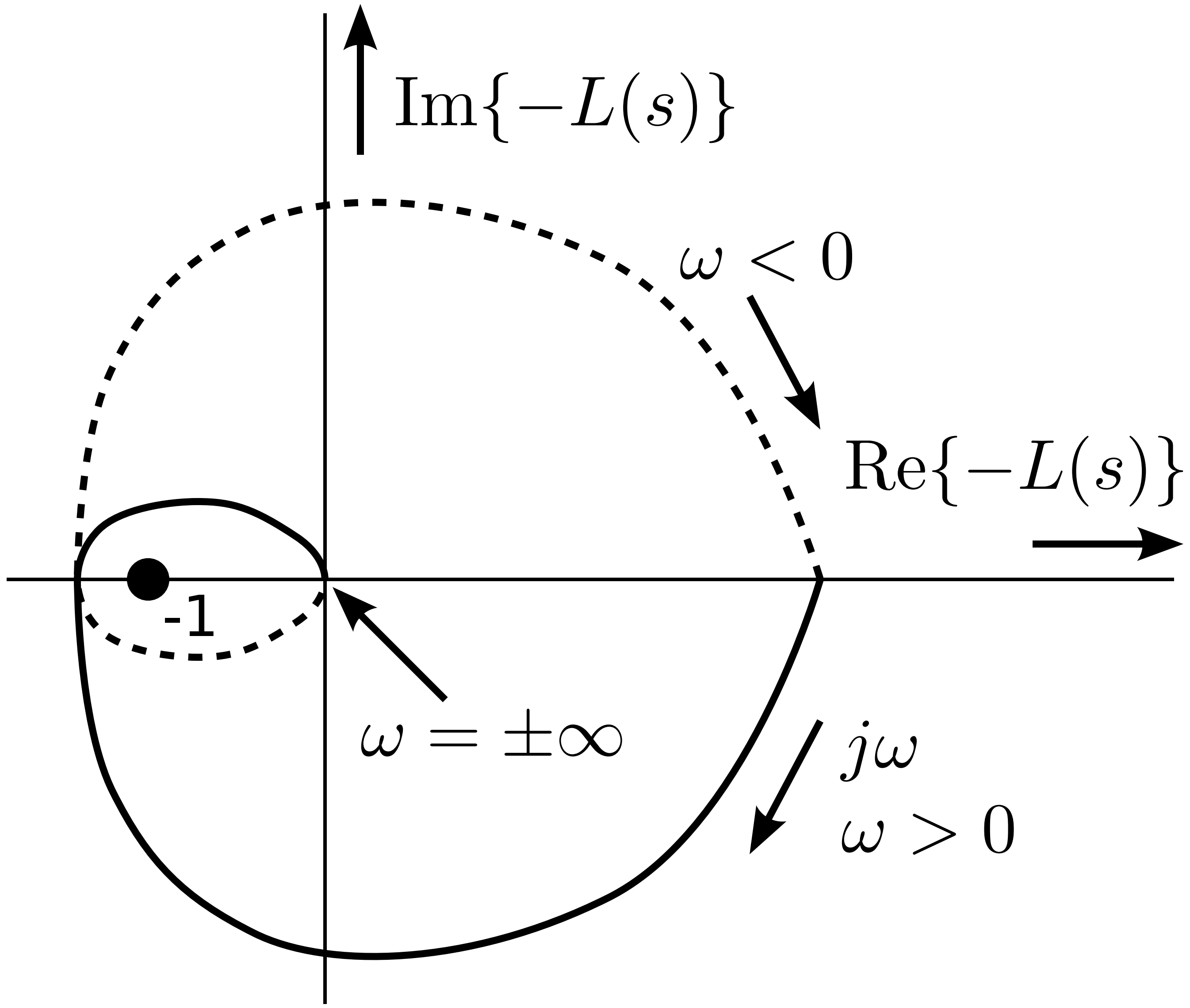

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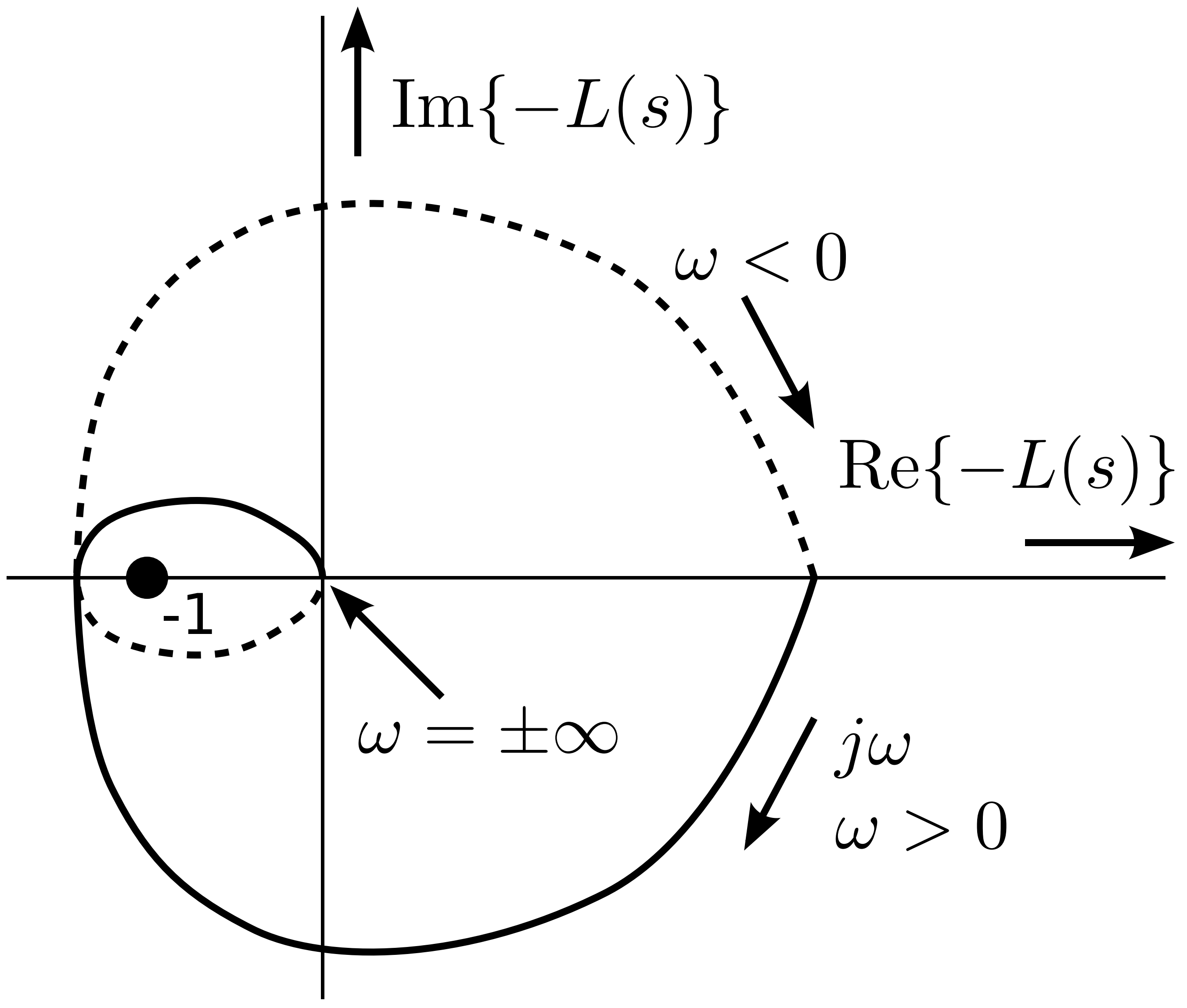

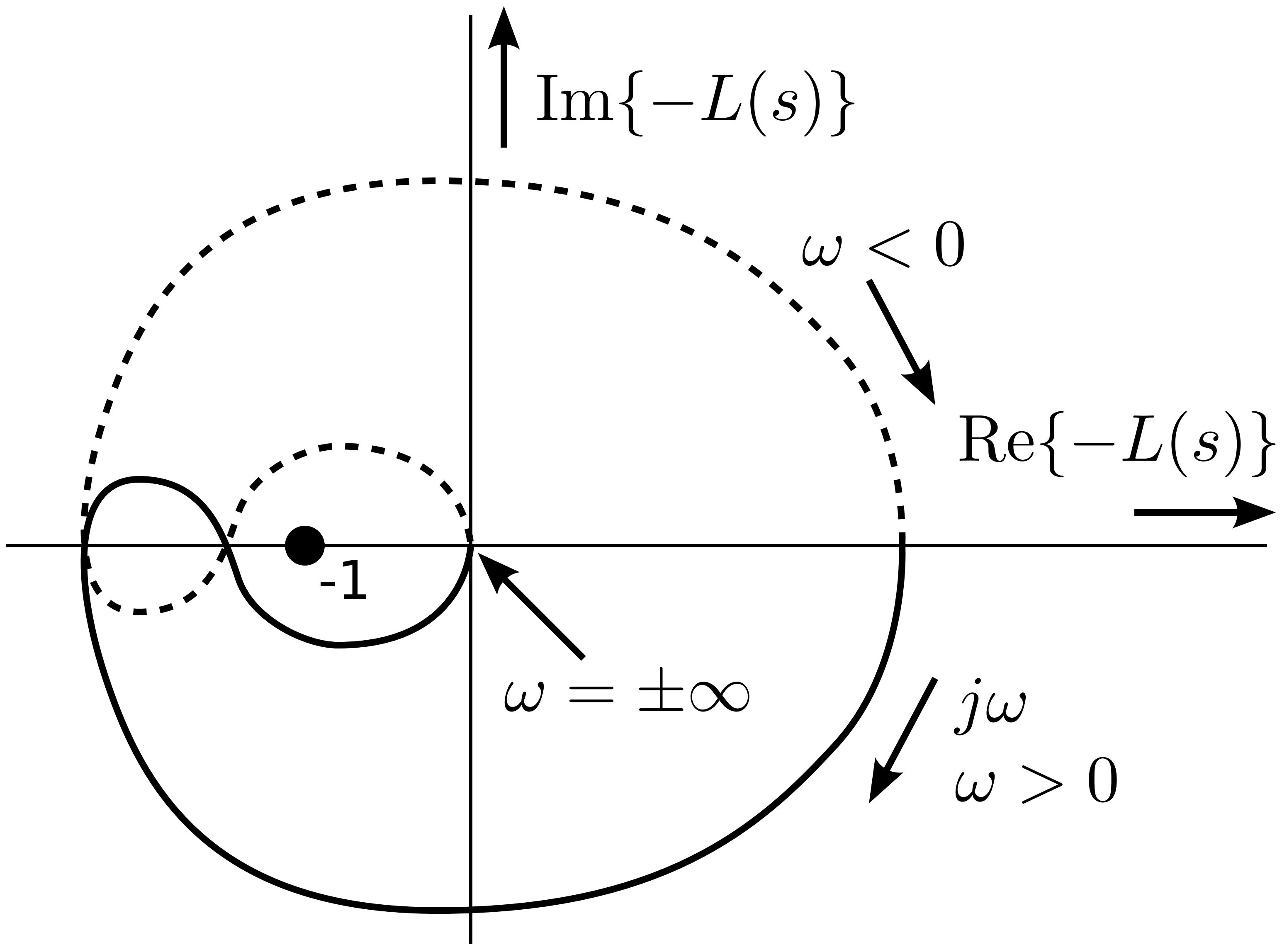

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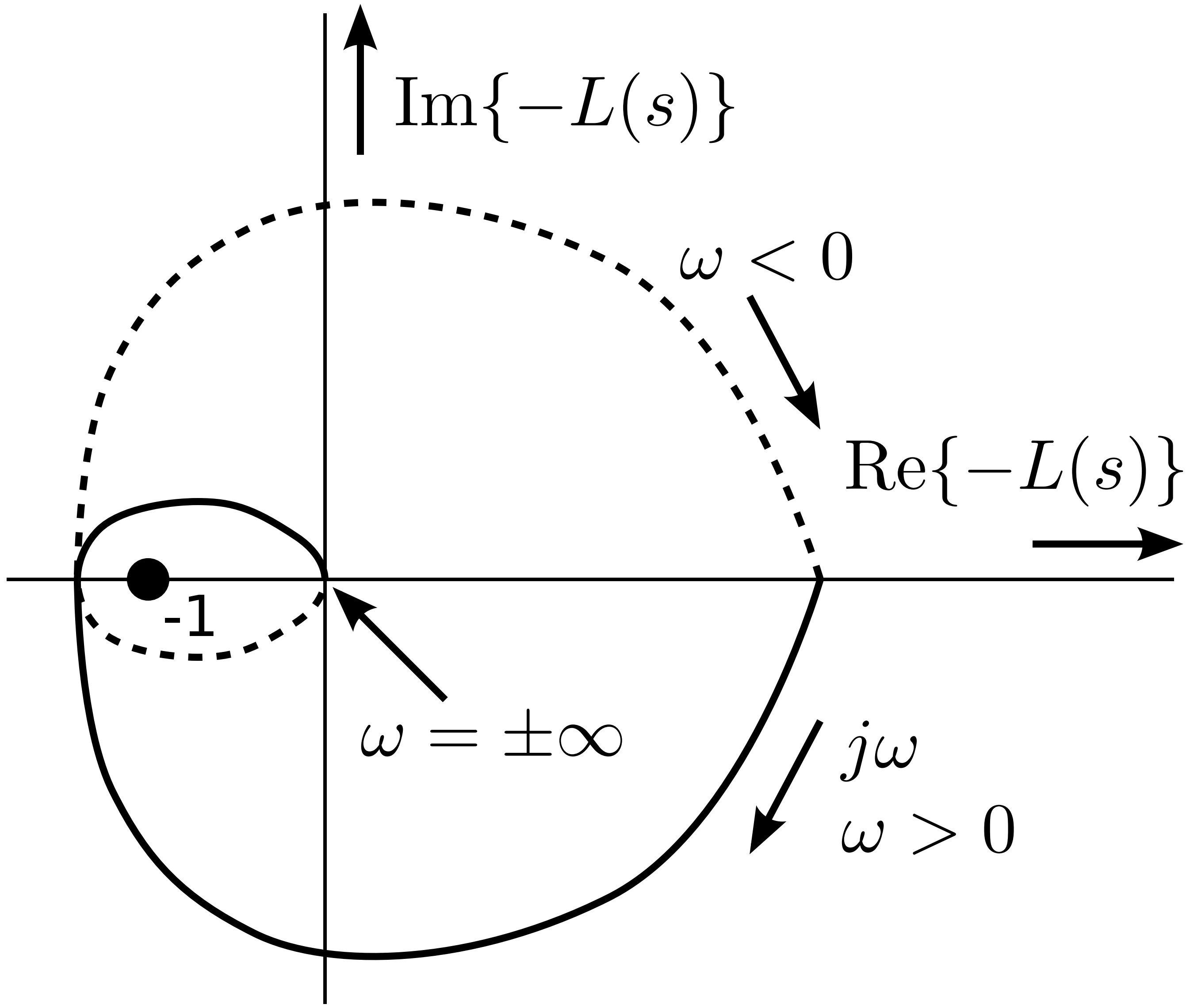

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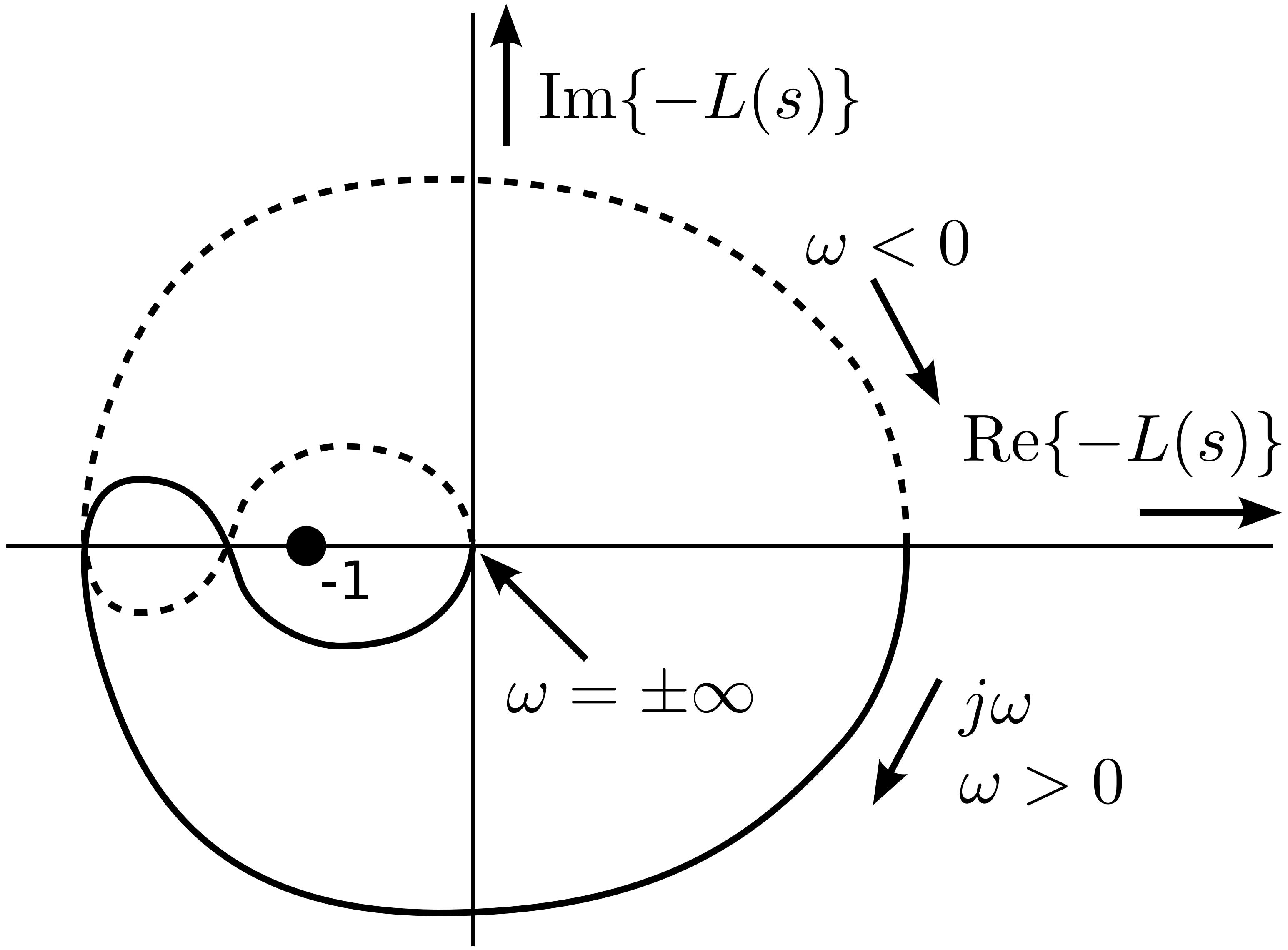

Zero clockwise encirclements

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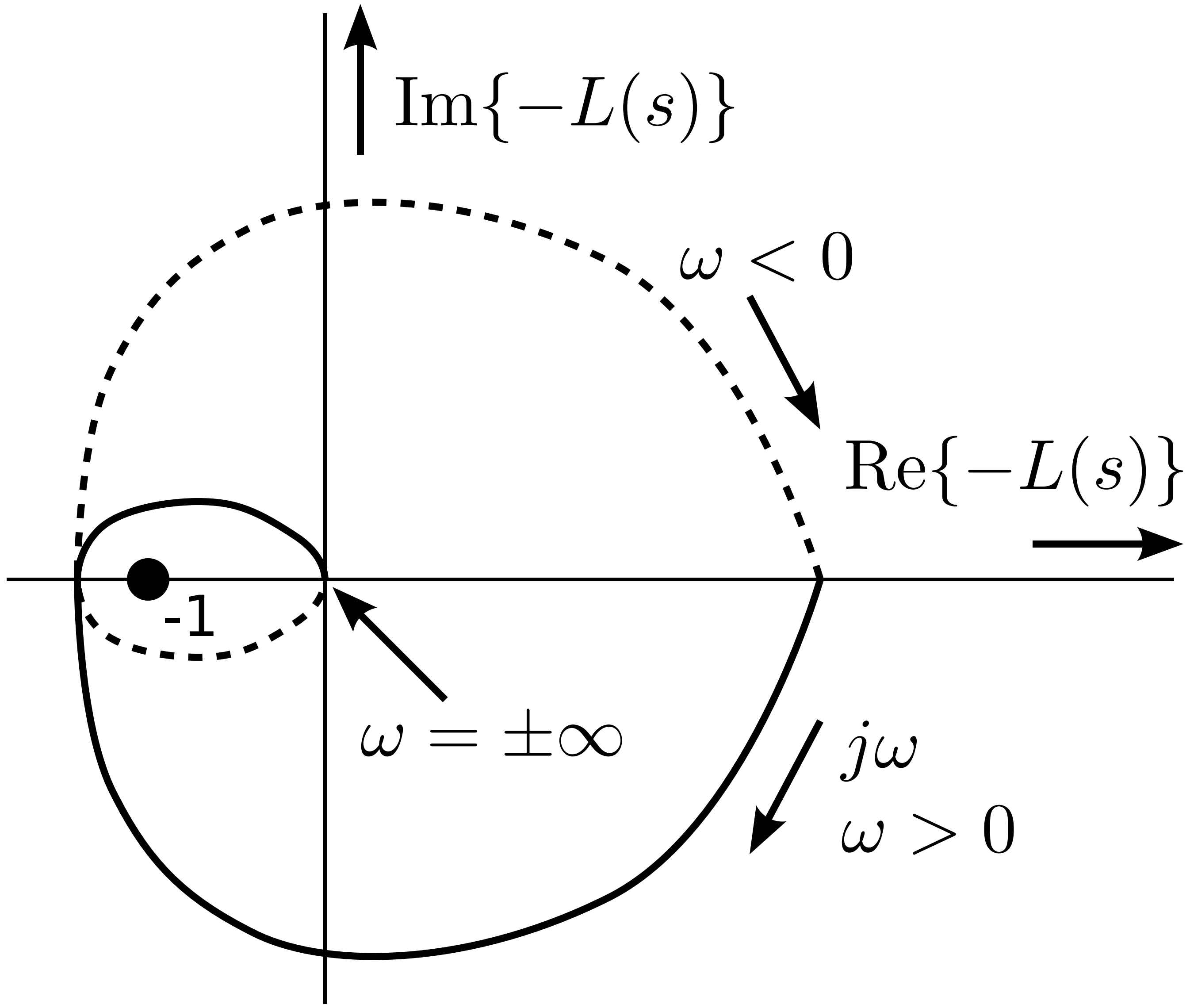

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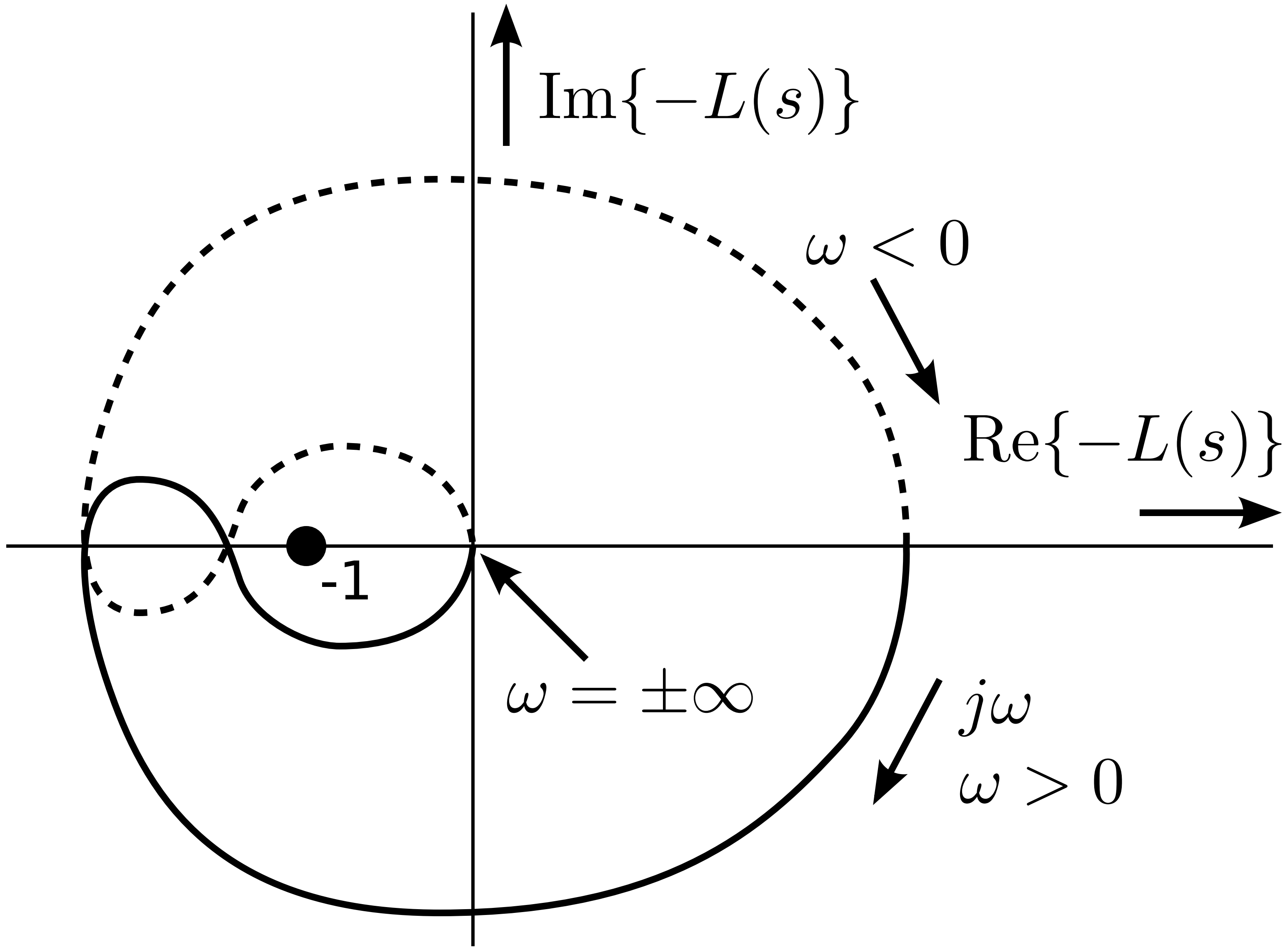

Zero clockwise encirclements

Gain margin and phase margin

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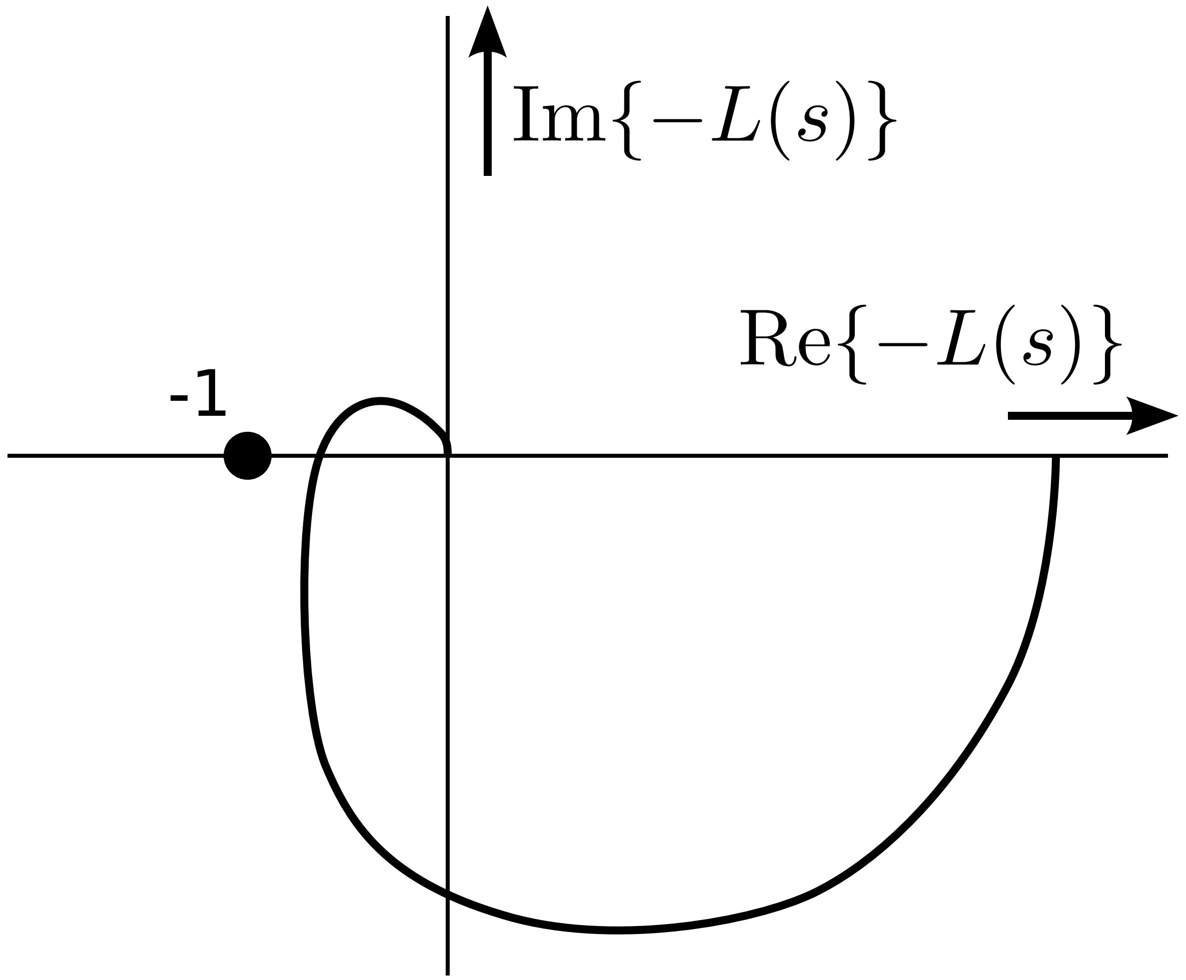
Indicative measure for stability



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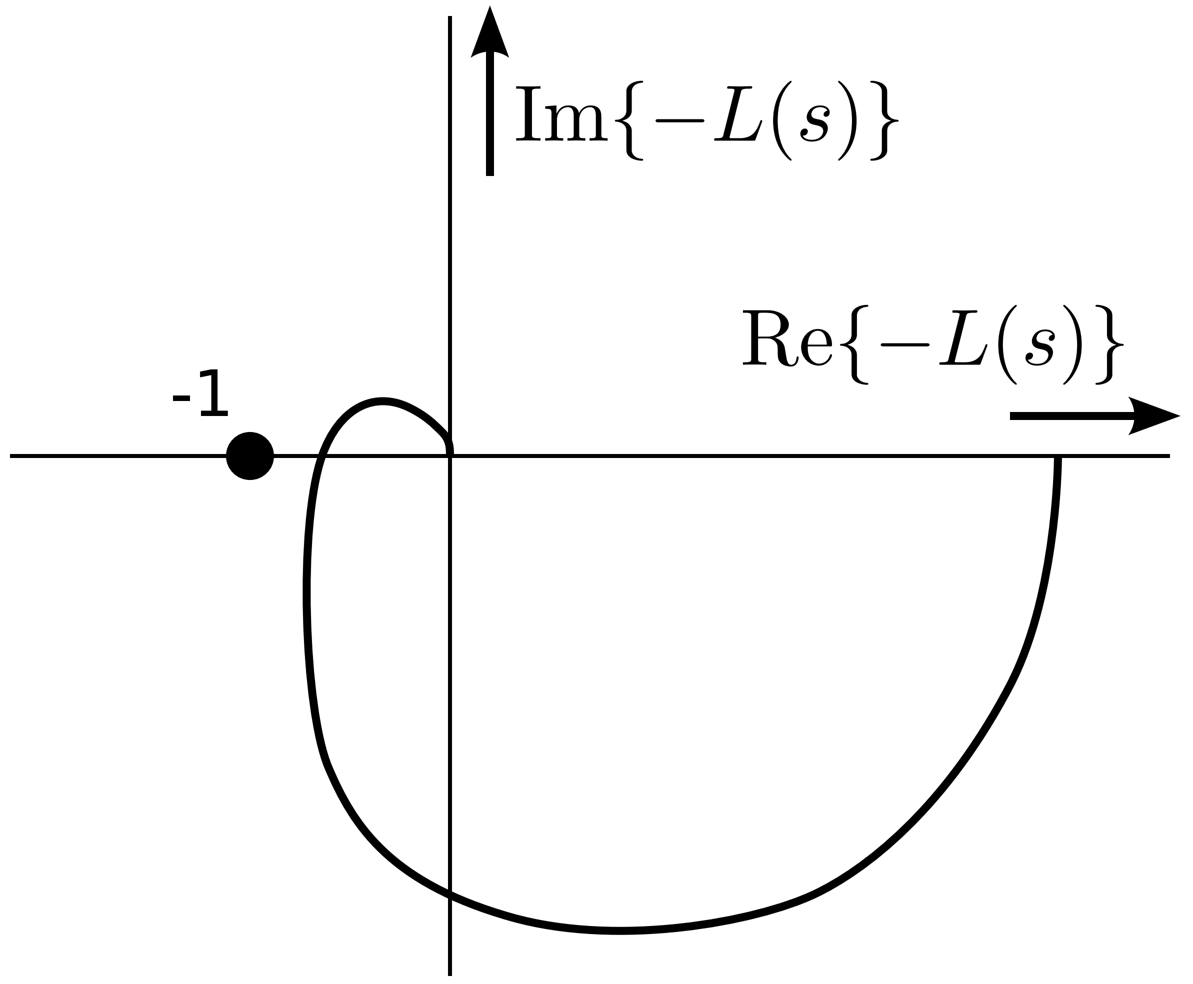

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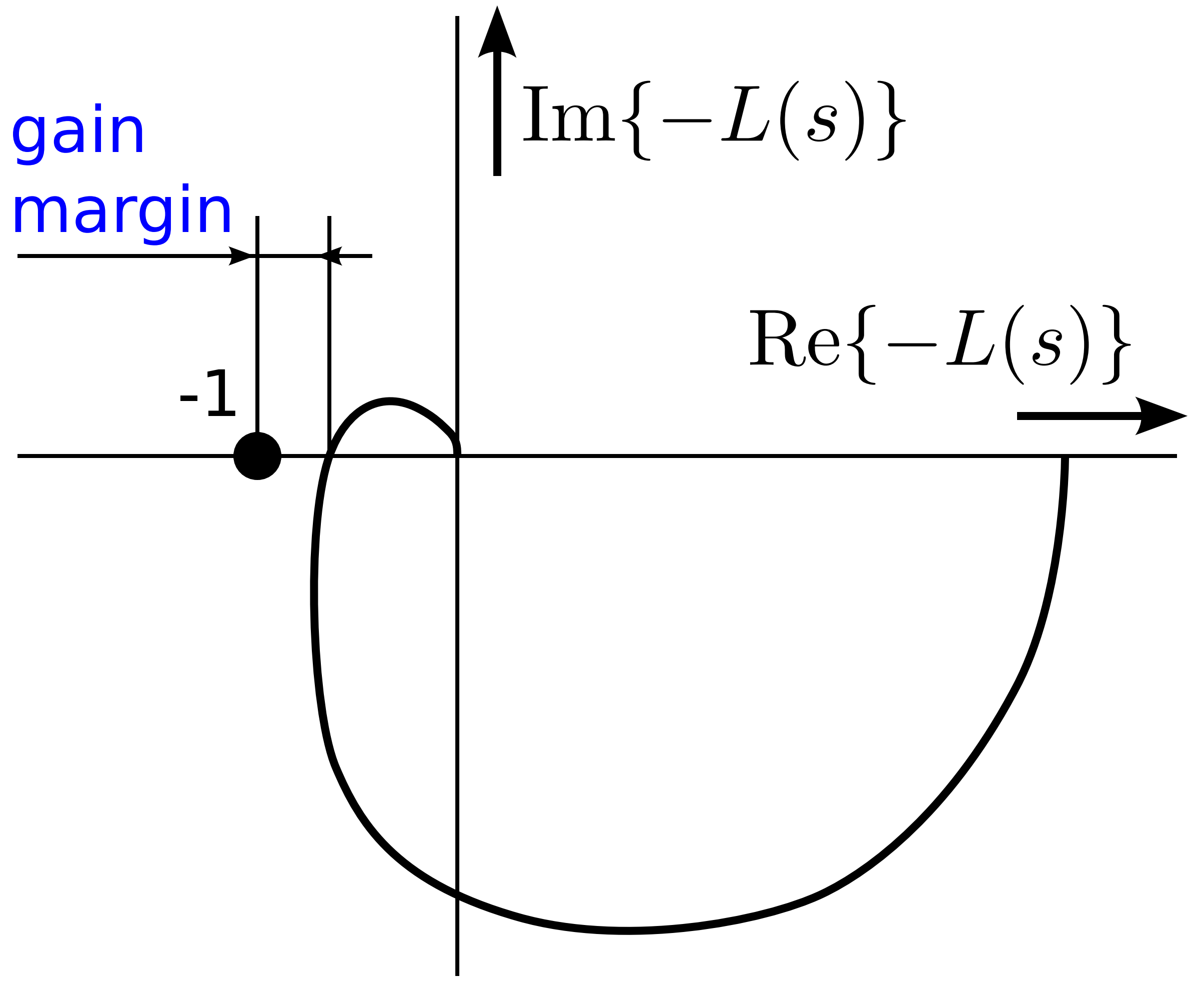

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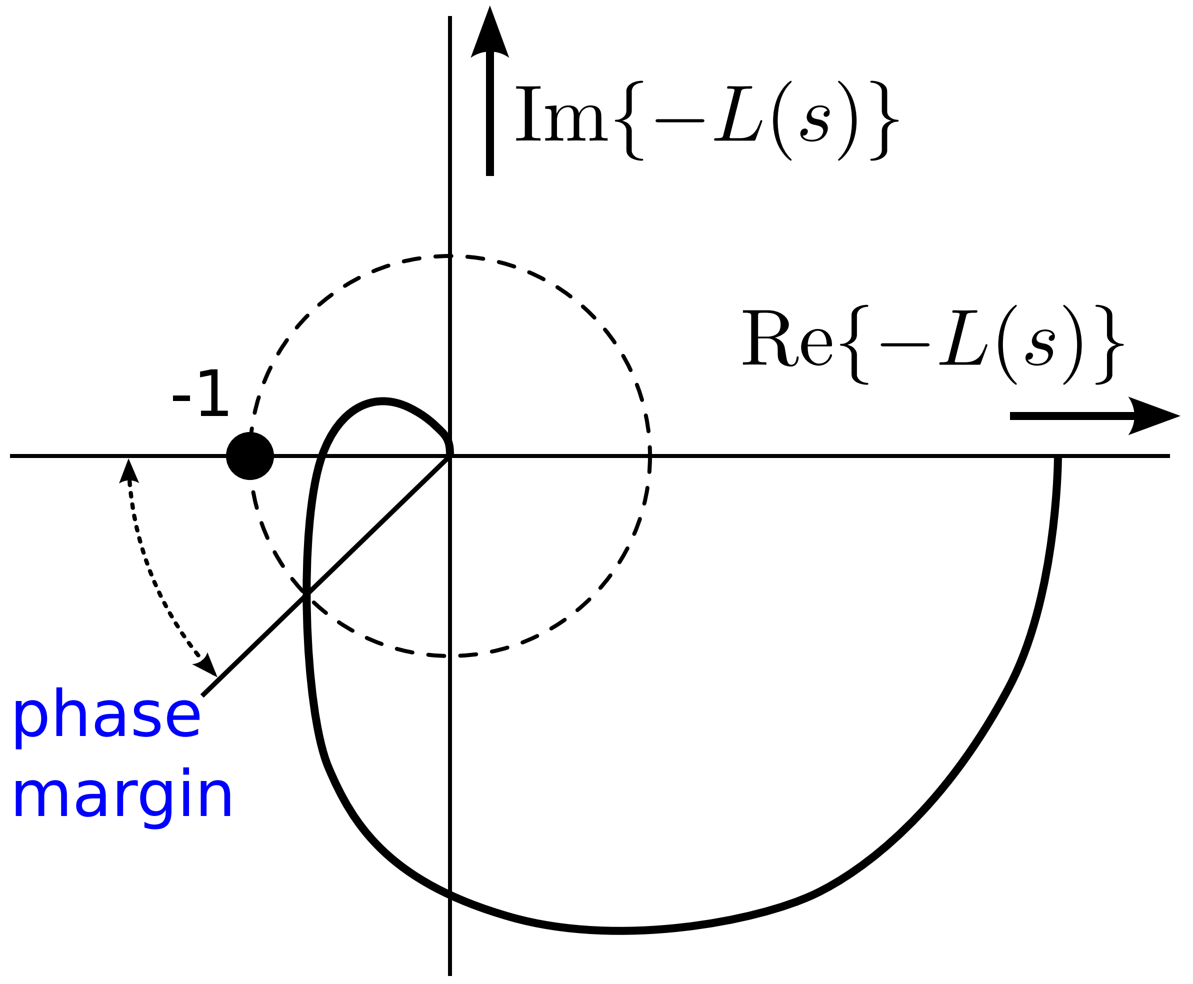

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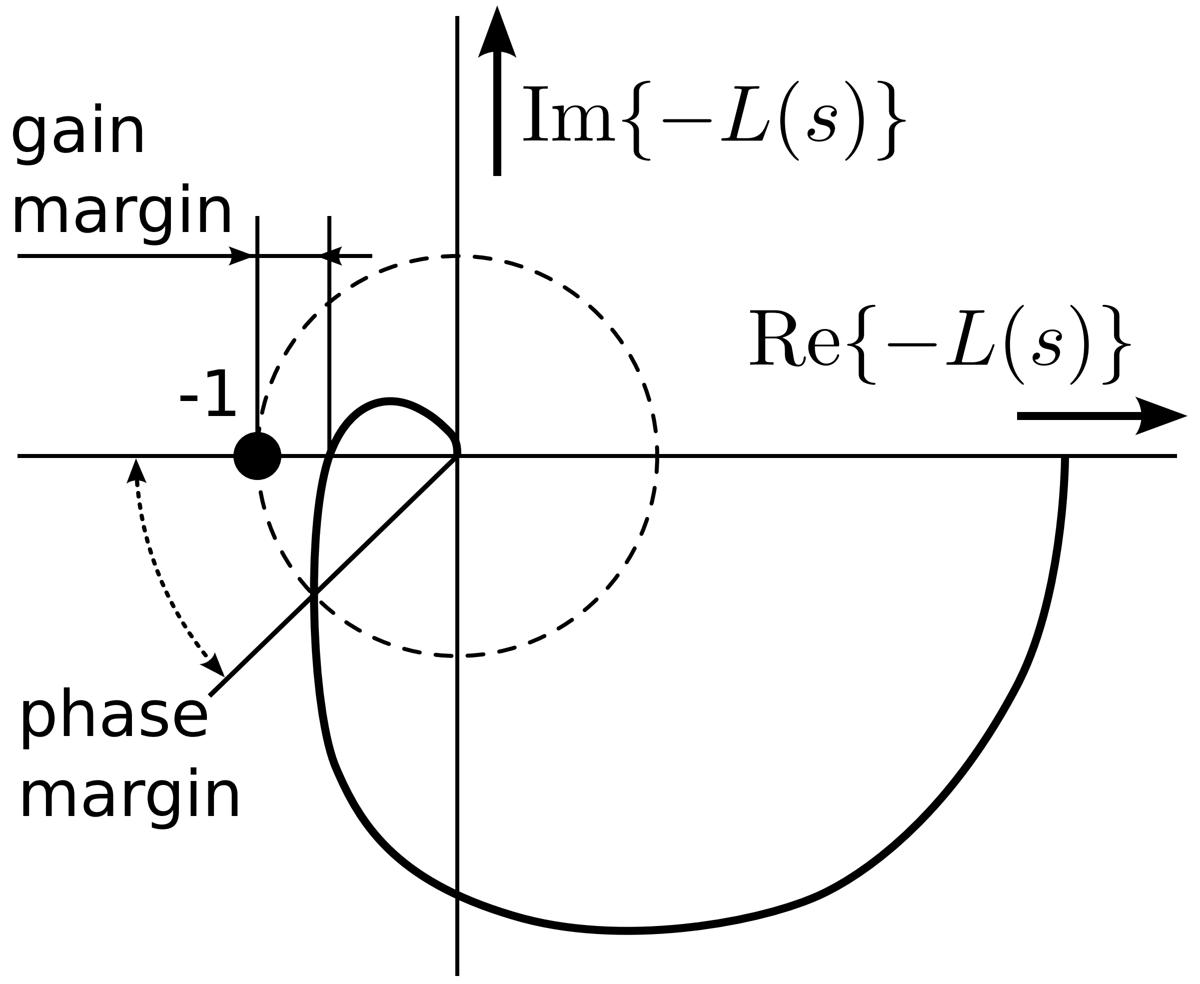

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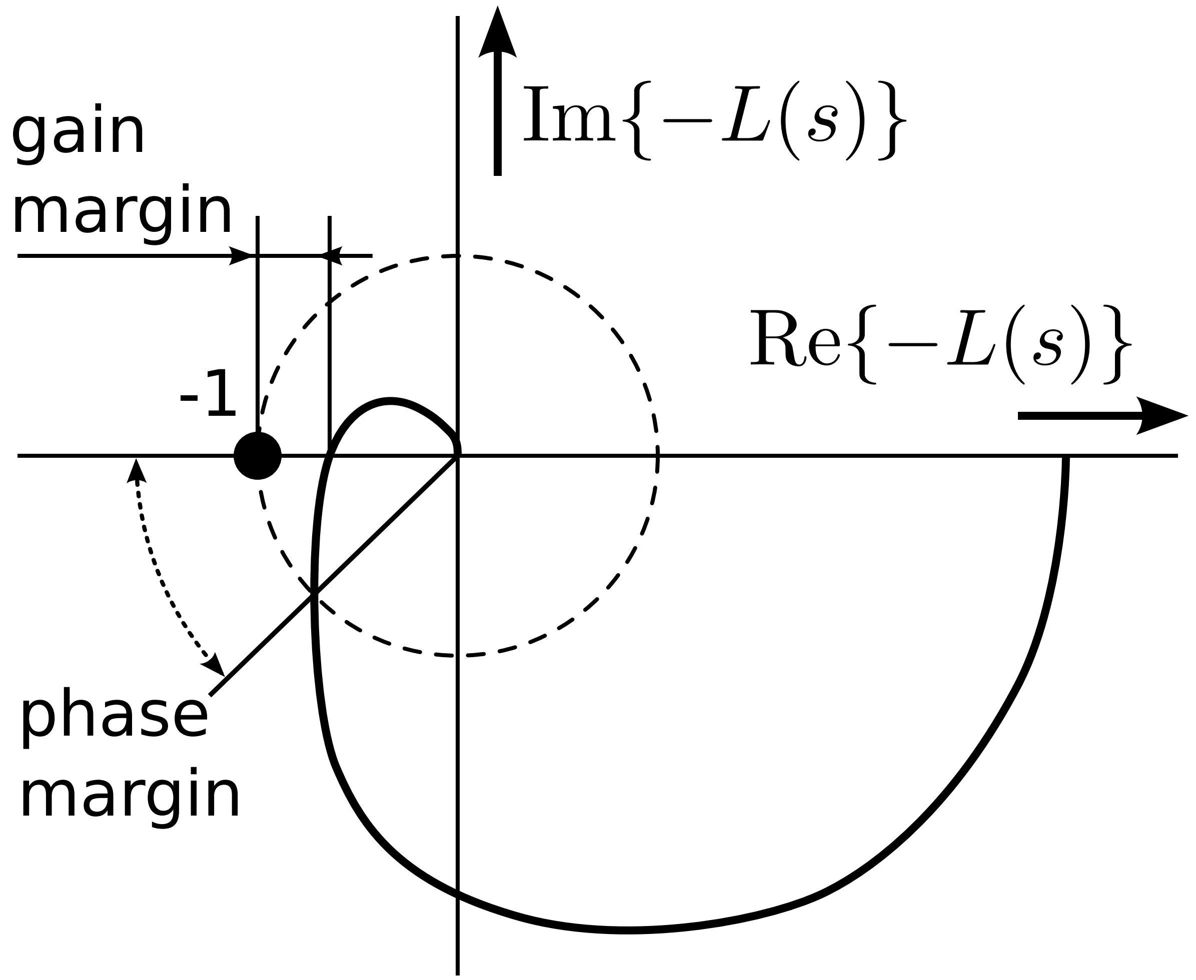

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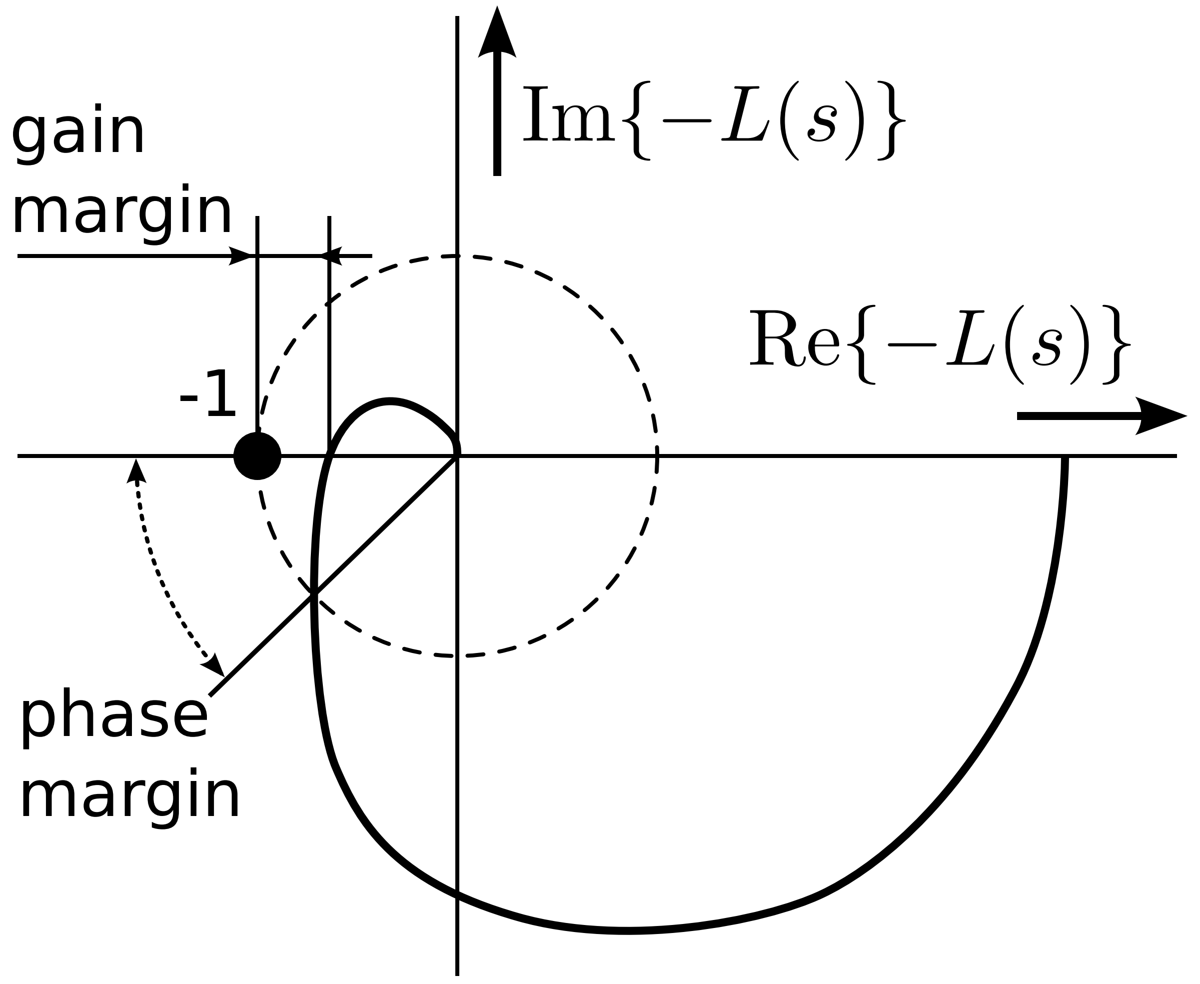

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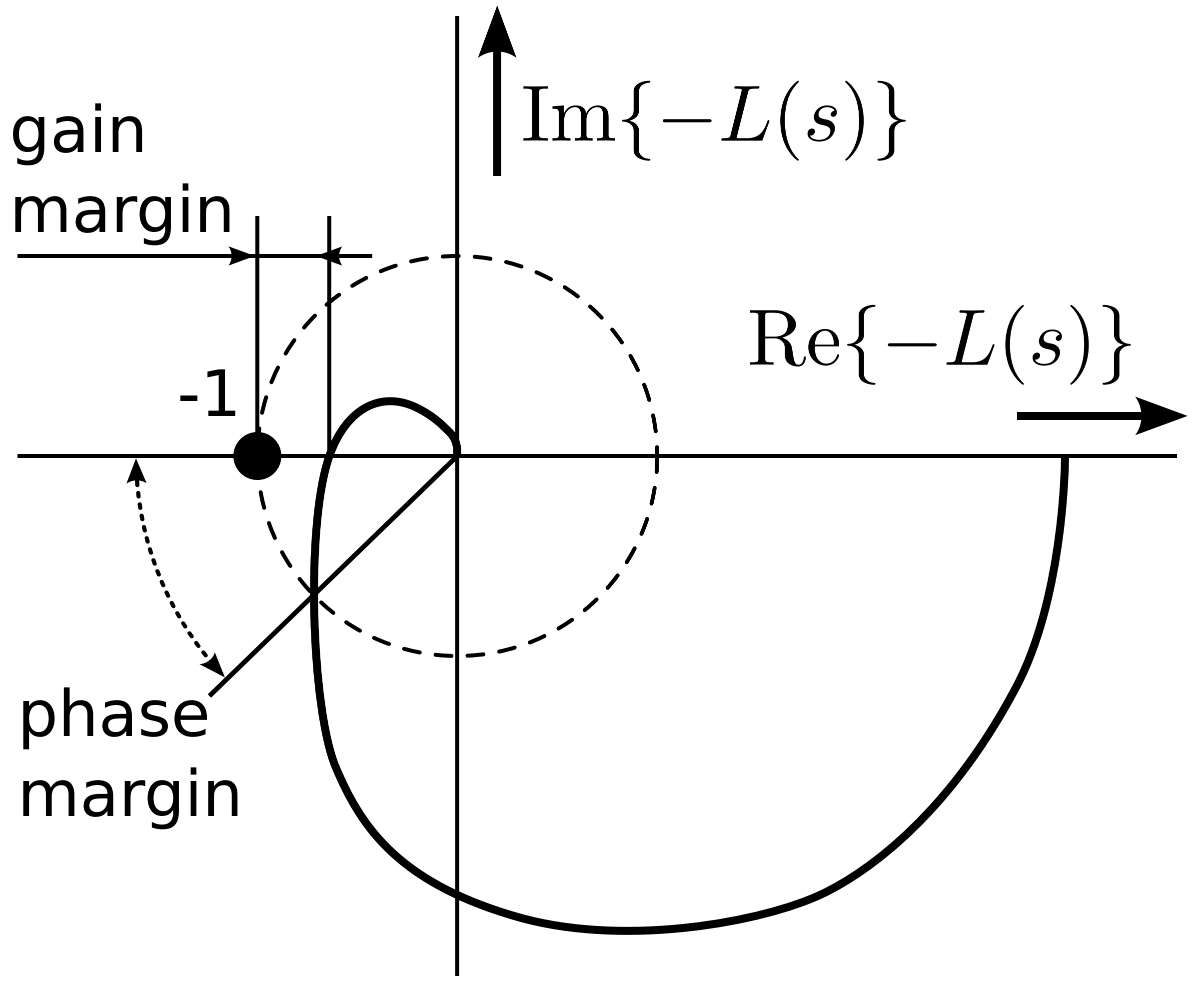

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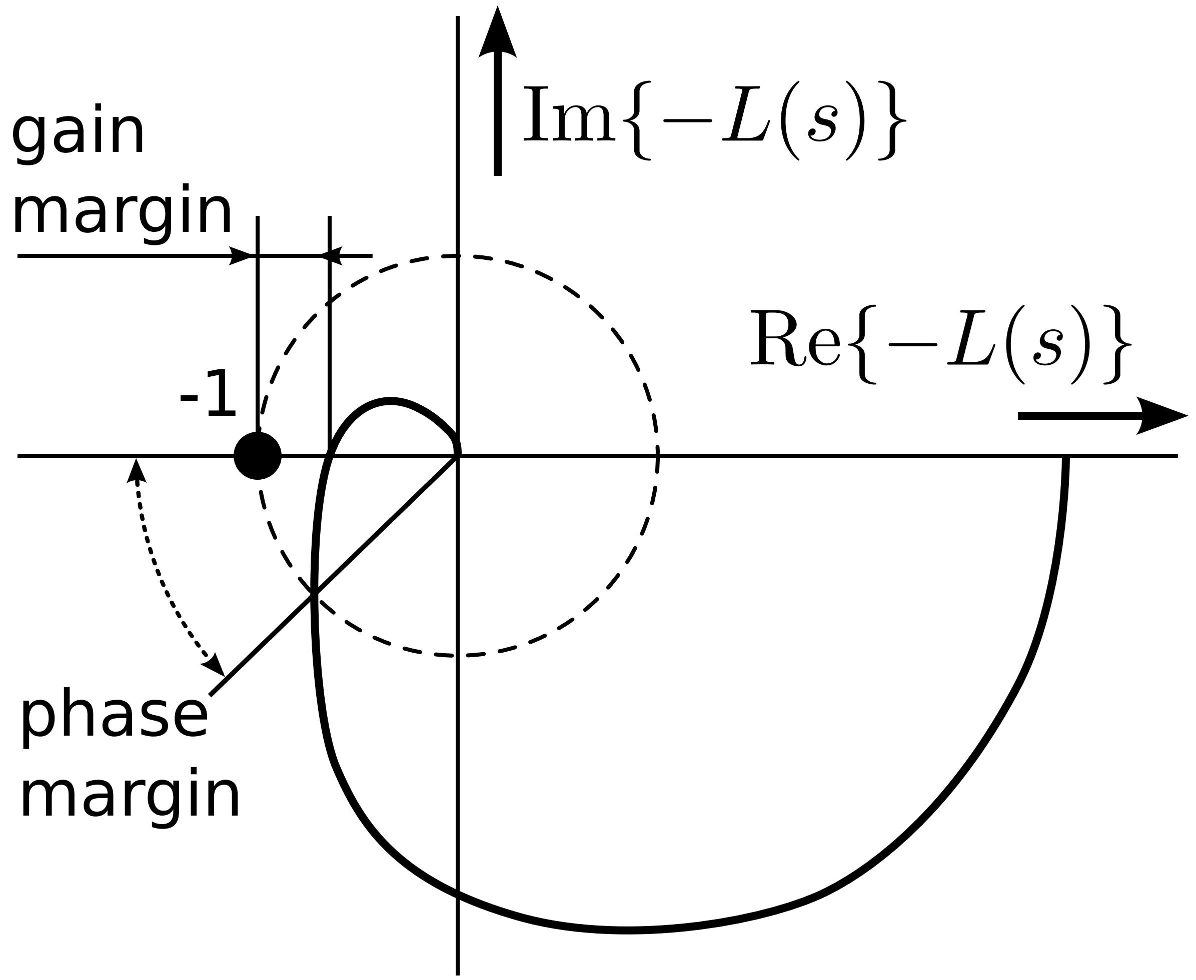

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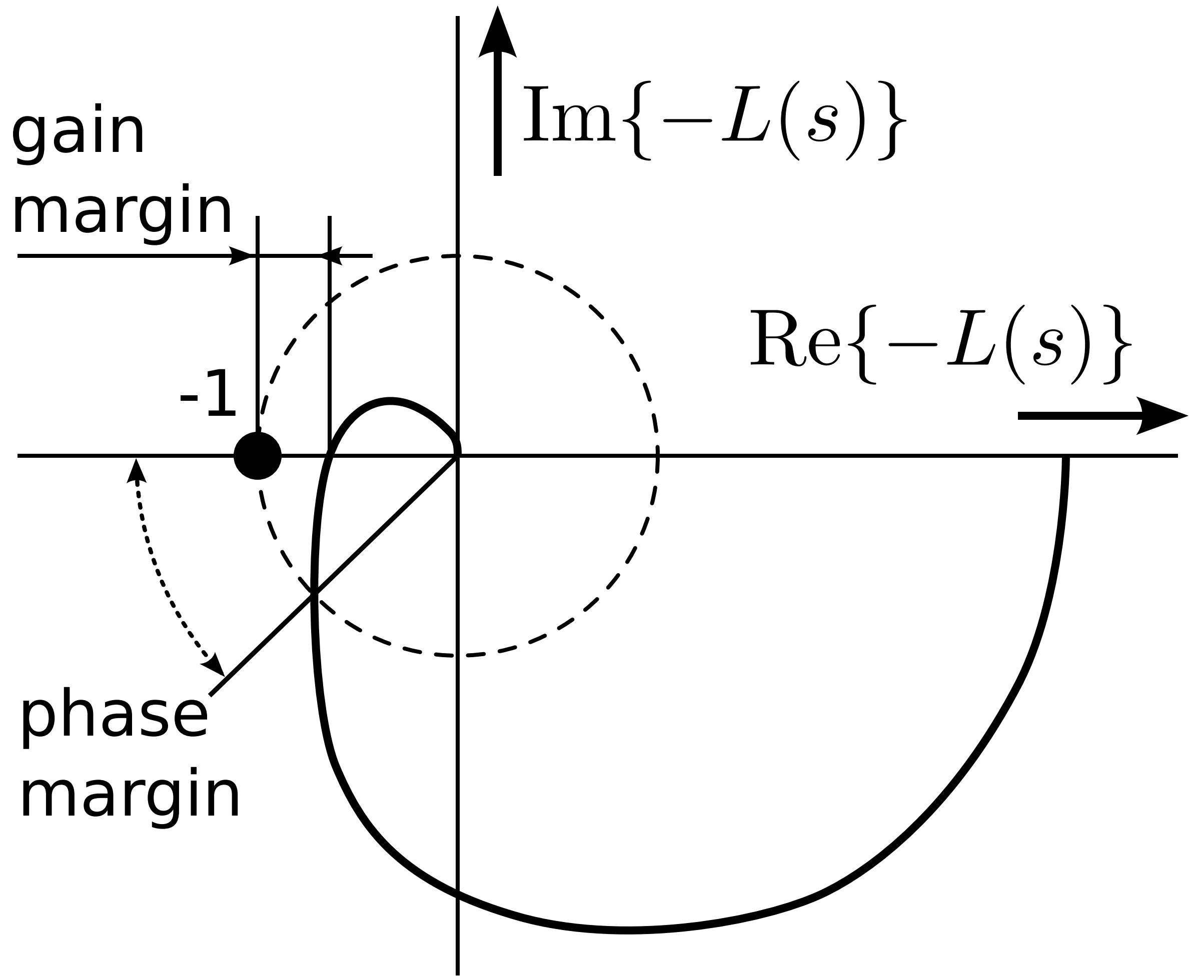

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Starting point: DC loop gain equals zero

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Paths traced out by the poles of the servo function while varying the DC loop gain

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1. Number of branches equals number of poles of the loop gain

Root locus technique

$$L(s) = L_{\mathrm{DC}} \frac{N(s)}{D(s)}$$

$L_{\mathrm{DC}} =$ DC value of the loop gain

$N(s) = 0 \Longrightarrow$ Zeros of the loop gain

$D(s) = 0 \Longrightarrow$ Poles of the loop gain

$$S(s) = \frac{-L(s)}{1-L(s)} = \frac{-L_{\mathrm{DC}} N(s)}{D(s)-L_{\mathrm{DC}} N(s)}$$

$D(s) - L_{\mathrm{DC}} N(s) = 0 \Longrightarrow$ Poles of servo function

Root locus:

Paths traced out by the poles of the servo function while varying the DC loop gain

Paths depend on:

1. Poles of the loop gain
2. Zeros of the loop gain
3. DC loop gain

Starting point: DC loop gain equals zero
End point: DC loop gain equals infinity
Actual servo poles: DC loop gain $= L_{\mathrm{DC}}$

1. Number of branches equals number of poles of the loop gain
2. Symmetrical with respect to the real axis

Root locus technique

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Actual servo poles: DC loop gain $= L_{\mathrm{DC}}$

1. Number of branches equals number of poles of the loop gain
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3. Branches start at poles of the loop gain

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6. n poles and m zeros, then n - m asymptotes

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$$\sigma = \frac{\sum_{k=1}^{n} p_k - \sum_{i=1}^{m} z_i}{n-m}$$

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8. Angle asymptotes equally spaced
9. Break away (and arrival) points

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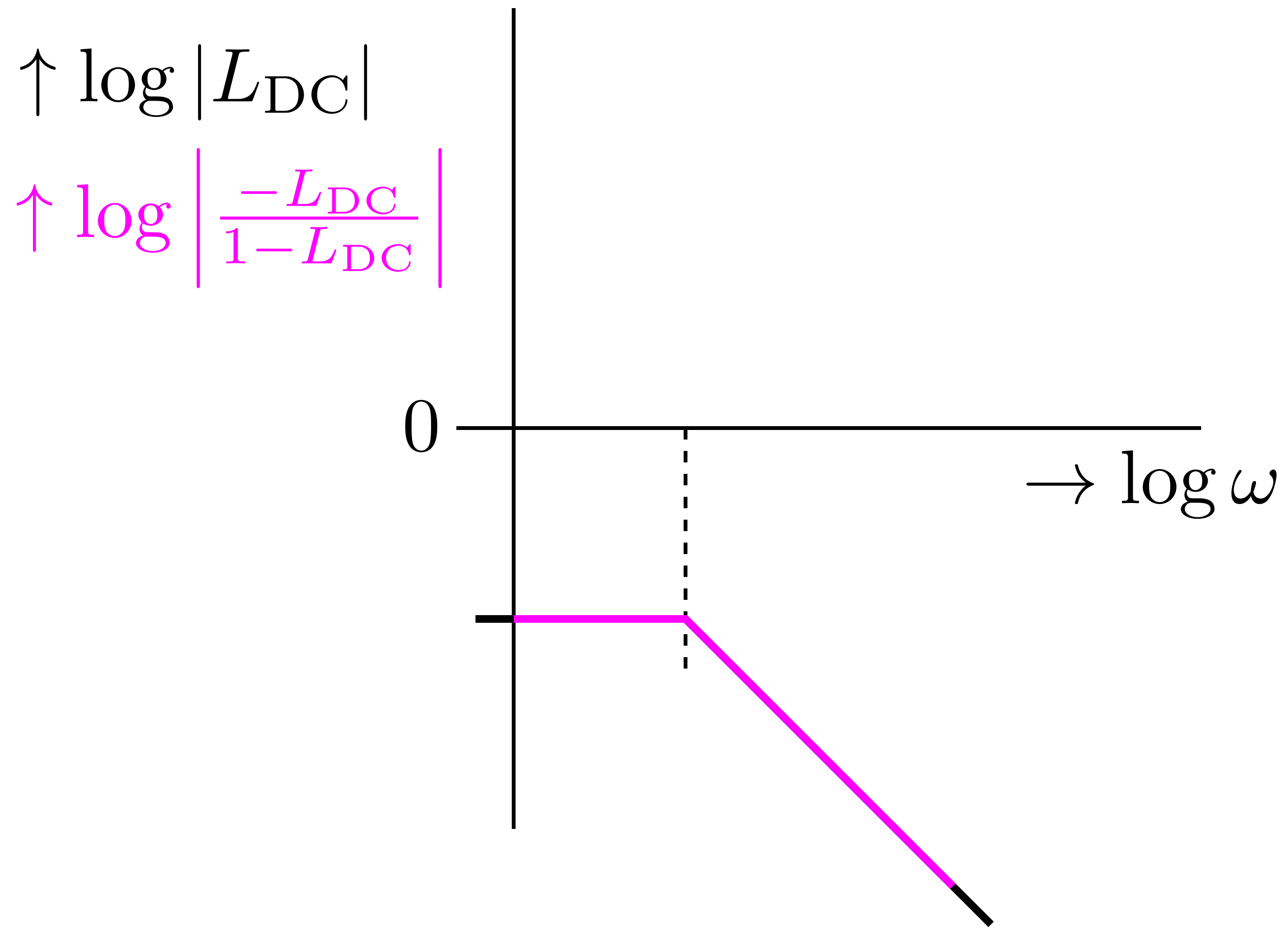
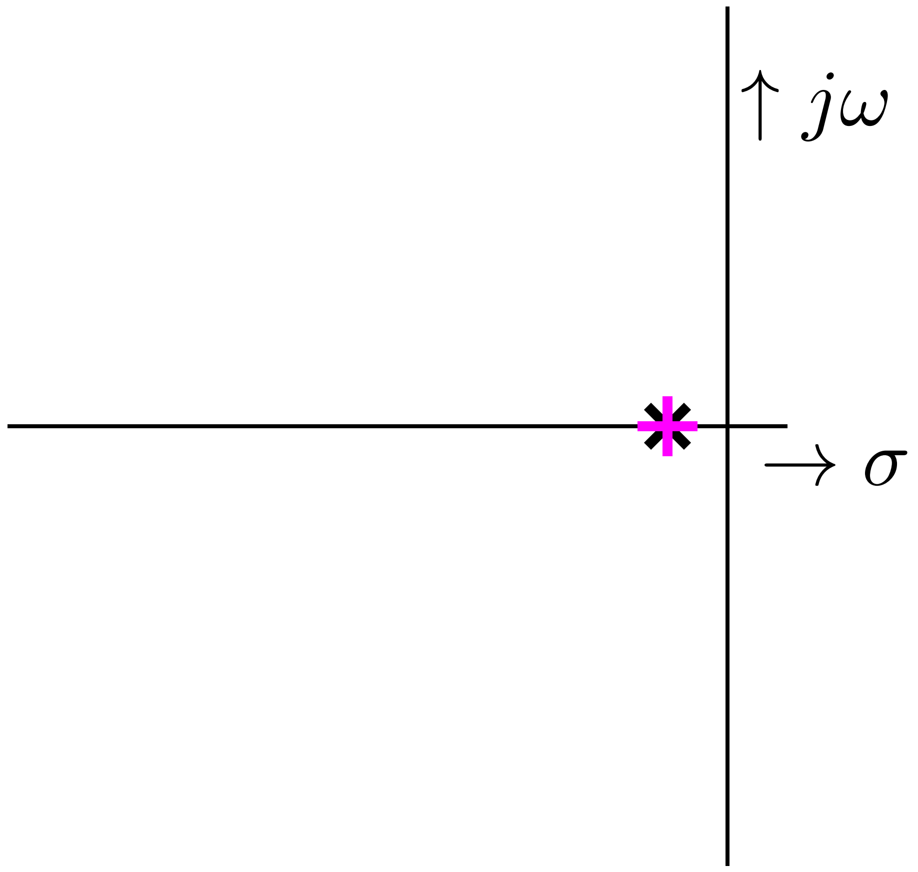
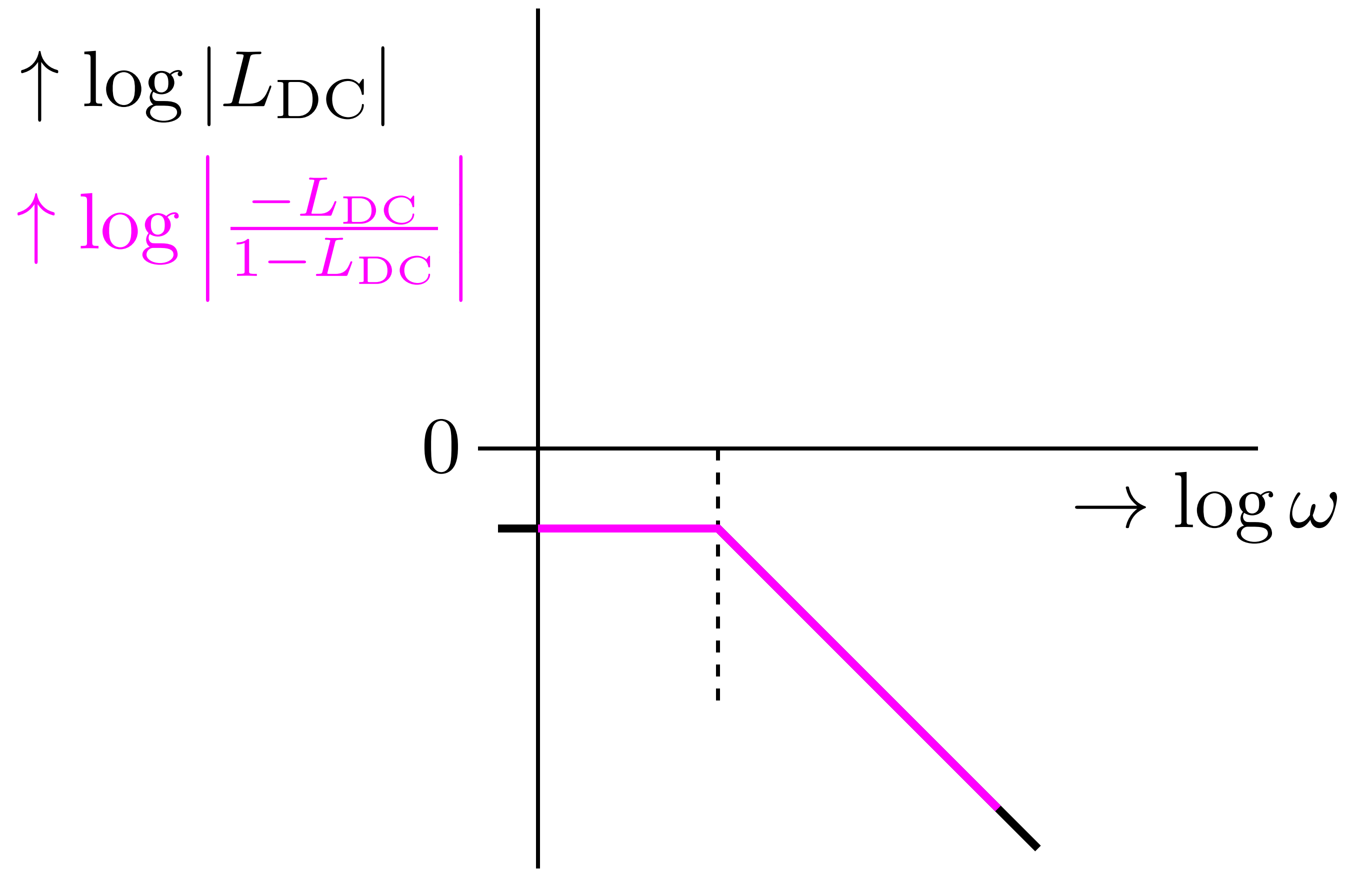
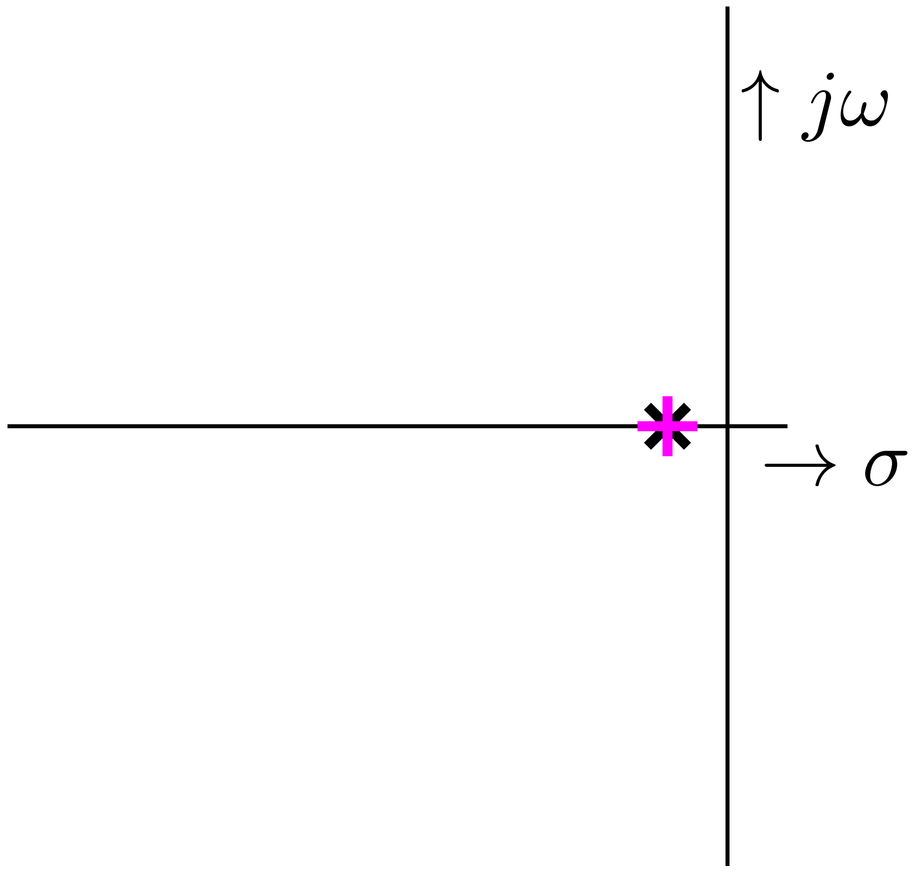
Root locus first order



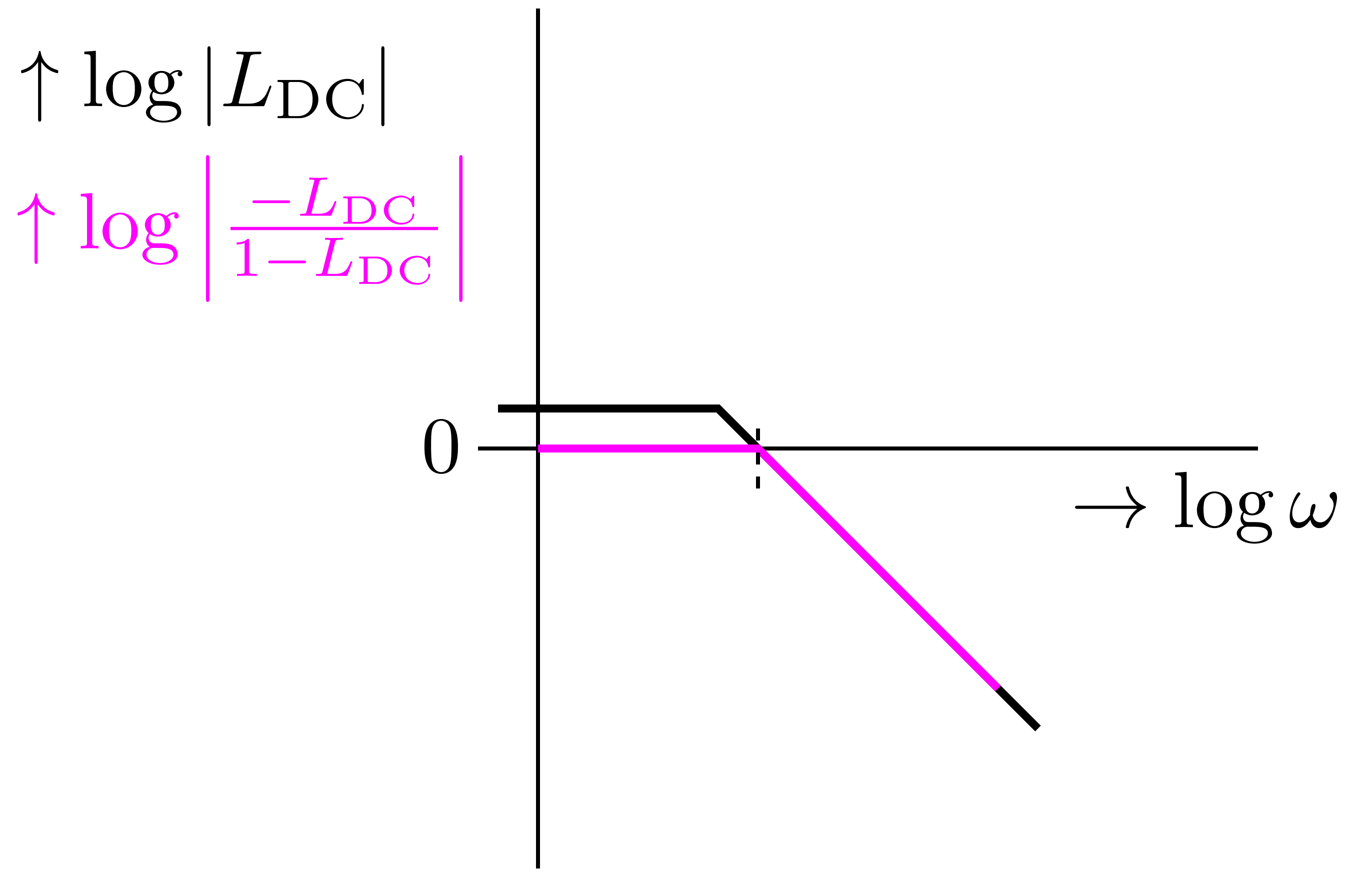


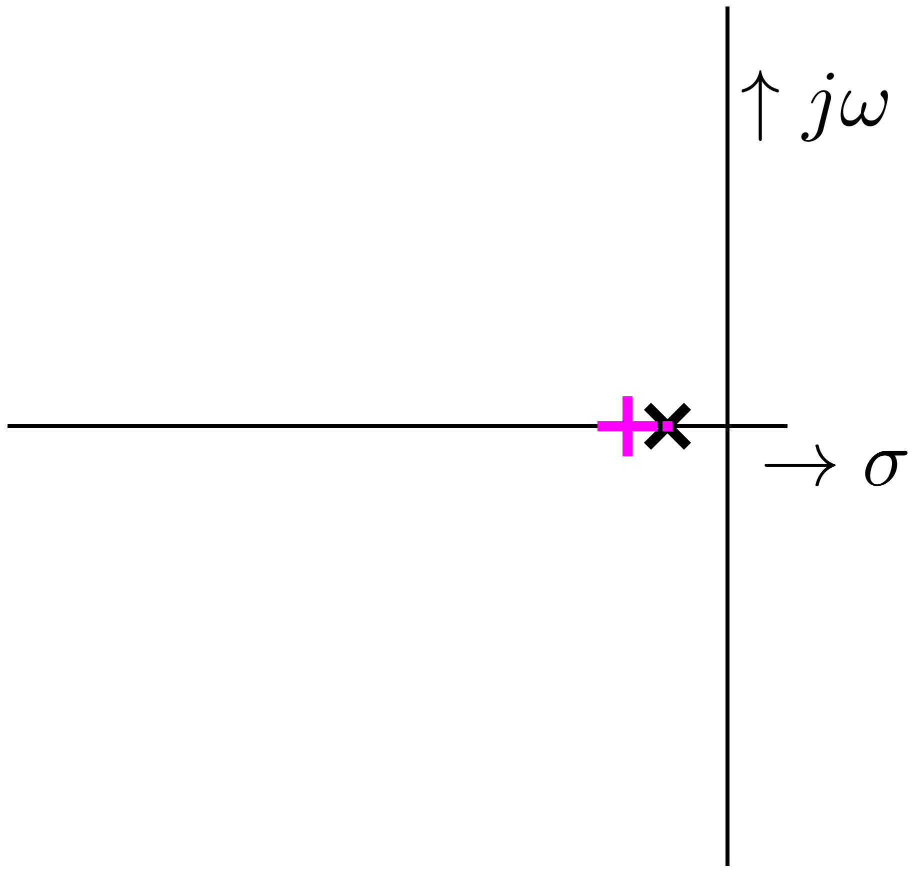
Root locus first order





Root locus first order





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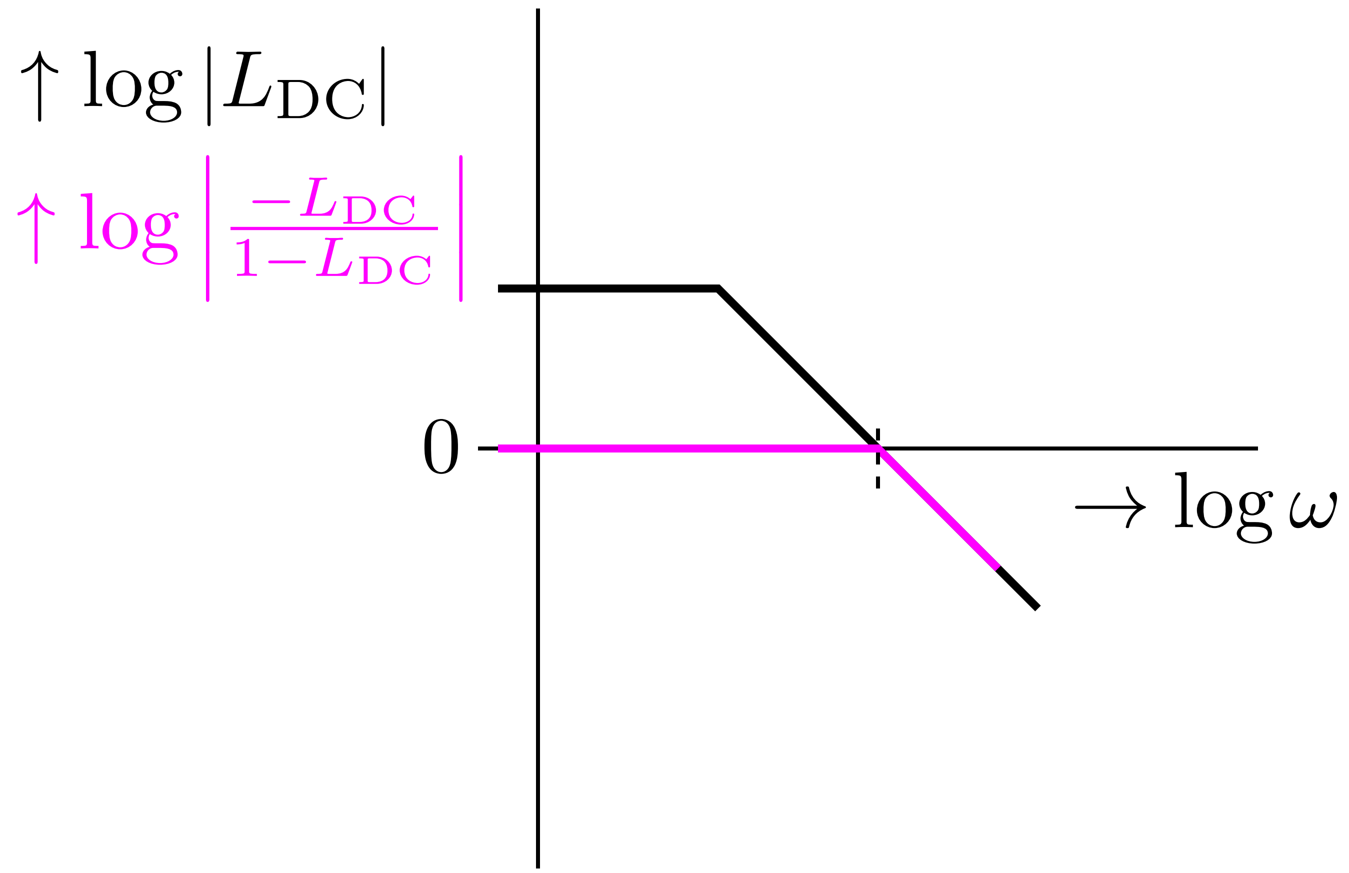

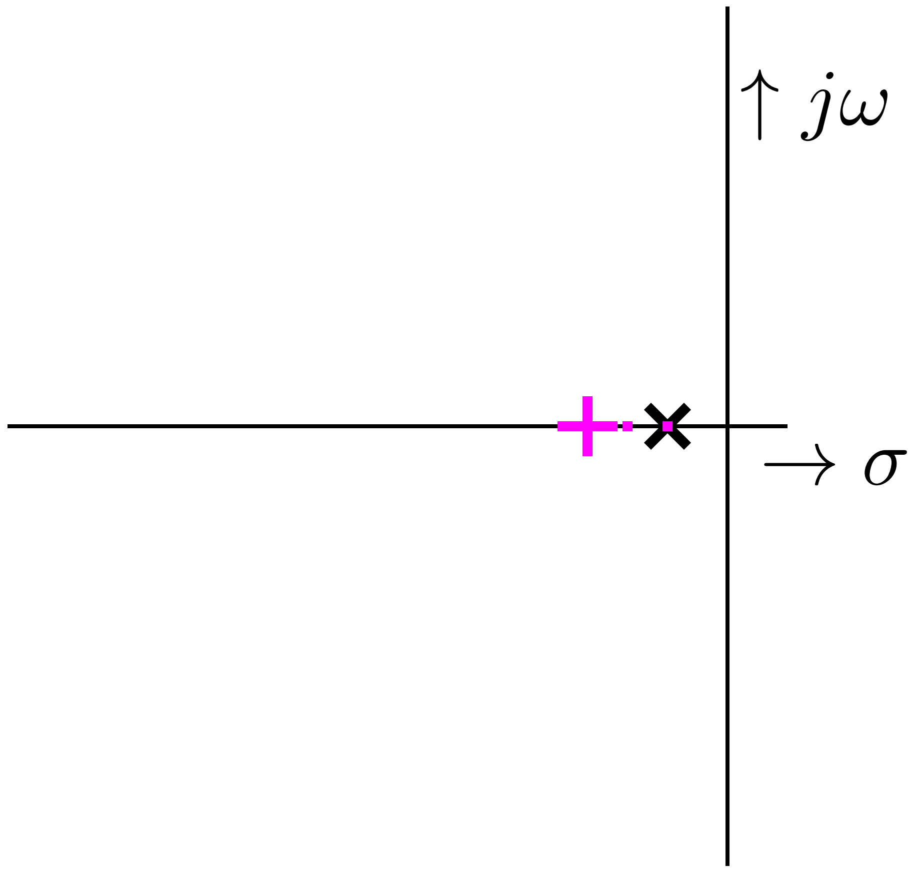

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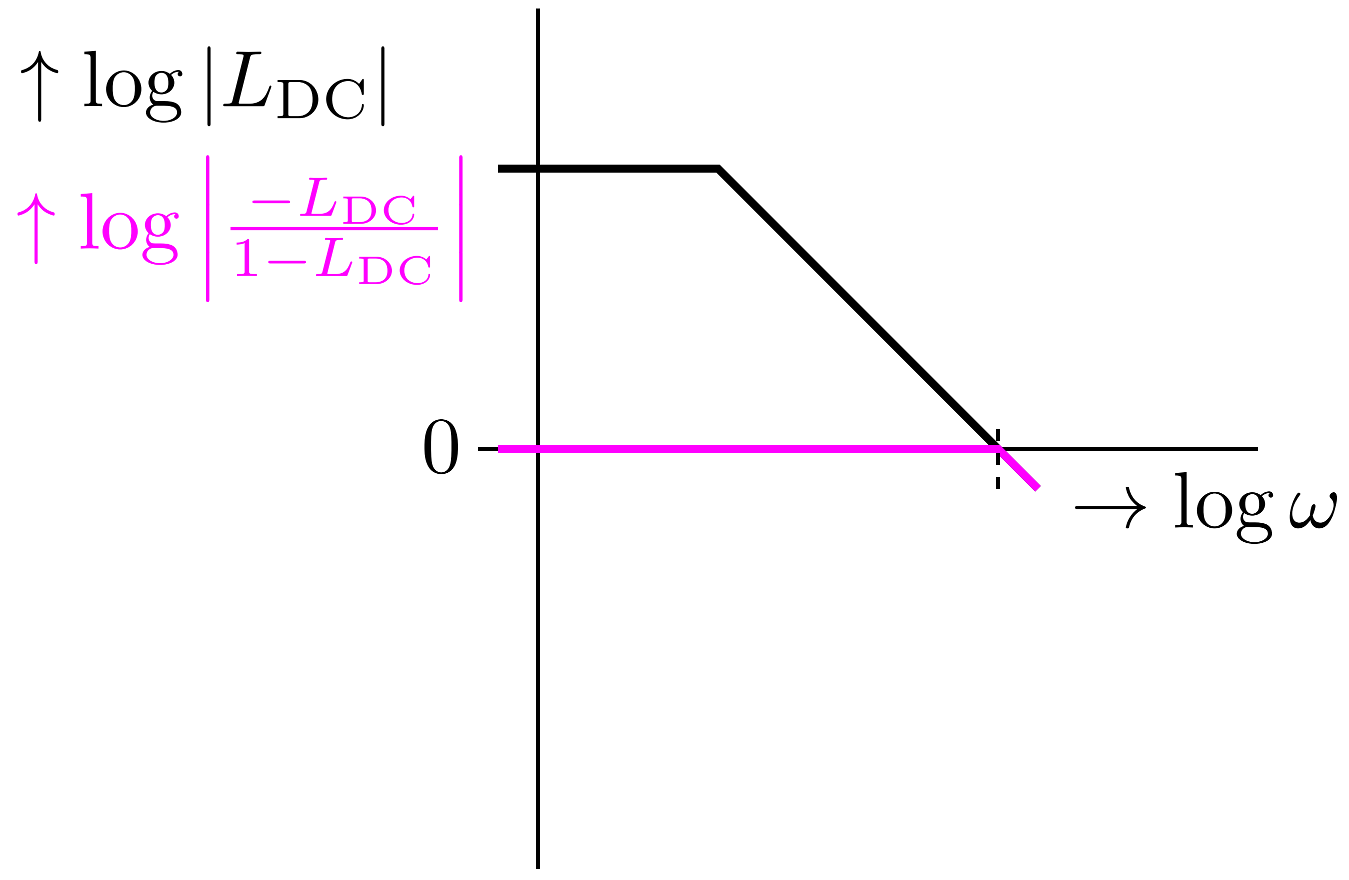
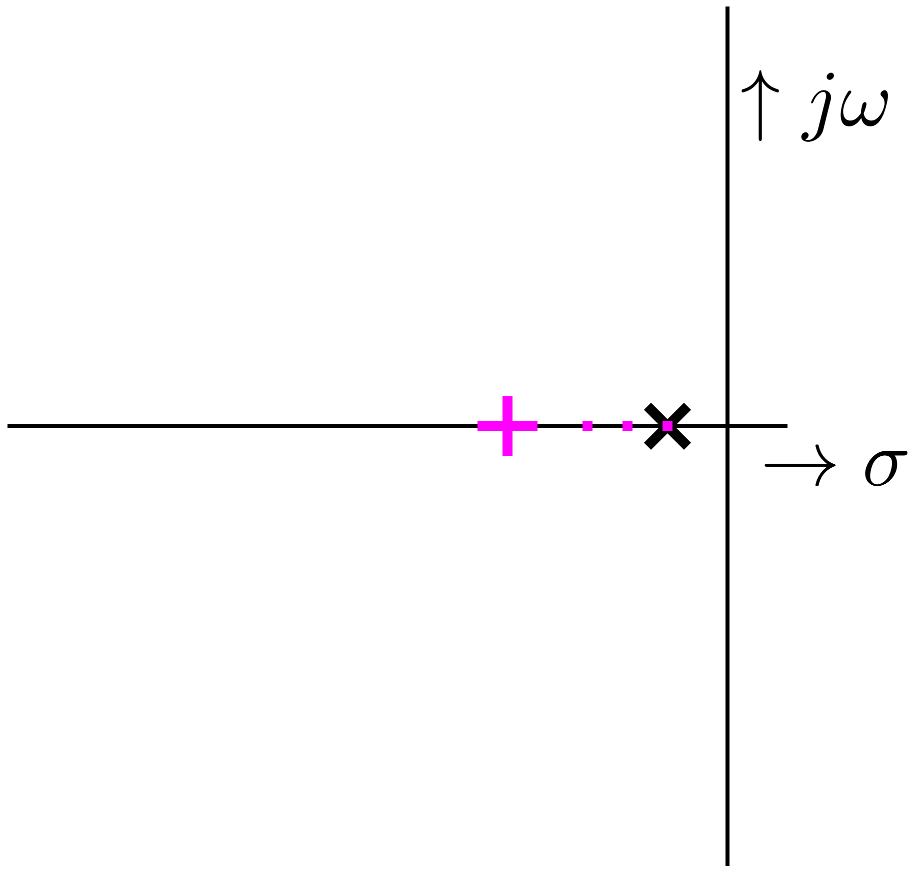
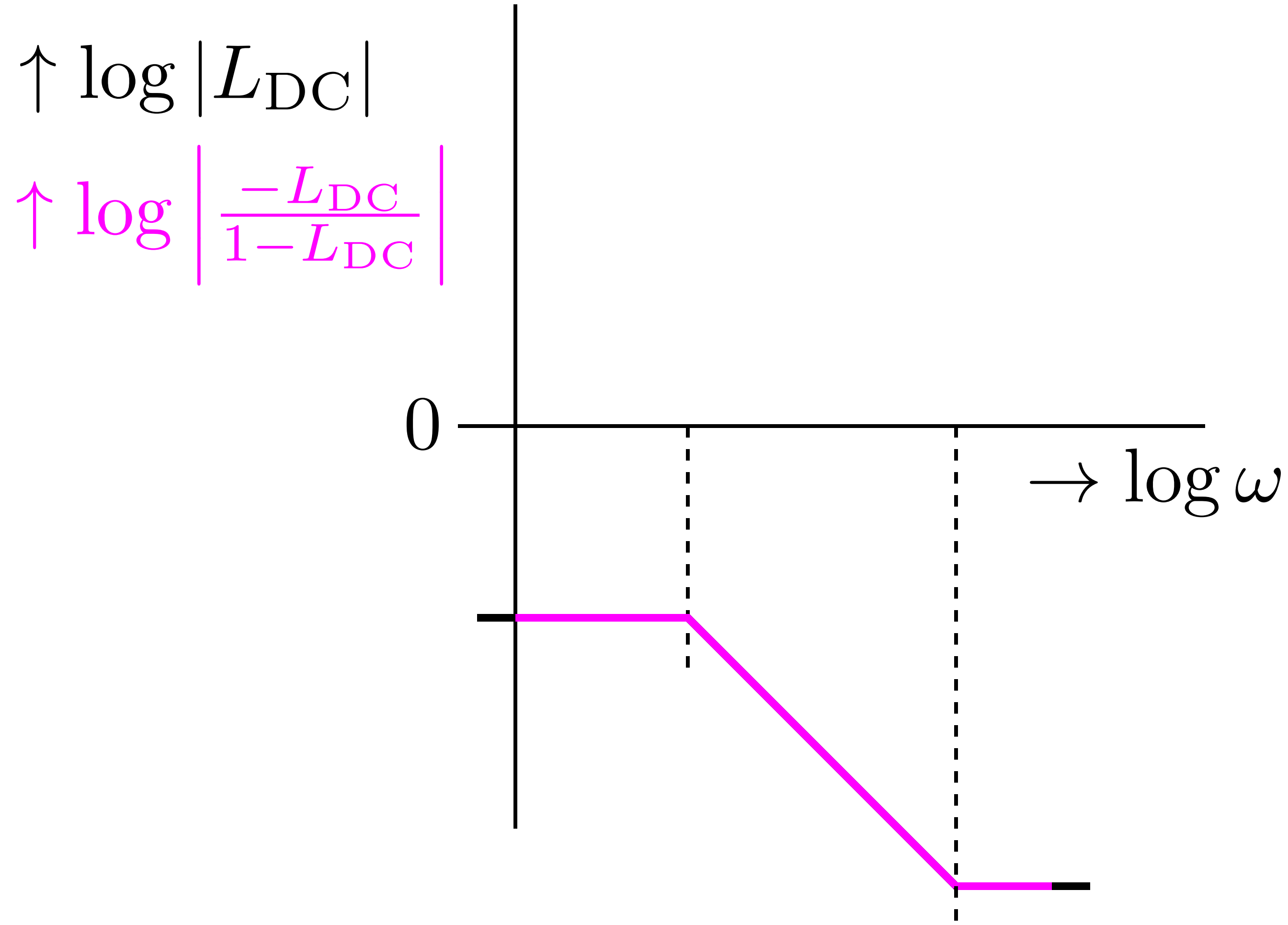
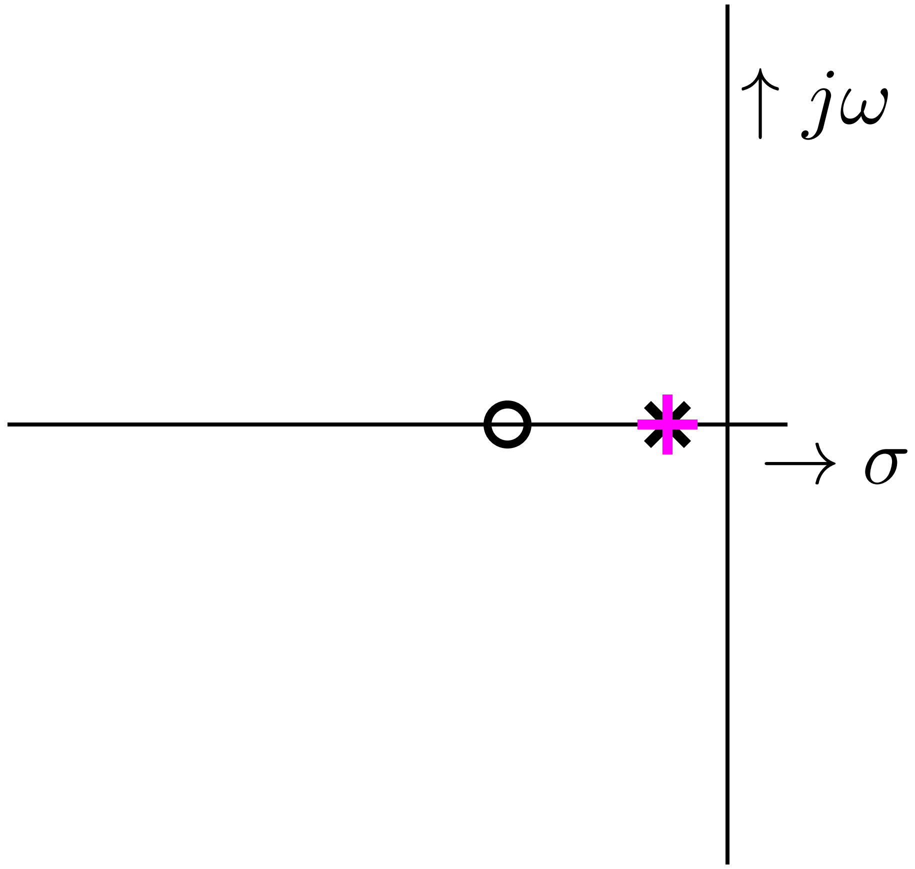




Root locus first order with zero left





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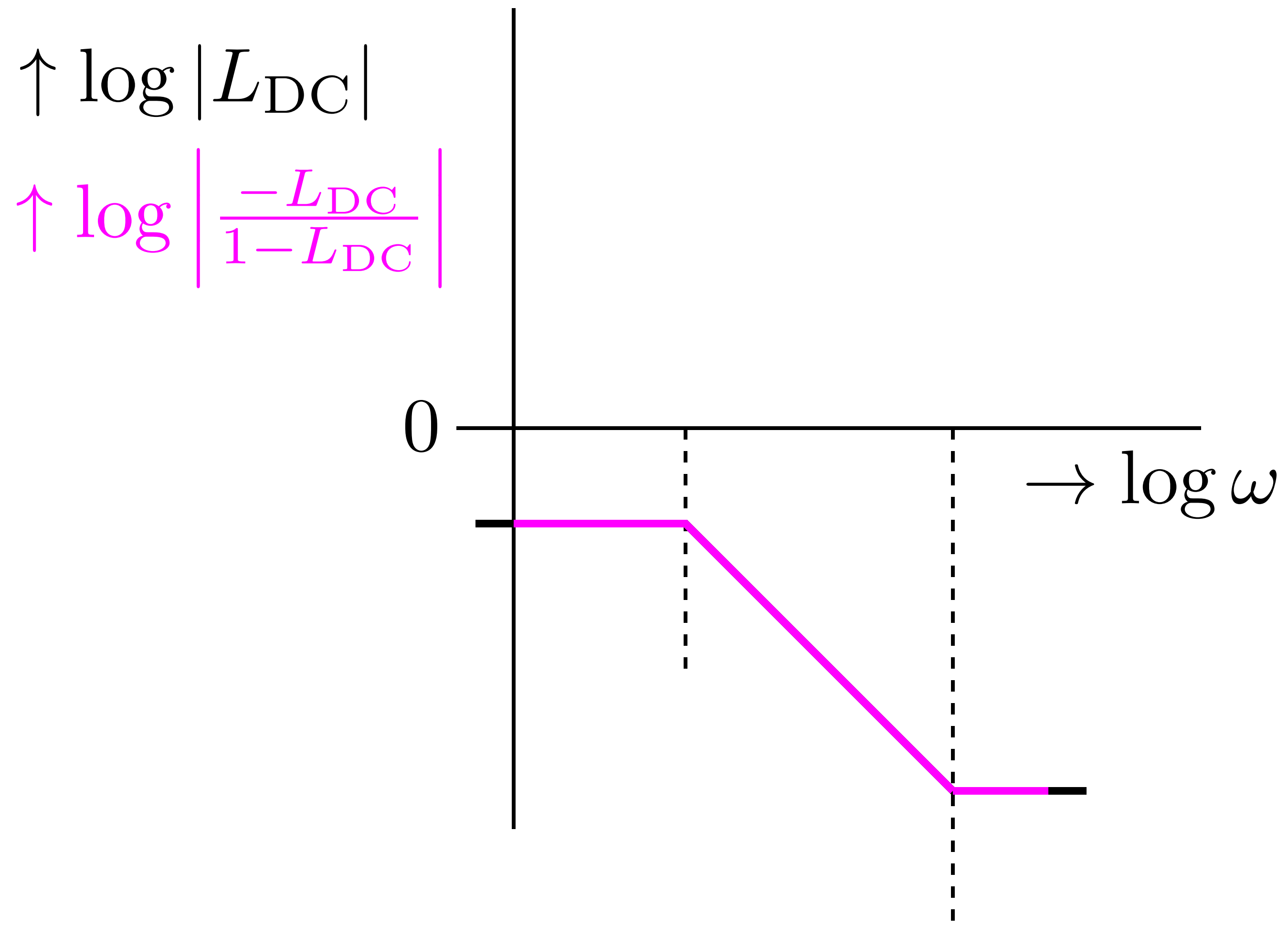

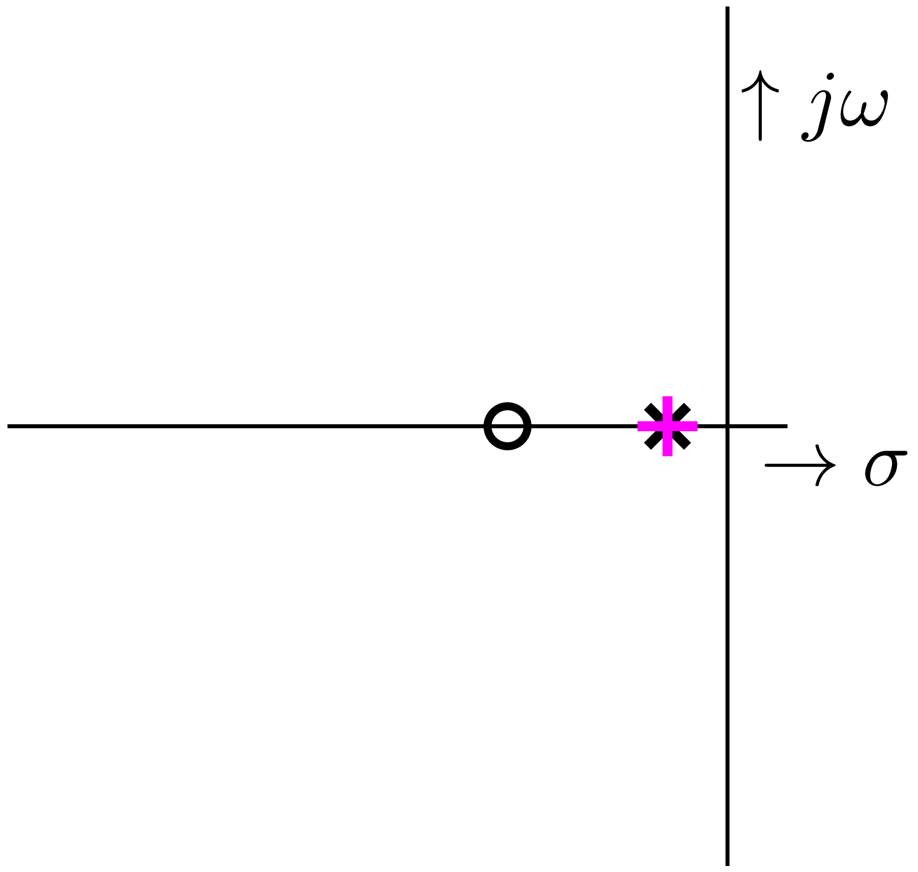

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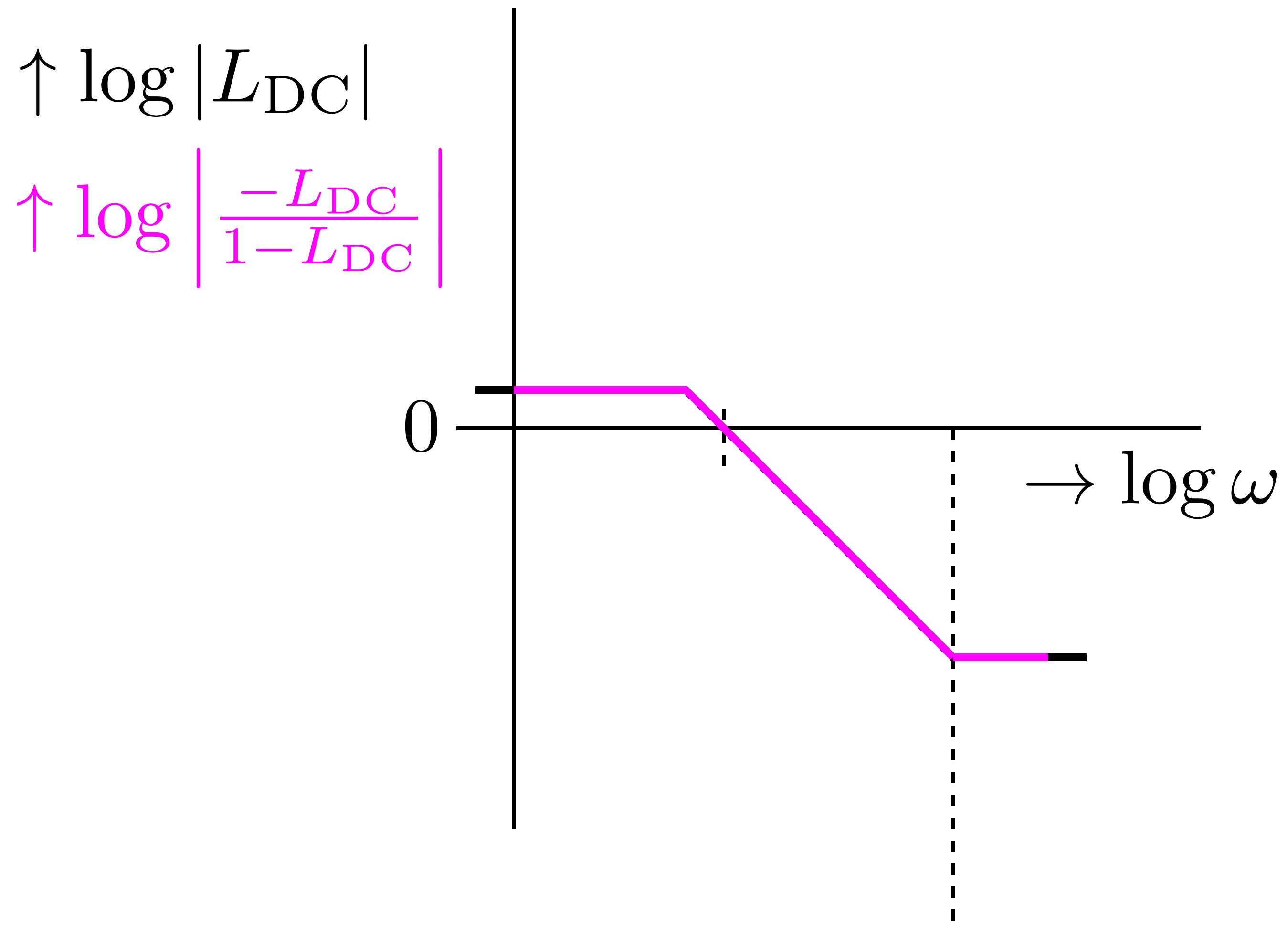

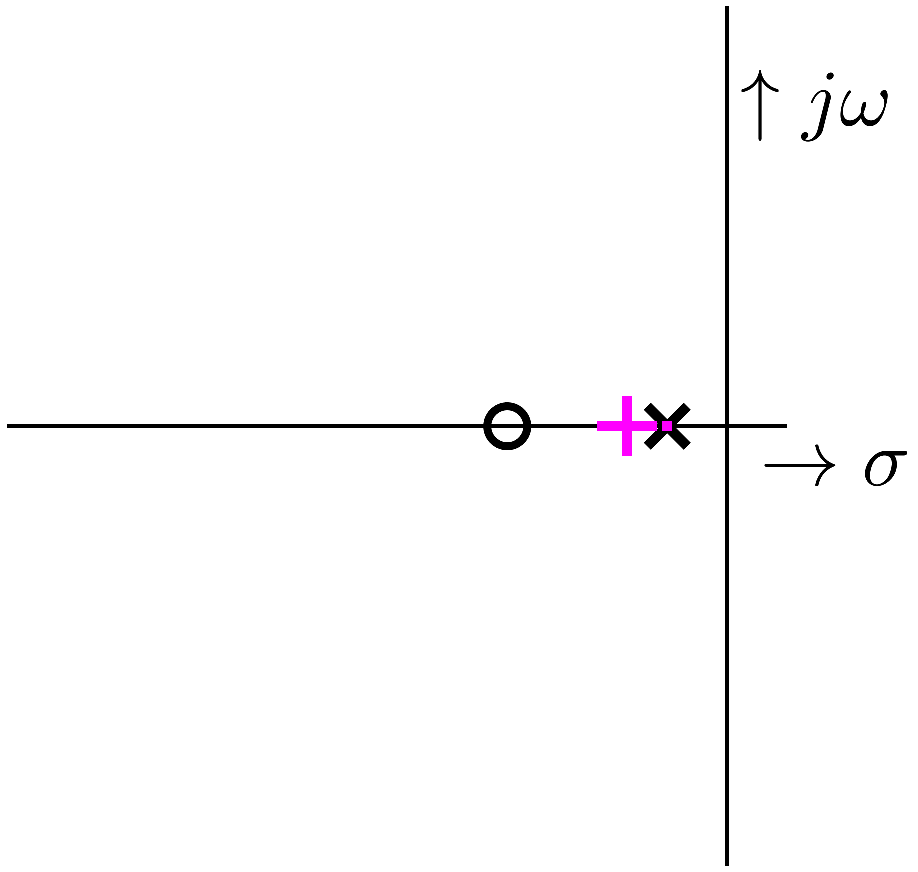

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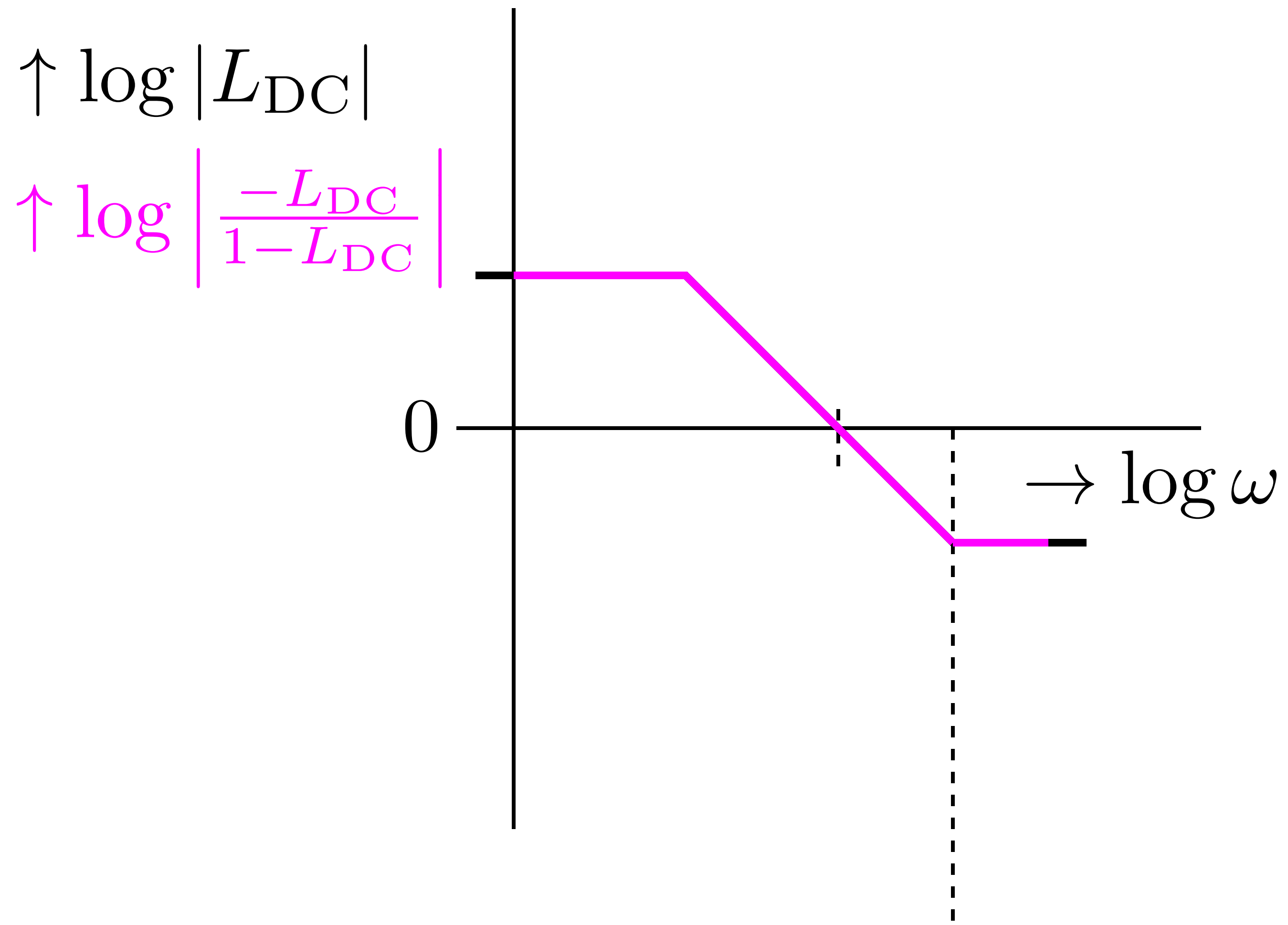

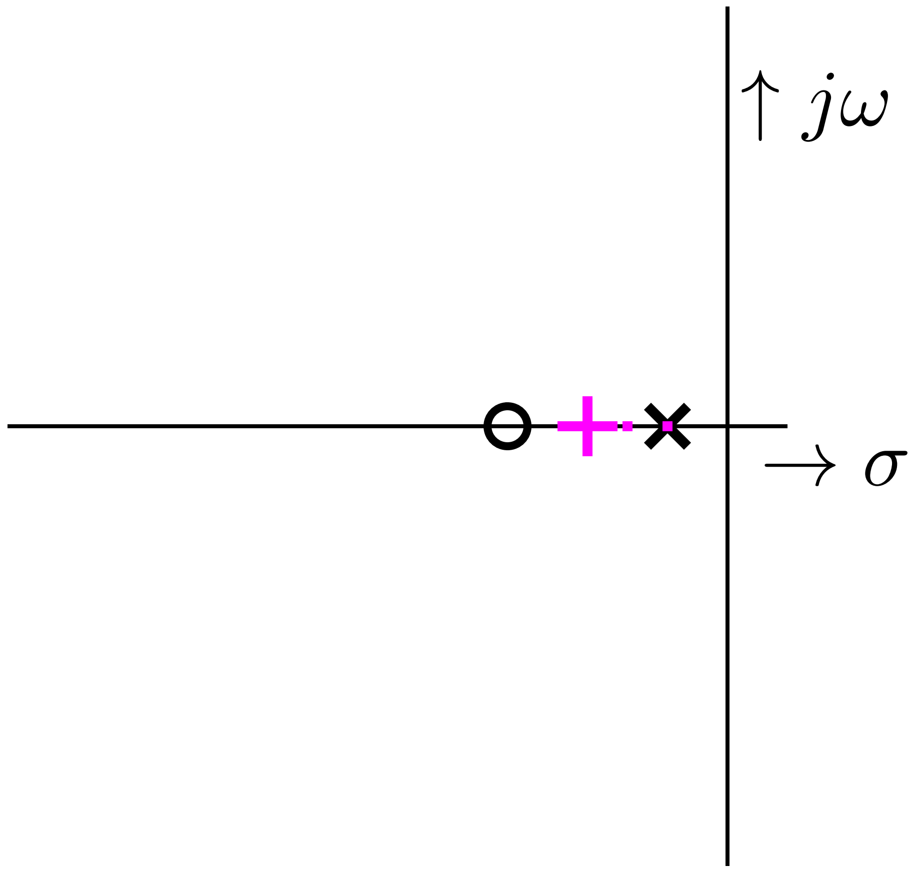

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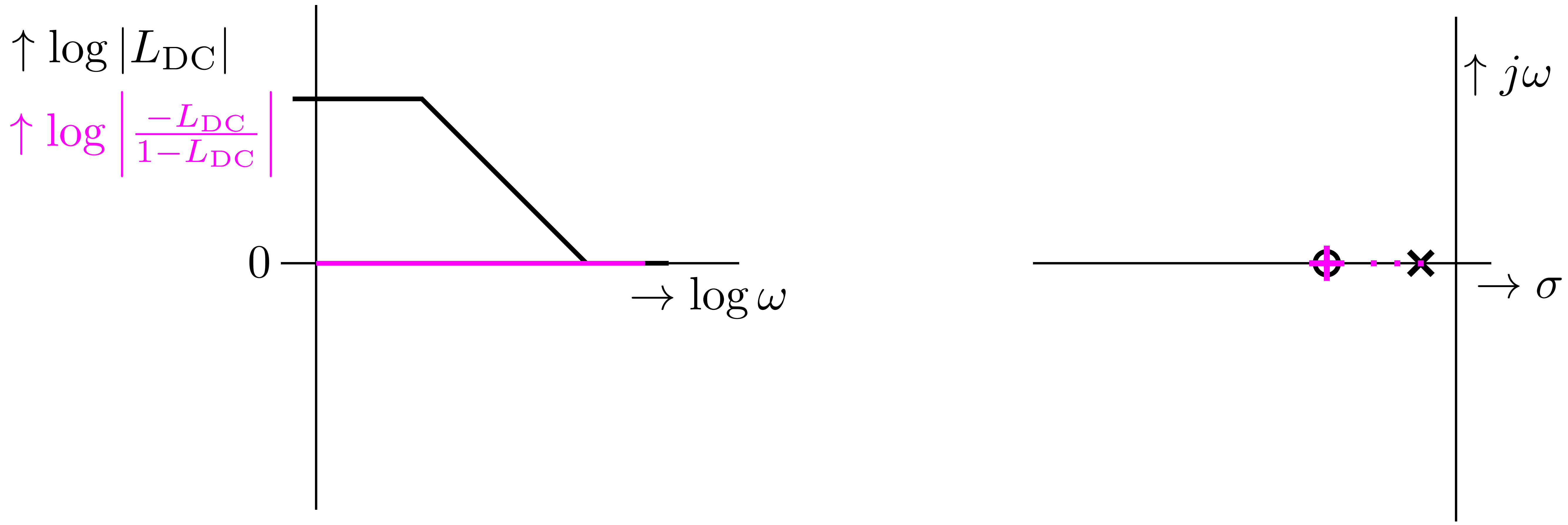

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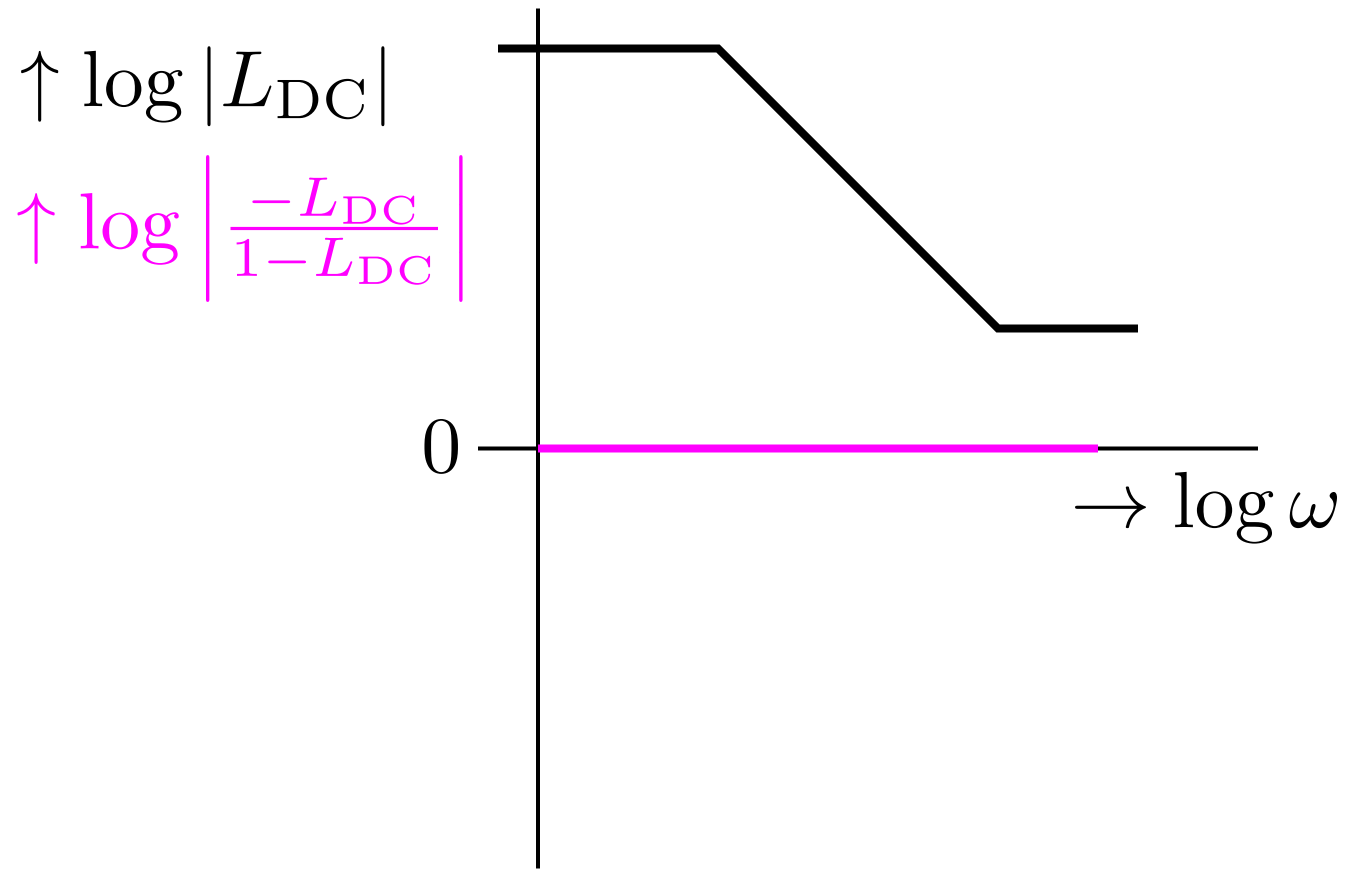

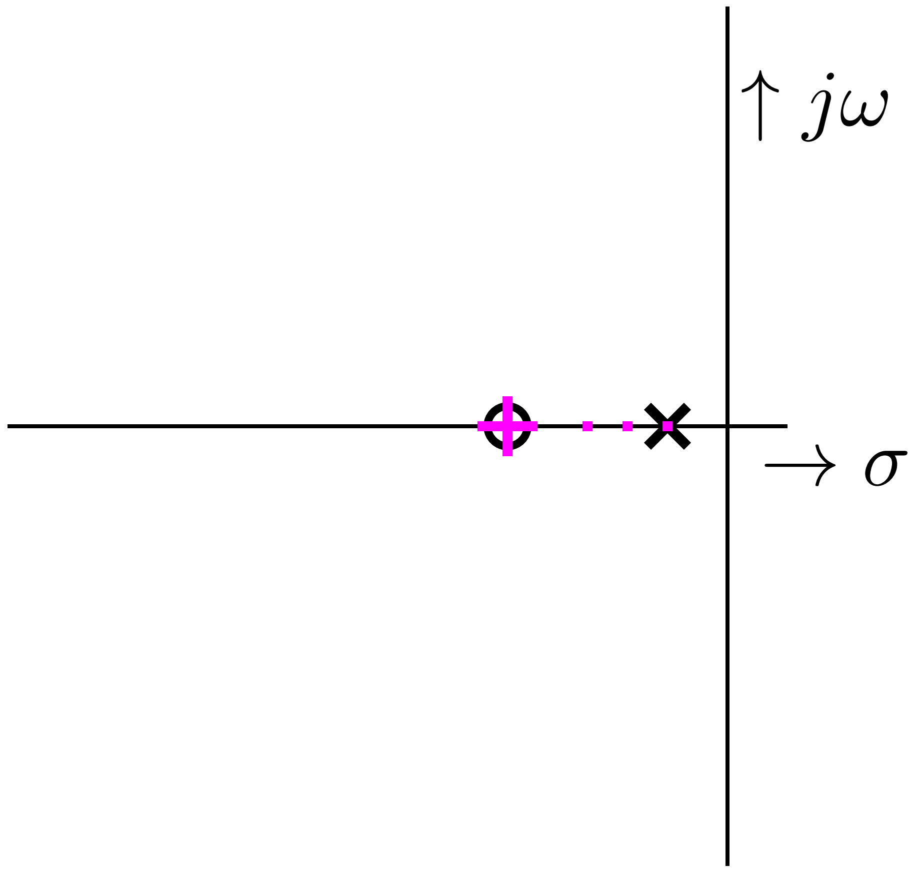

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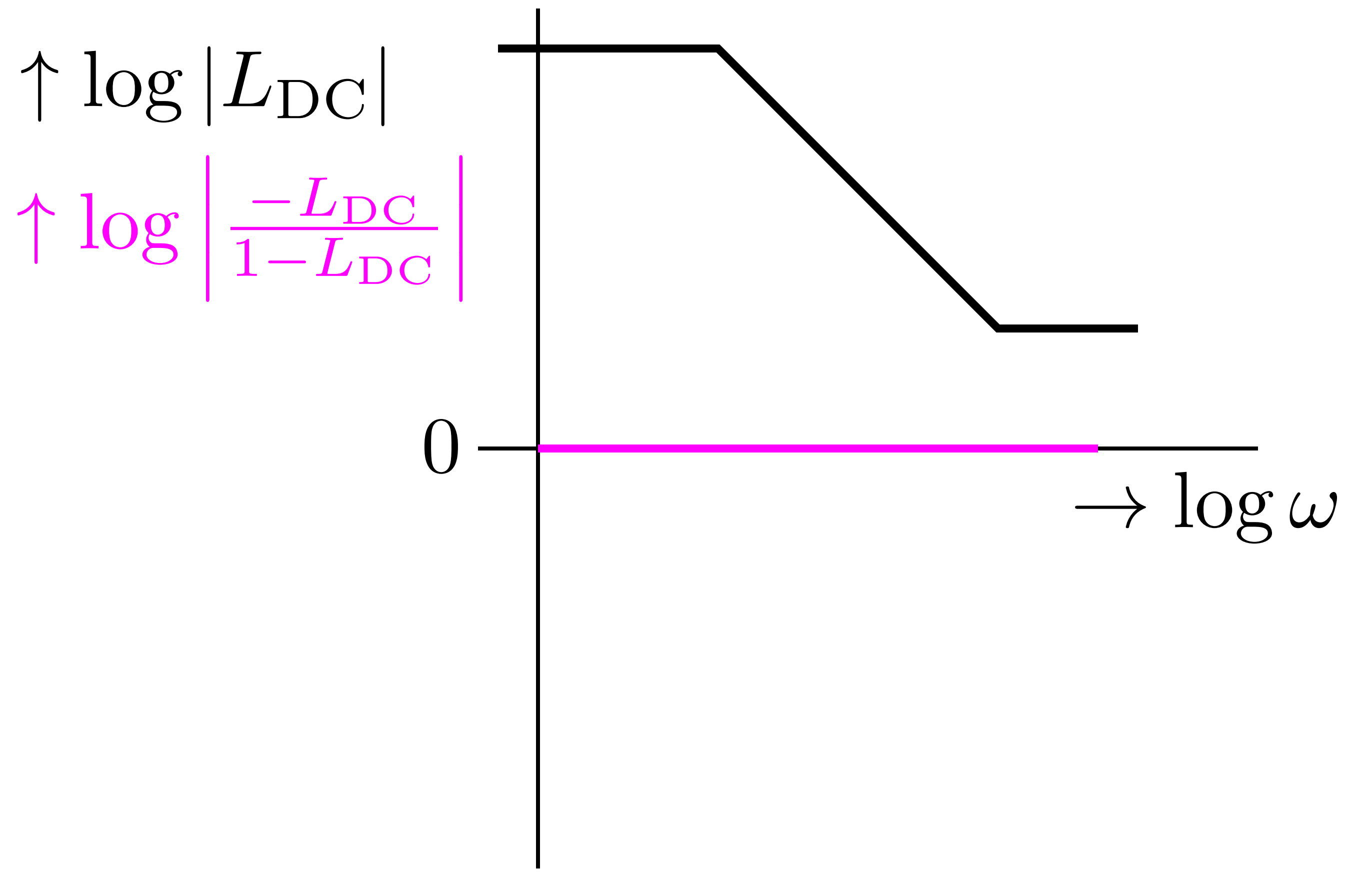

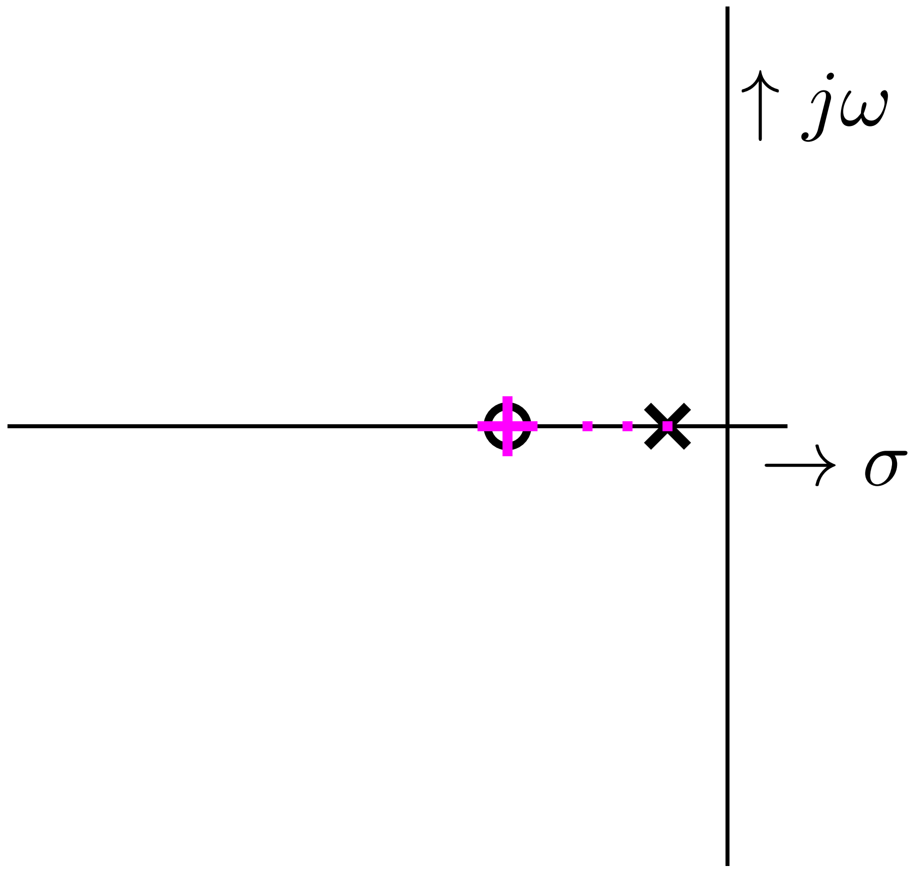

Note: pole only drops on the zero if DC loop gain is infinite!

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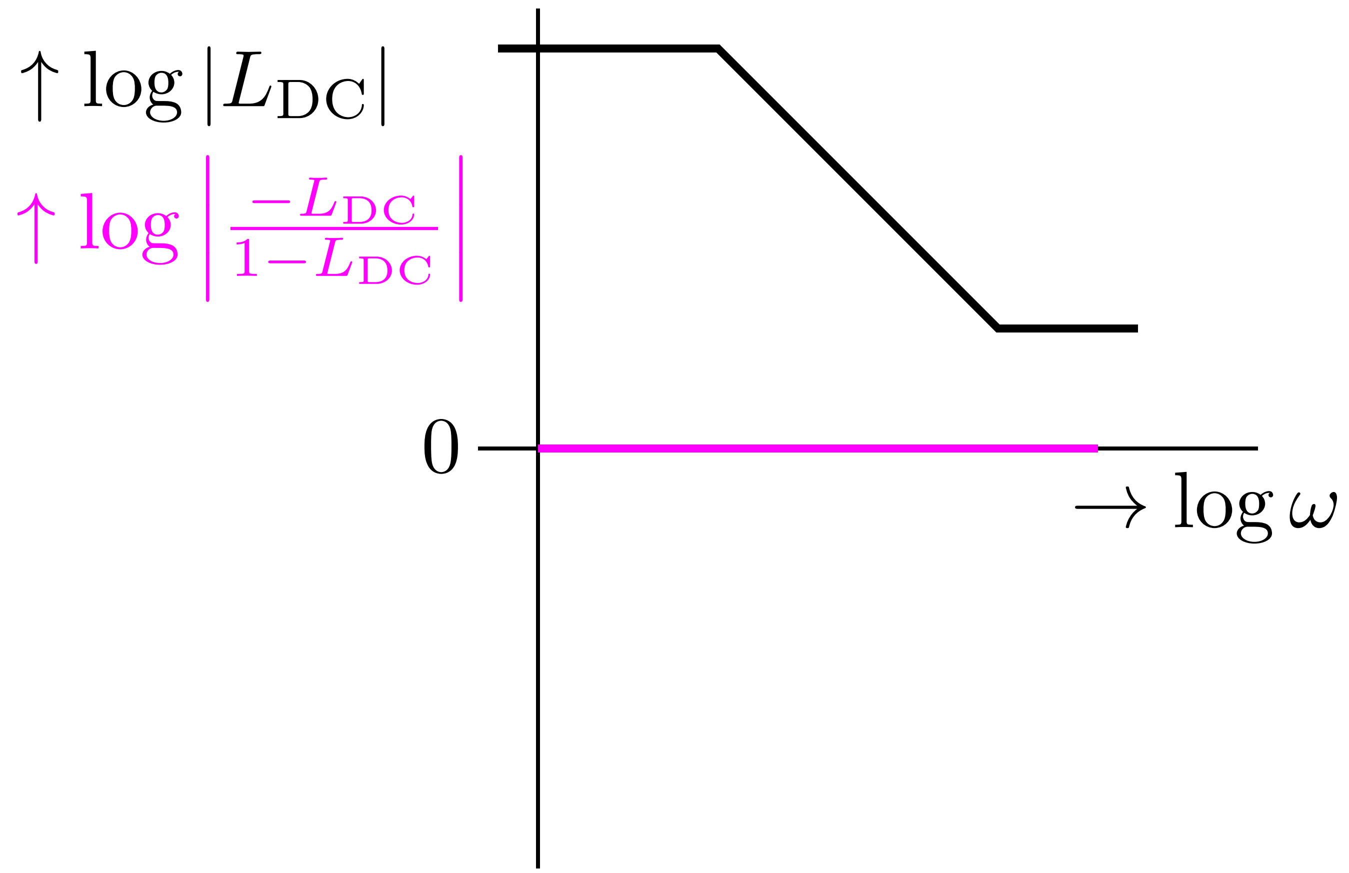

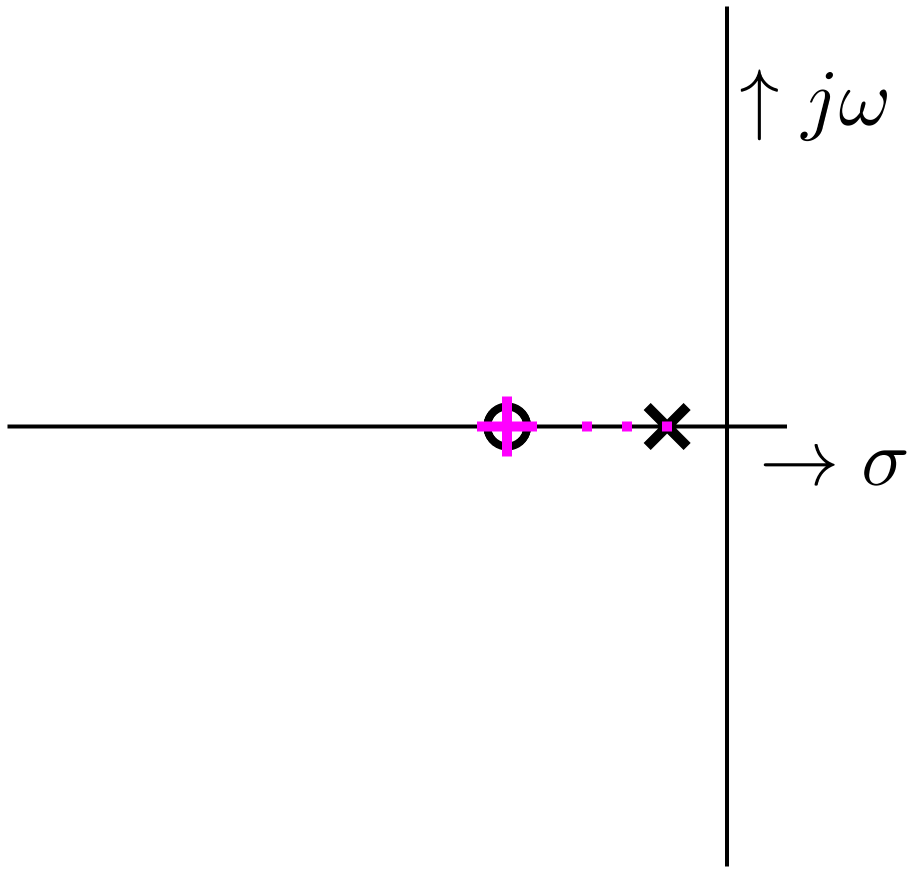

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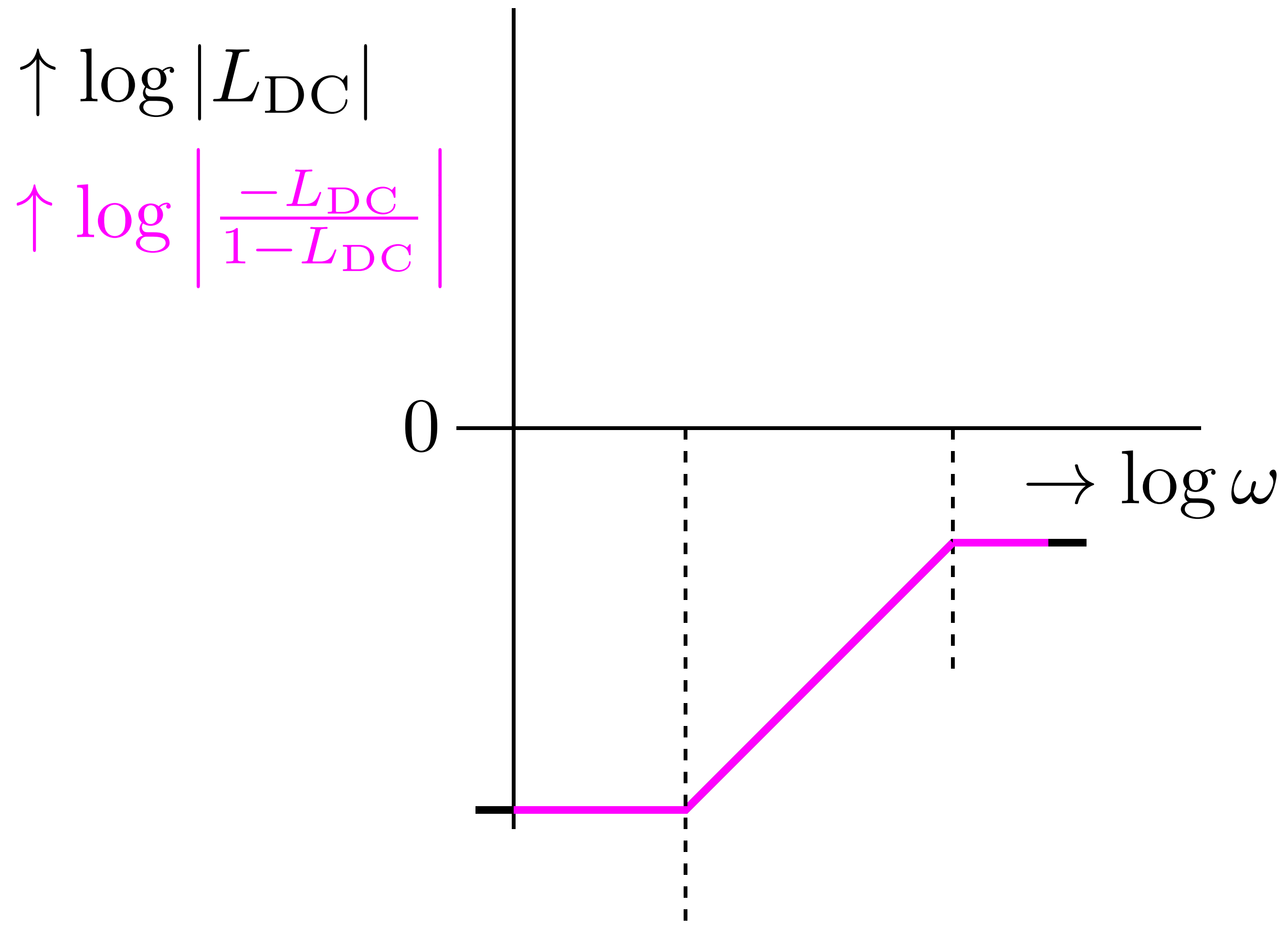
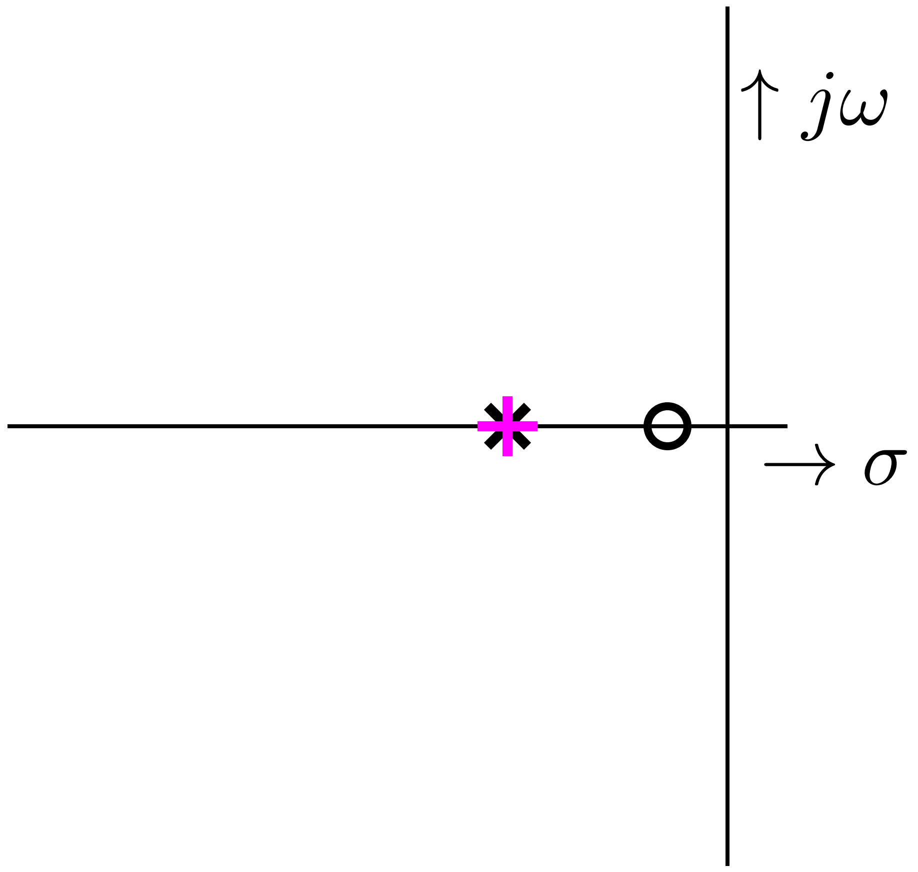
Root locus first order with zero right





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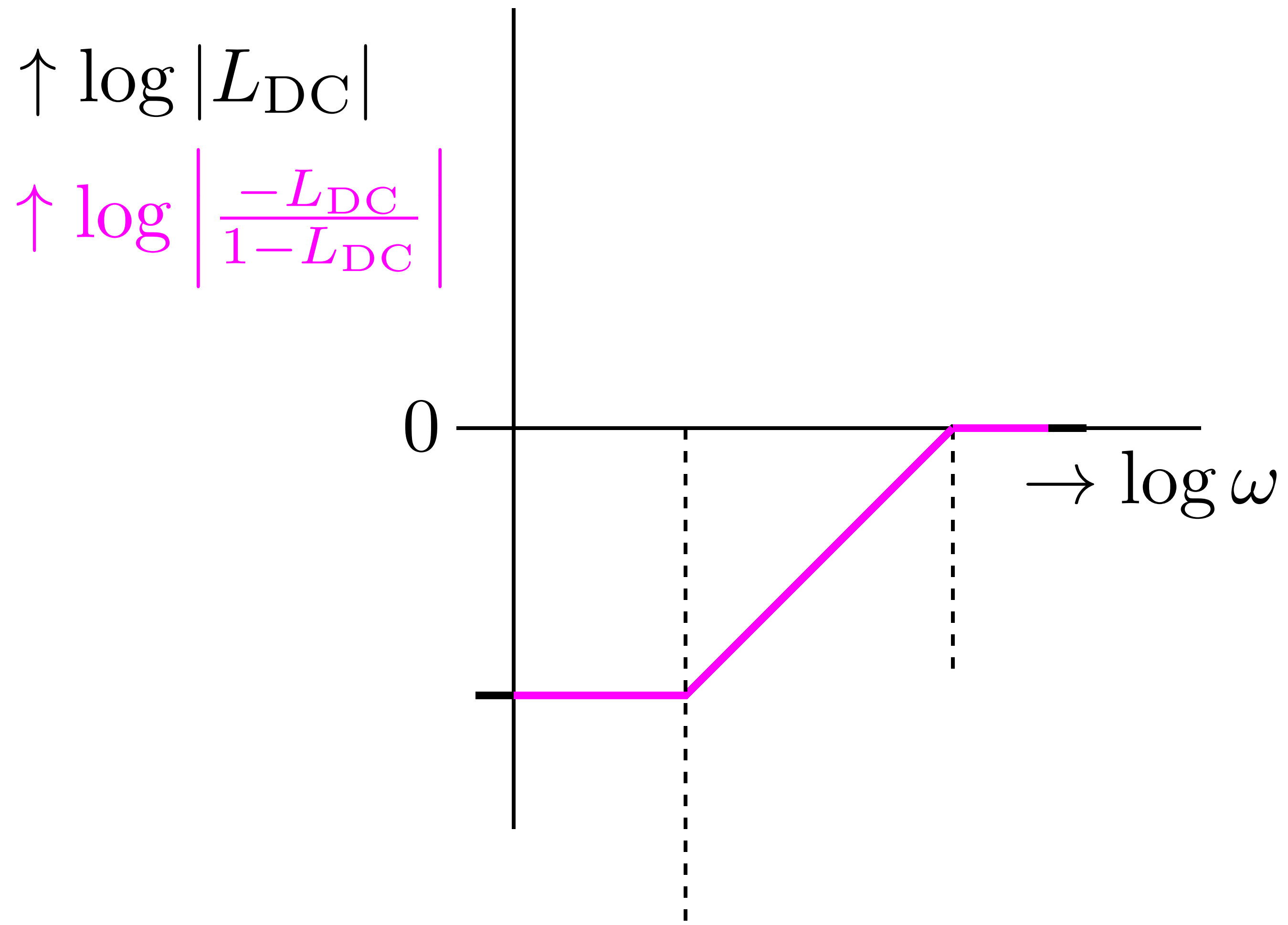

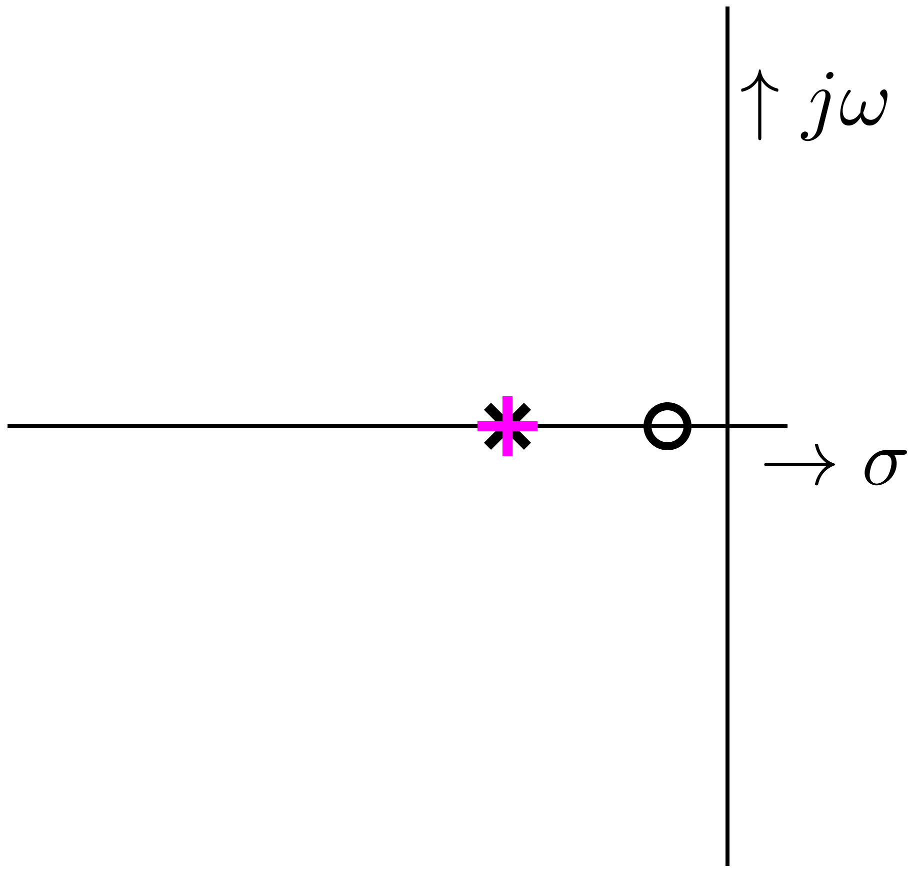

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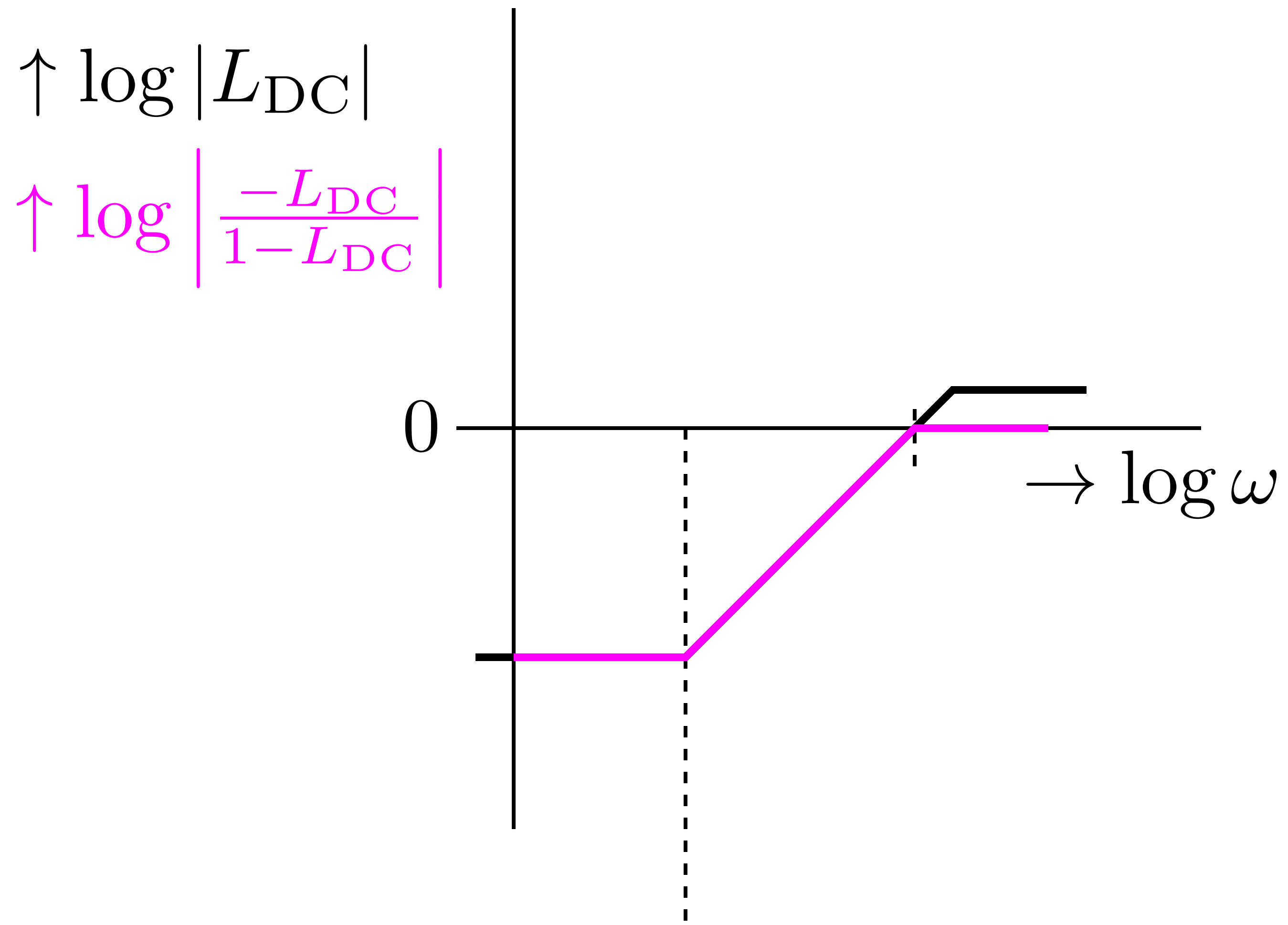

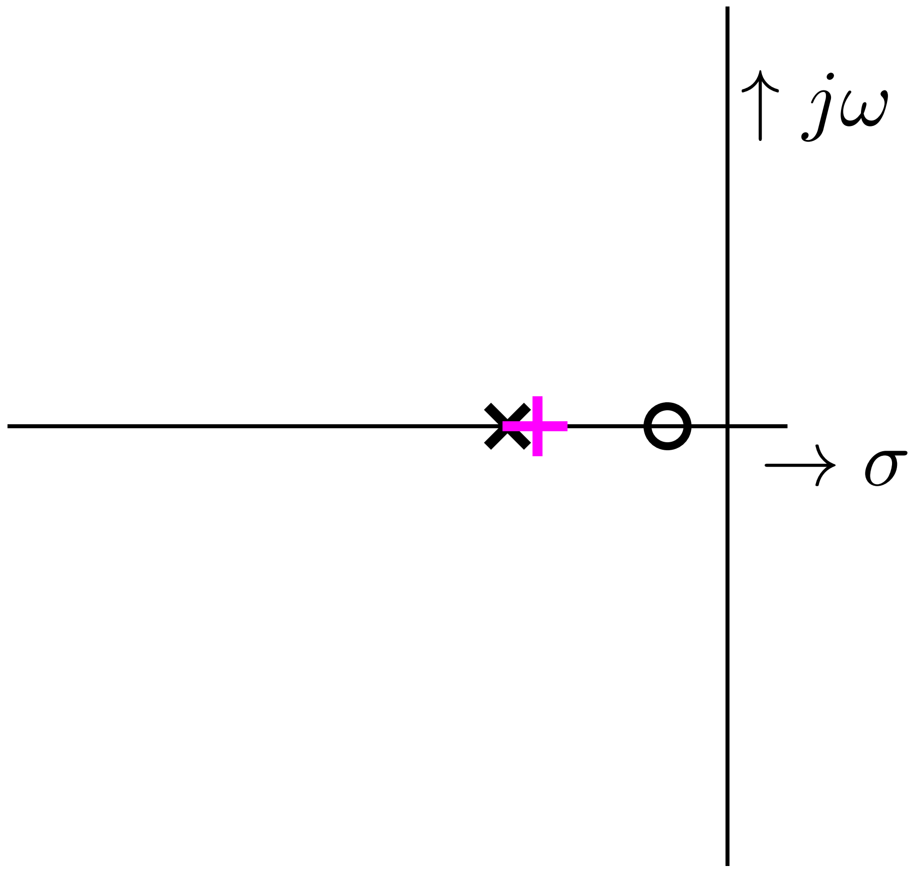

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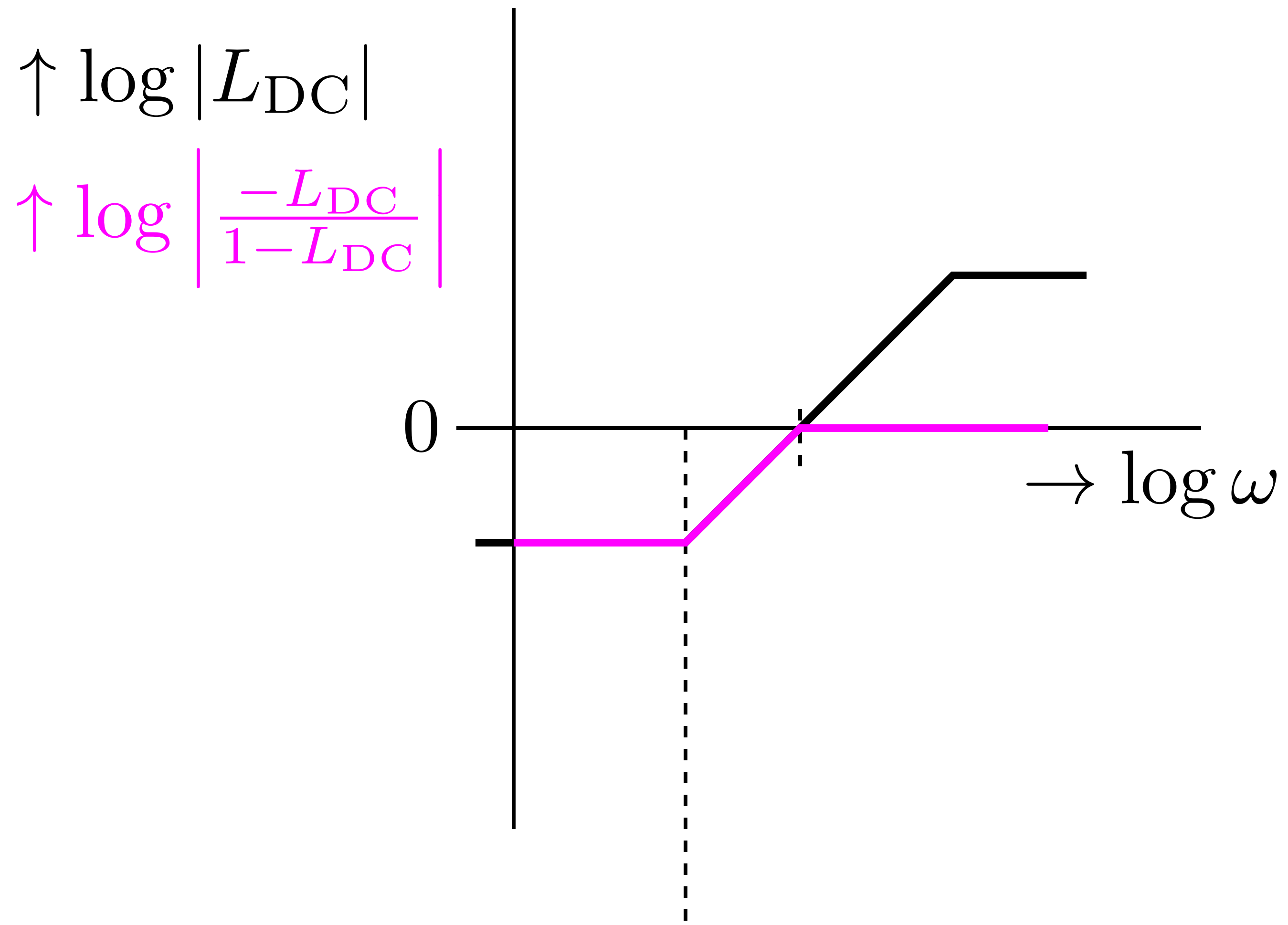
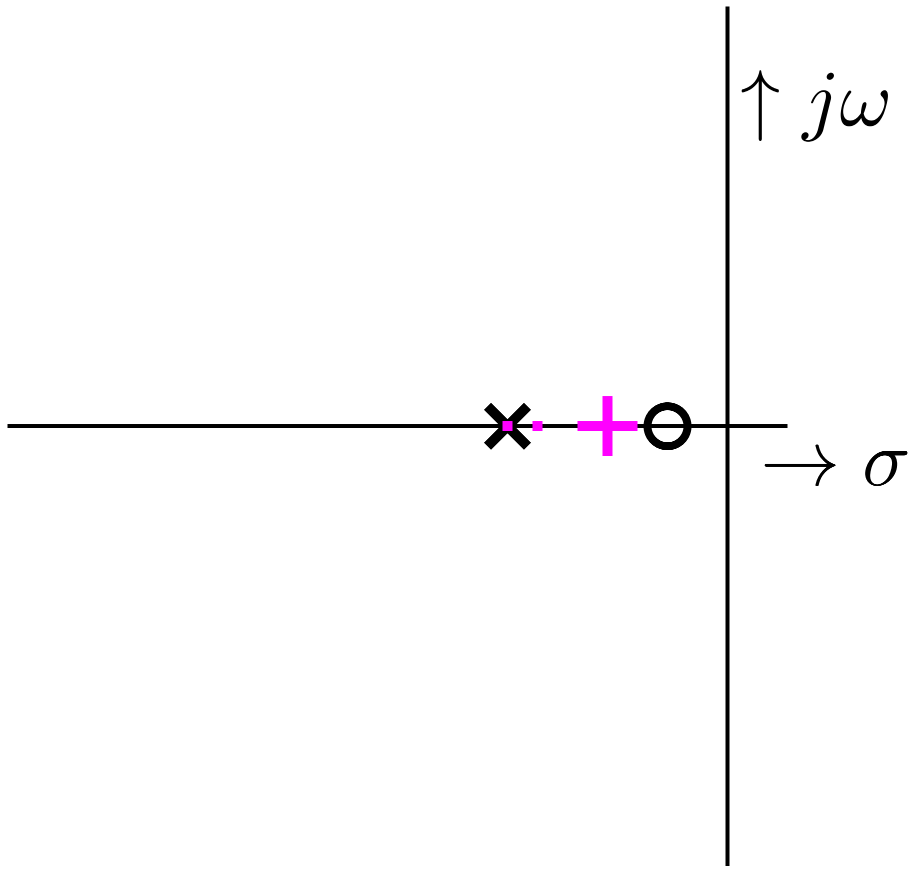
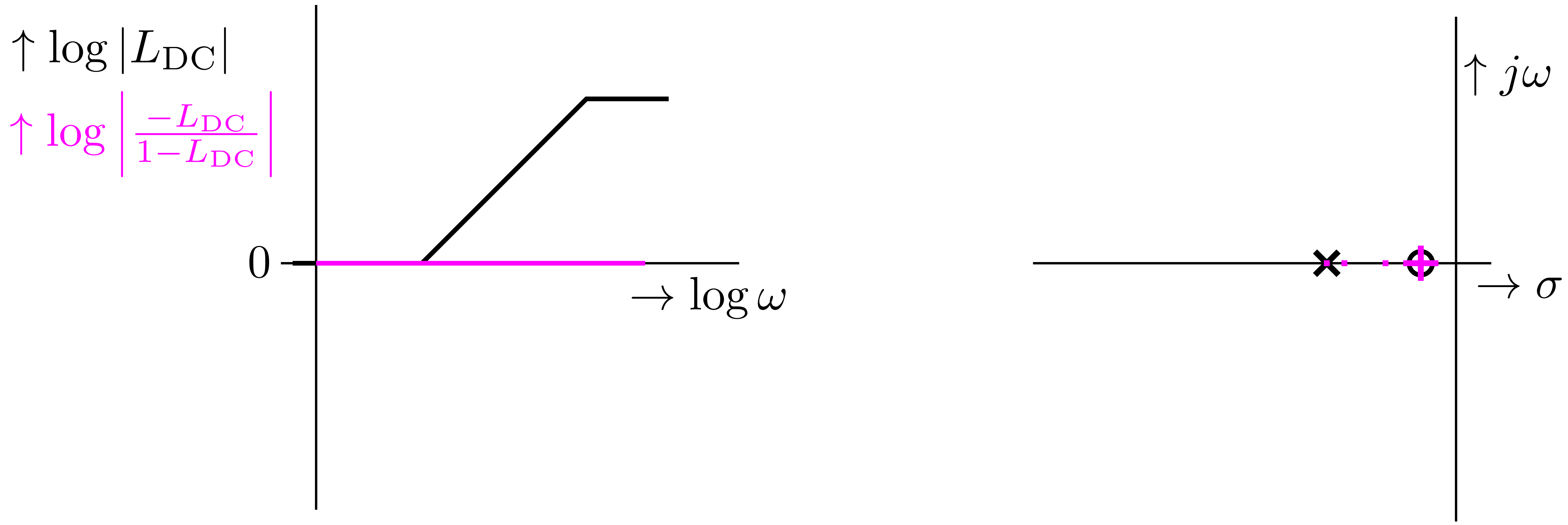
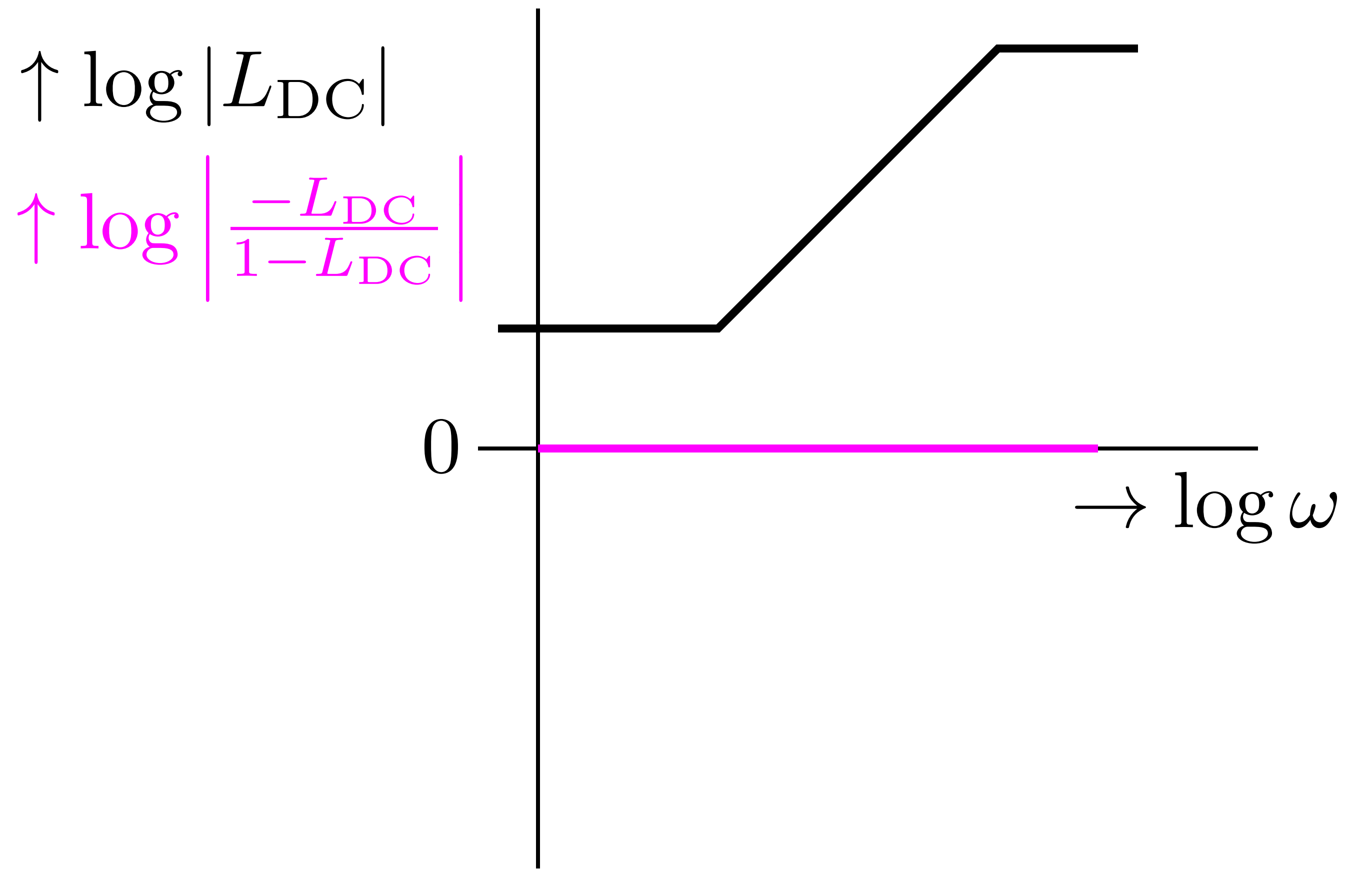


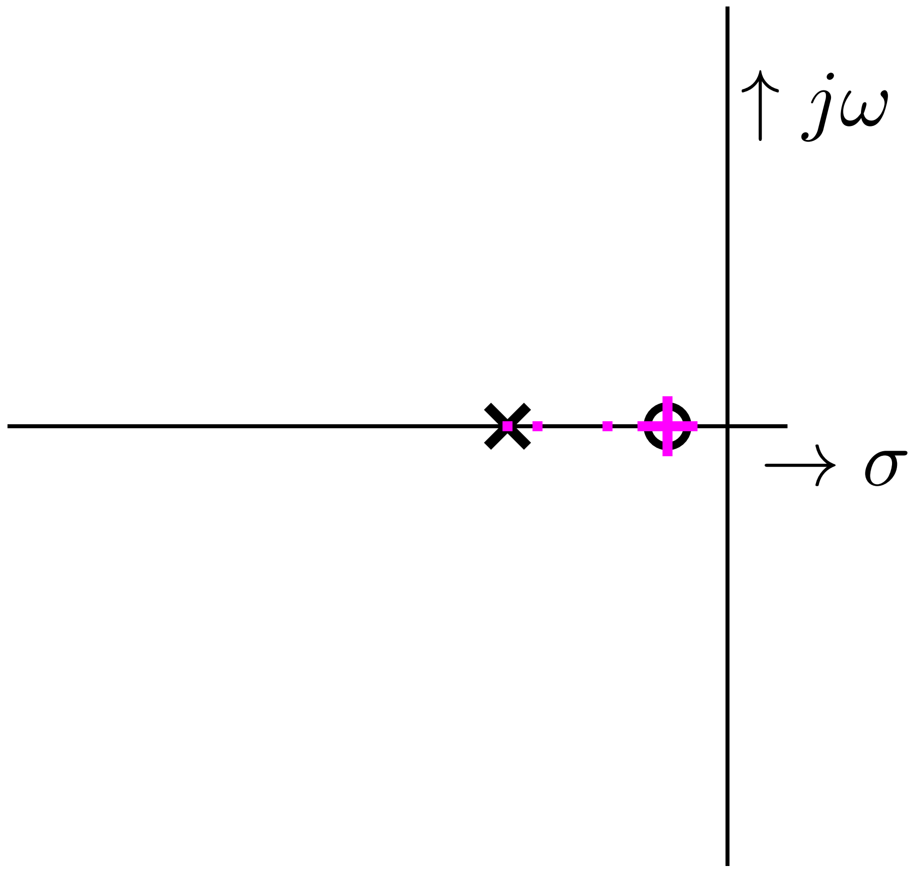


Root locus first order with zero right

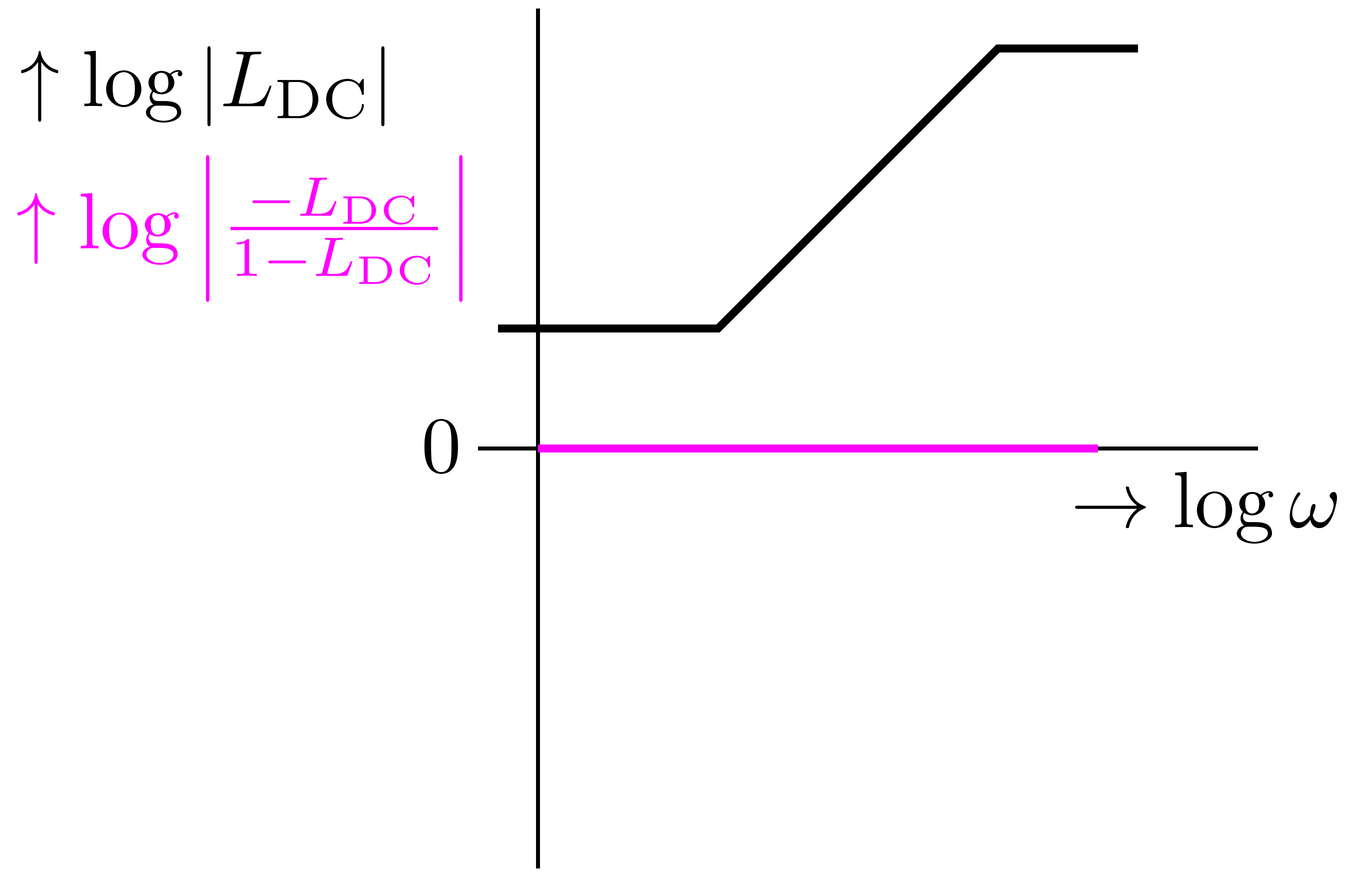


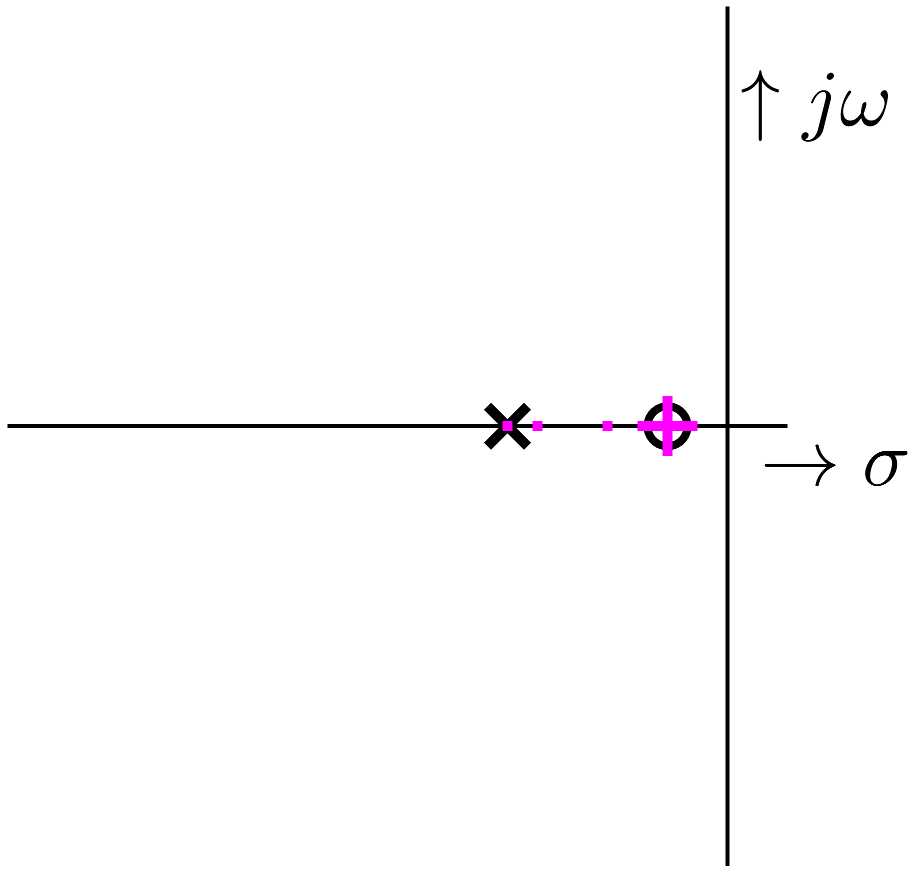
Root locus first order with zero right





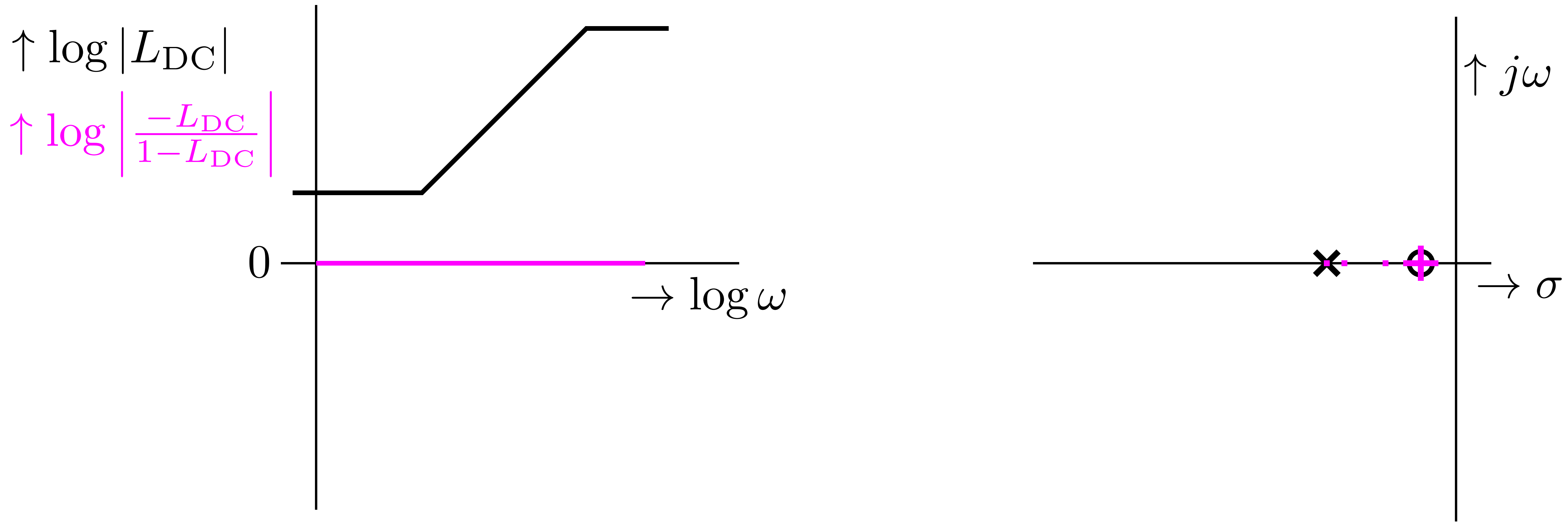
Root locus first order with zero right





Note: pole only drops on the zero if DC loop gain is infinite!

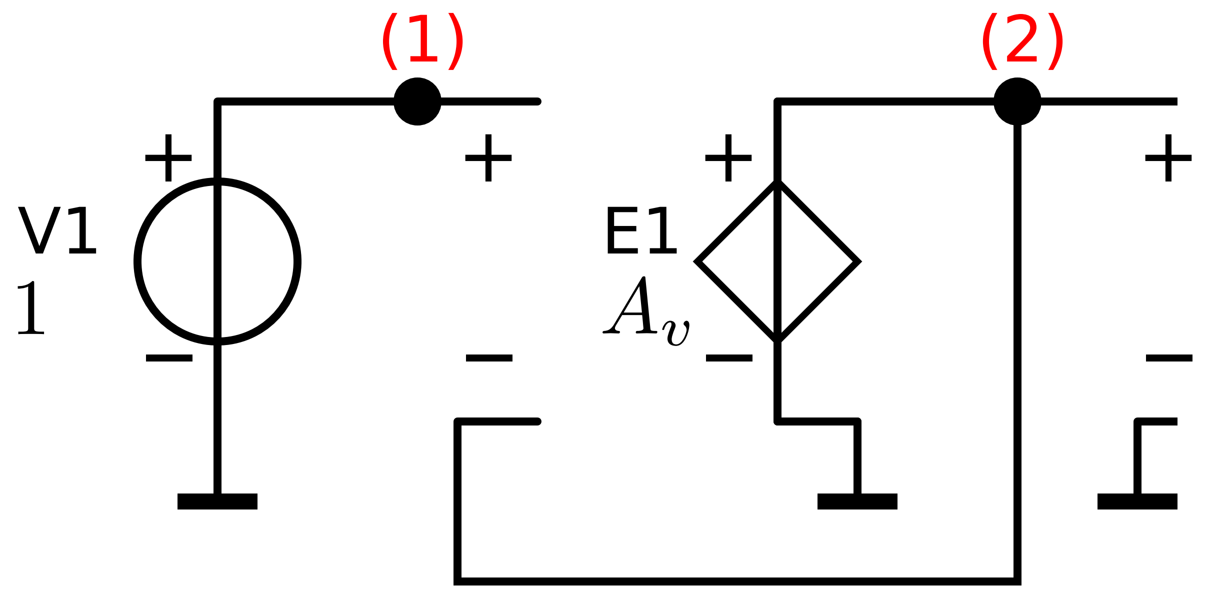
Root locus first order with zero right



Note: pole only drops on the zero if DC loop gain is infinite!

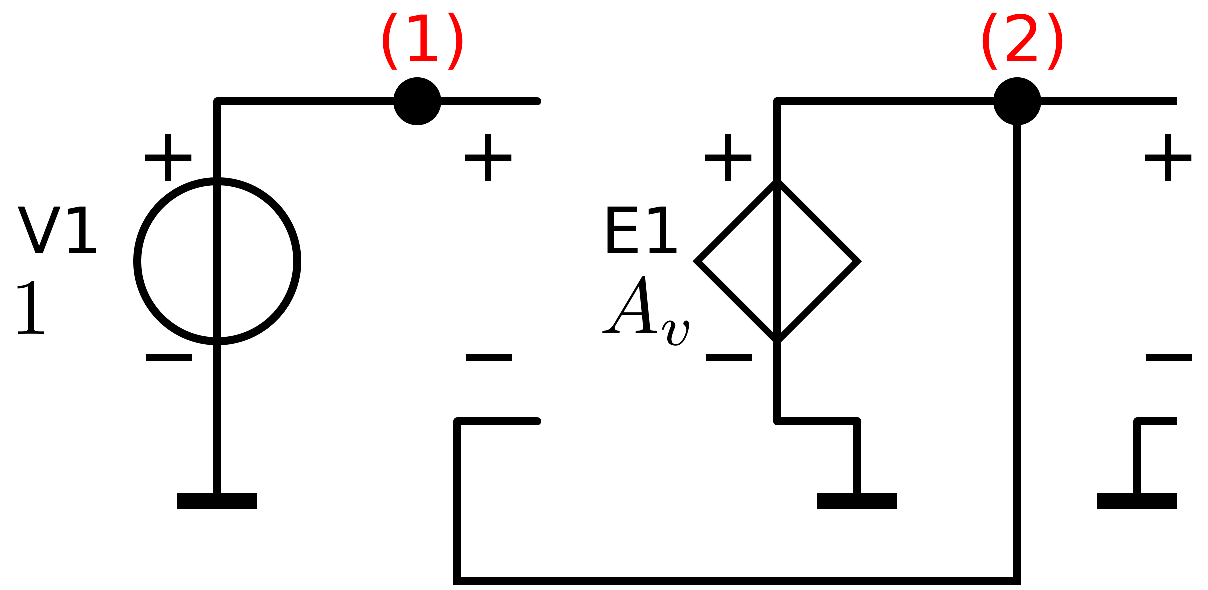
Root locus SLiCAP

Circuit for plotting root locus



Root locus SLiCAP

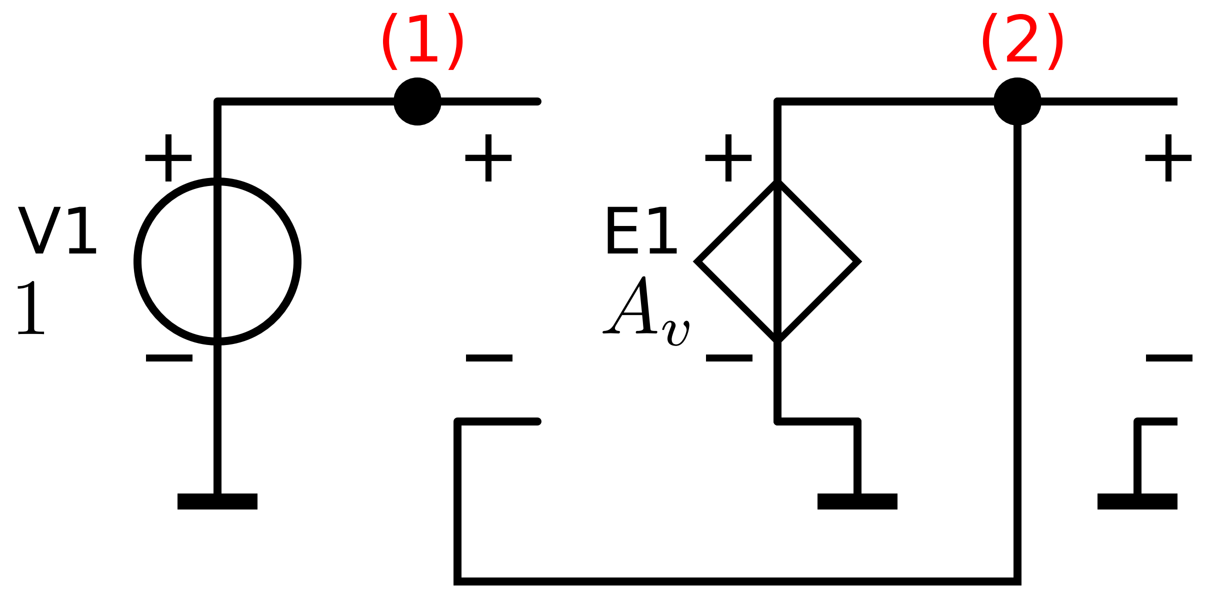
Circuit for plotting root locus



E1 = loop gain reference

Root locus SLiCAP

Circuit for plotting root locus

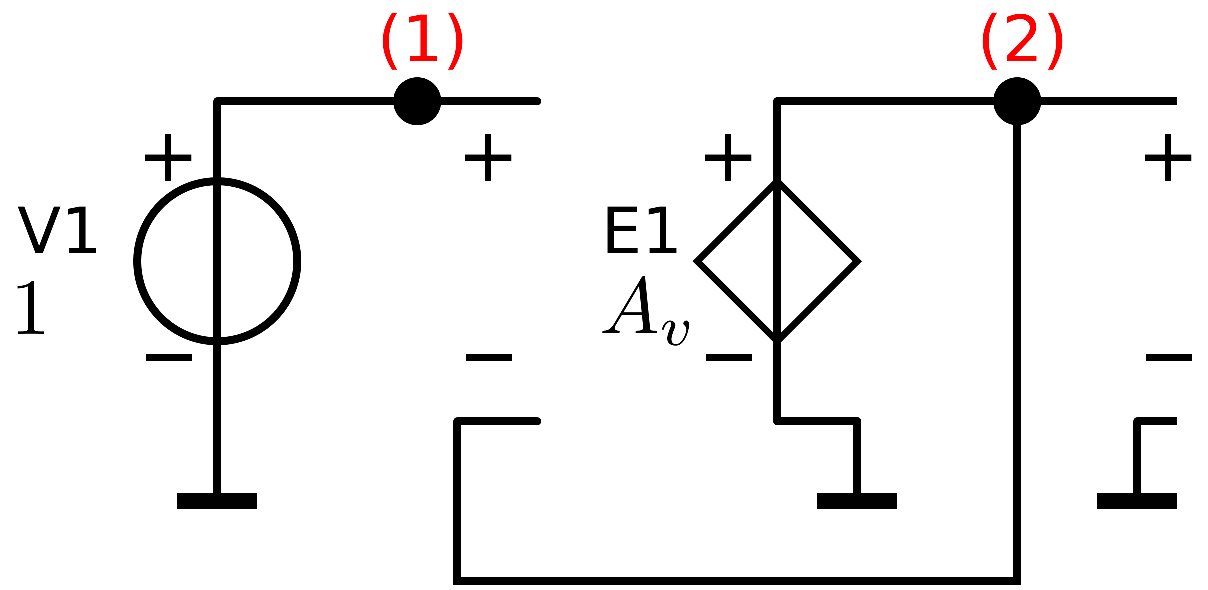


E1 = loop gain reference

Loop gain equals voltage gain of E1

Root locus SLiCAP

Circuit for plotting root locus



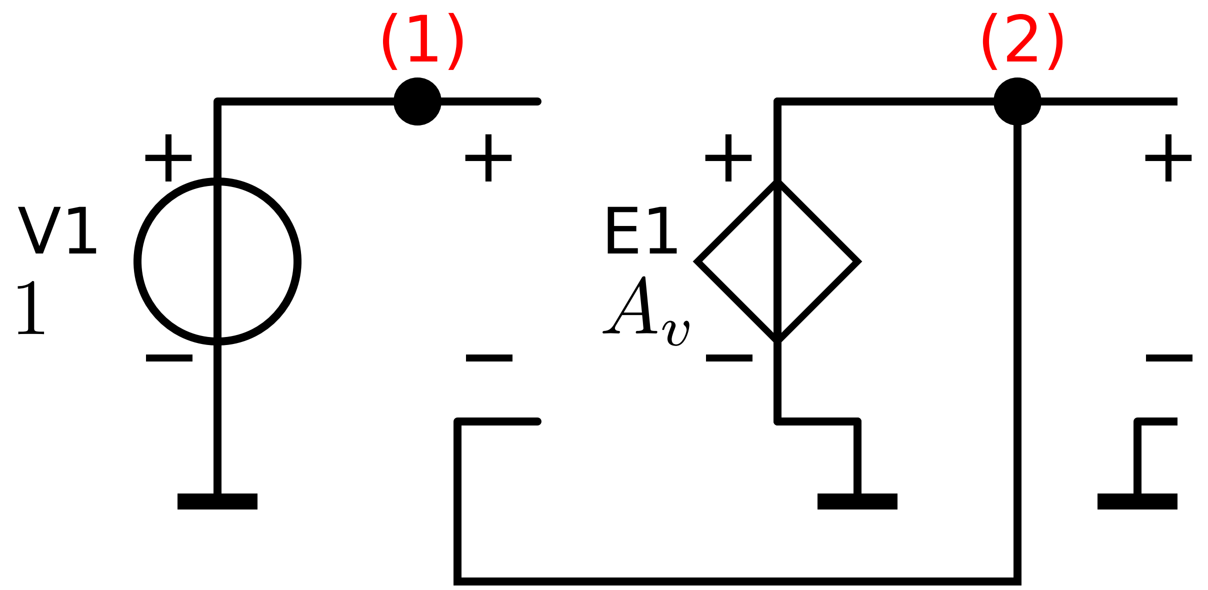
E1 = loop gain reference

Loop gain equals voltage gain of E1

Transfer of E1 has DC gain, poles and zeros

Root locus SLiCAP

Circuit for plotting root locus

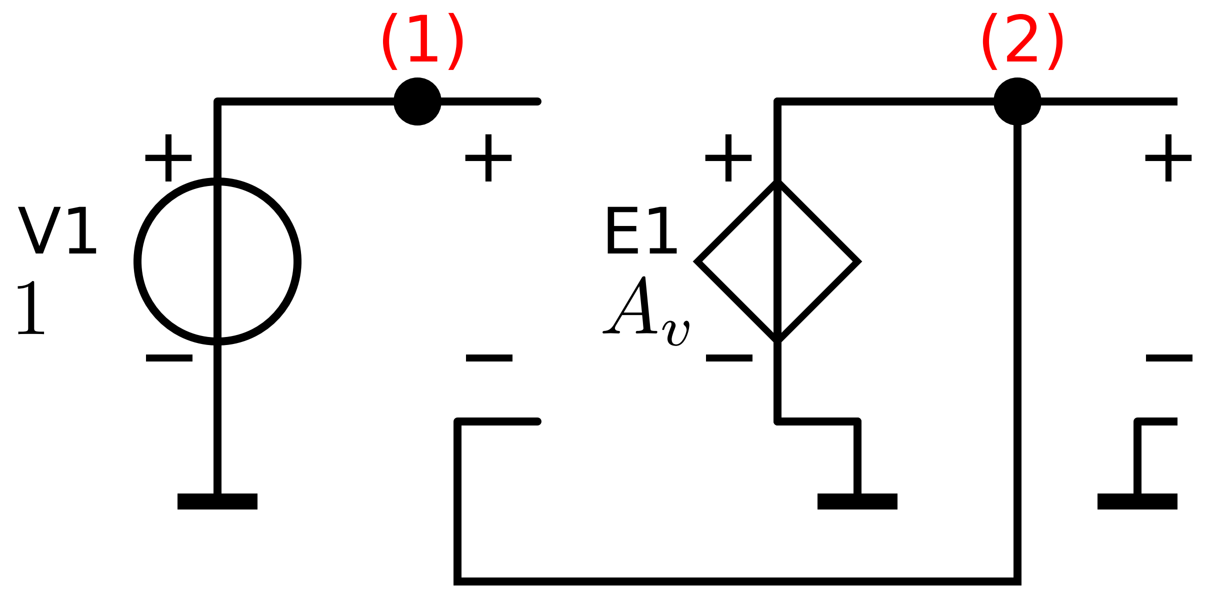


Root locus plot:

E1 = loop gain reference
Loop gain equals voltage gain of E1
Transfer of E1 has DC gain, poles and zeros

Root locus SLiCAP

Circuit for plotting root locus



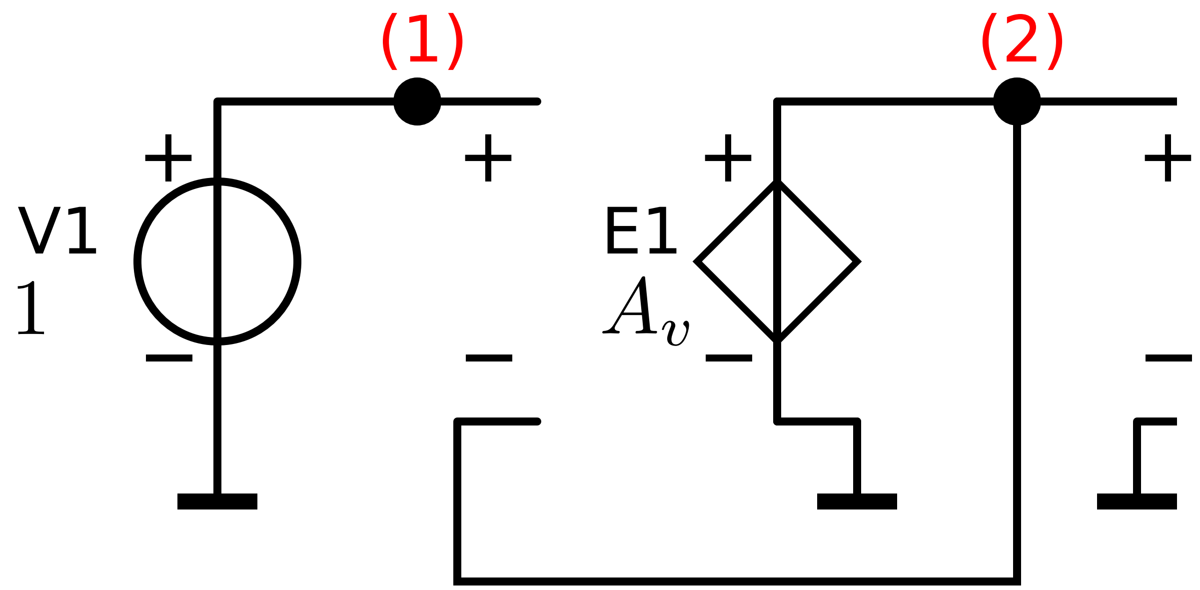
E1 = loop gain reference
Loop gain equals voltage gain of E1
Transfer of E1 has DC gain, poles and zeros

Root locus plot:

1. Poles of the loop gain

Root locus SLiCAP

Circuit for plotting root locus



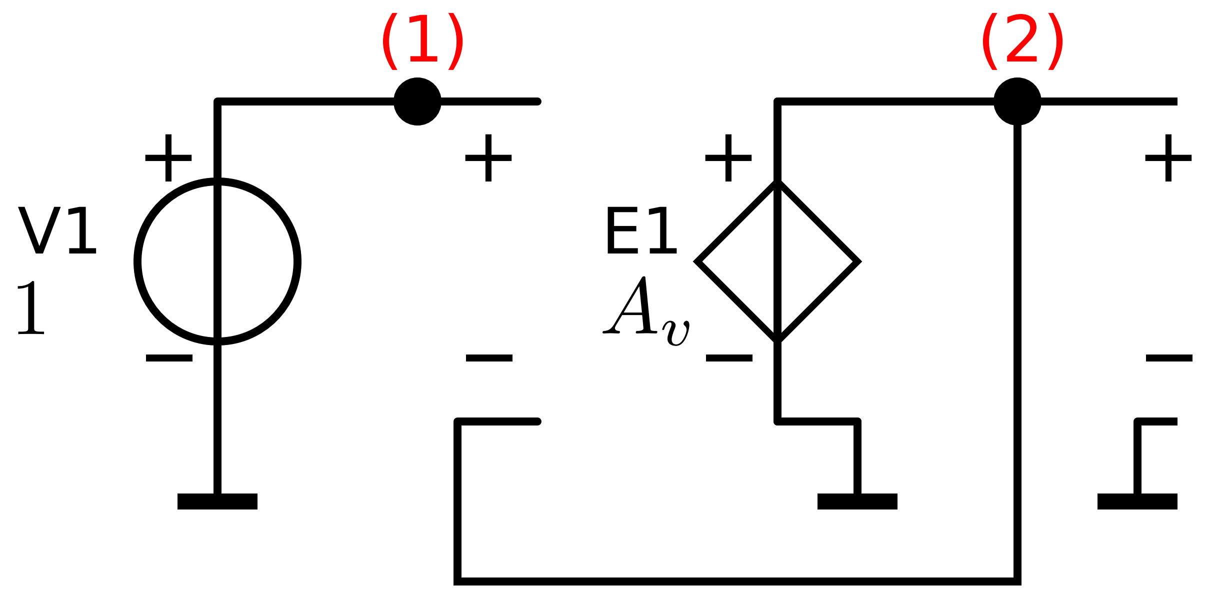
E1 = loop gain reference
Loop gain equals voltage gain of E1
Transfer of E1 has DC gain, poles and zeros

Root locus plot:

1. Poles of the loop gain
2. Zeros of the loop gain

Root locus SLiCAP

Circuit for plotting root locus



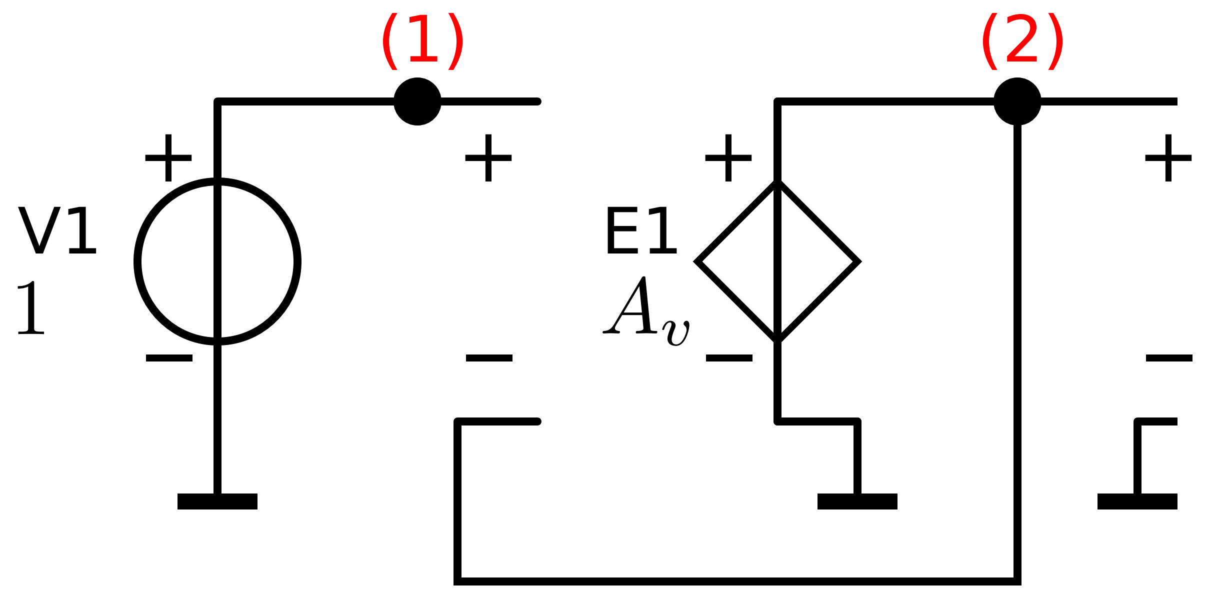
E1 = loop gain reference
Loop gain equals voltage gain of E1
Transfer of E1 has DC gain, poles and zeros

Root locus plot:

1. Poles of the loop gain
2. Zeros of the loop gain
3. Poles of the servo function while stepping the DC gain of E1

Root locus SLiCAP

Circuit for plotting root locus



E1 = loop gain reference

Loop gain equals voltage gain of E1

Transfer of E1 has DC gain, poles and zeros

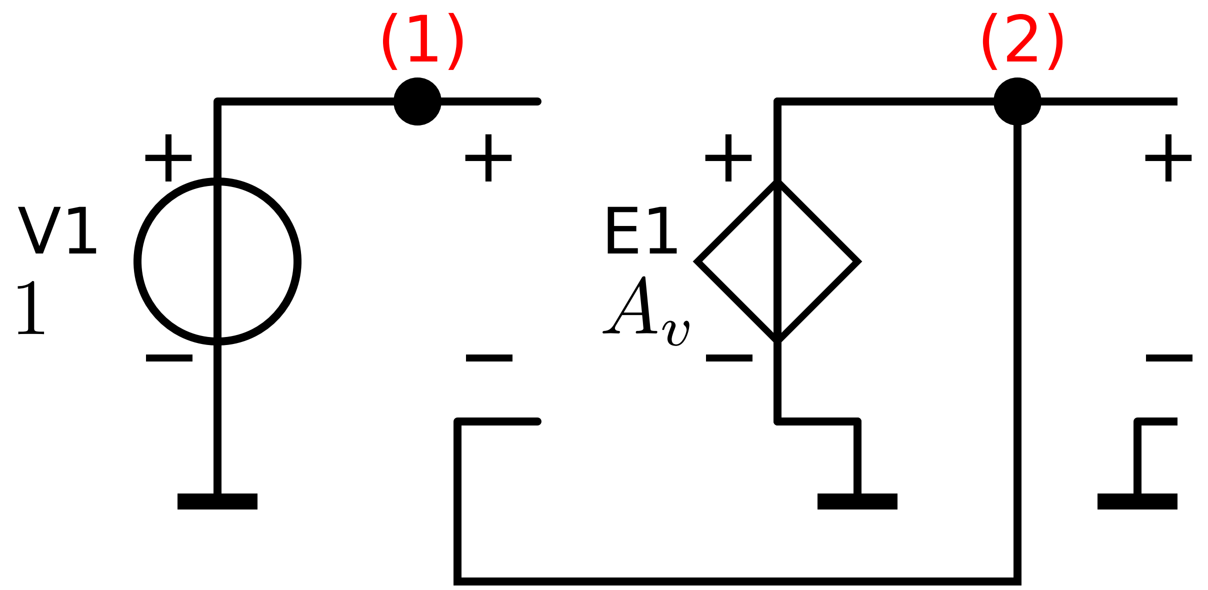
Root locus plot:

1. Poles of the loop gain
2. Zeros of the loop gain
3. Poles of the servo function while stepping the DC gain of E1

See section 11.5.3

Root locus SLiCAP

Circuit for plotting root locus



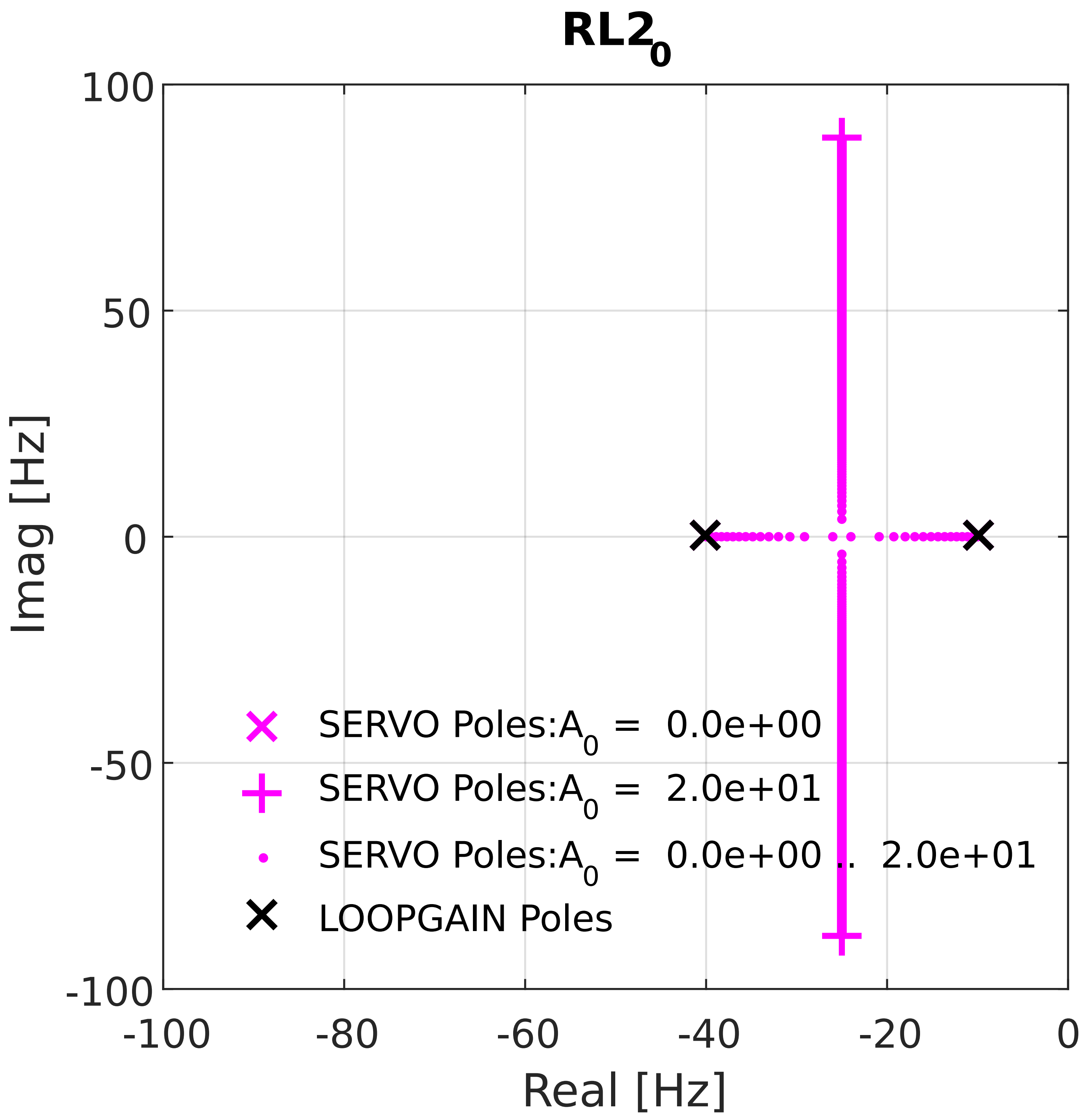
E1 = loop gain reference
Loop gain equals voltage gain of E1
Transfer of E1 has DC gain, poles and zeros

Root locus plot:

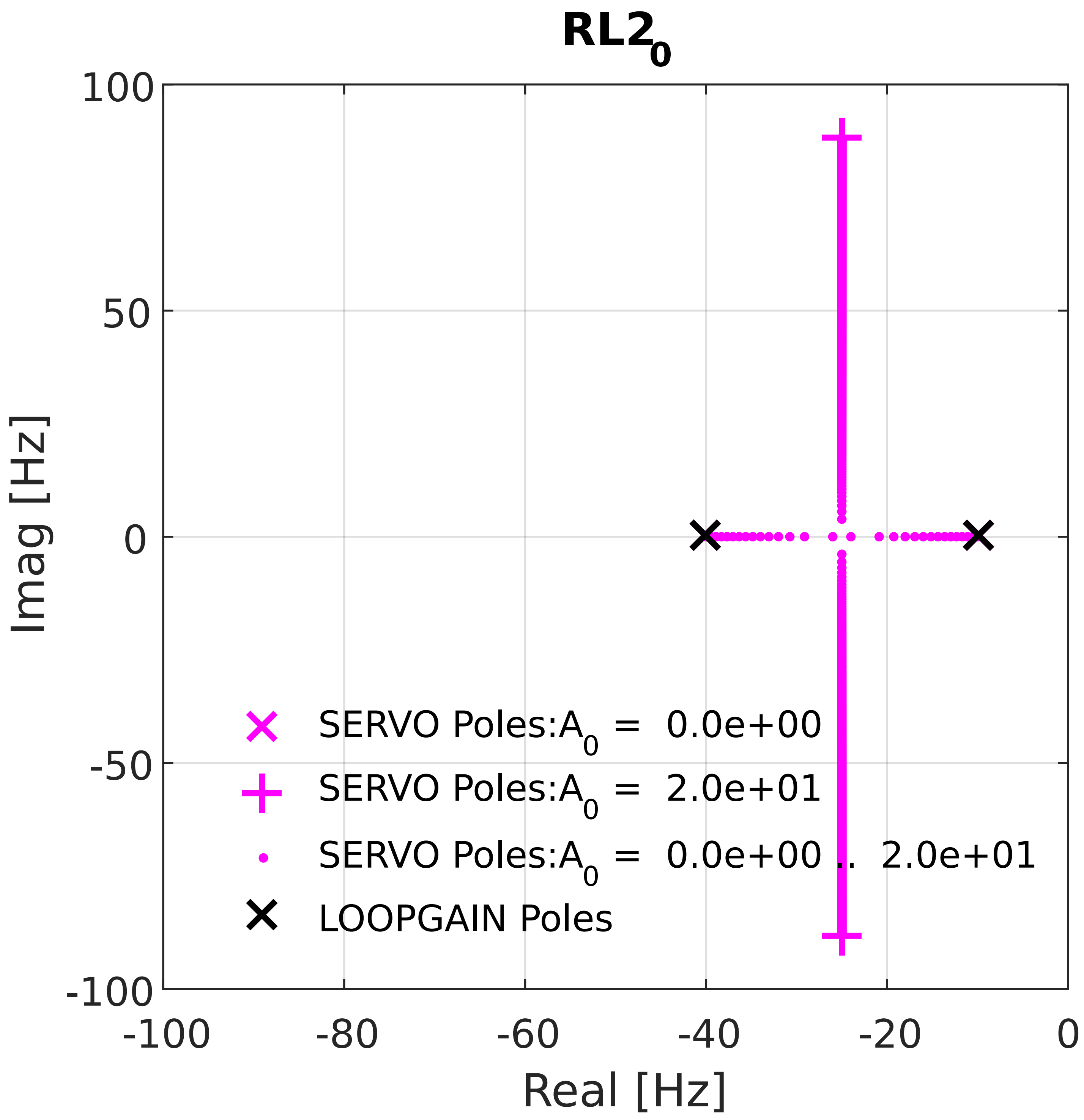
1. Poles of the loop gain
2. Zeros of the loop gain
3. Poles of the servo function while stepping the DC gain of E1

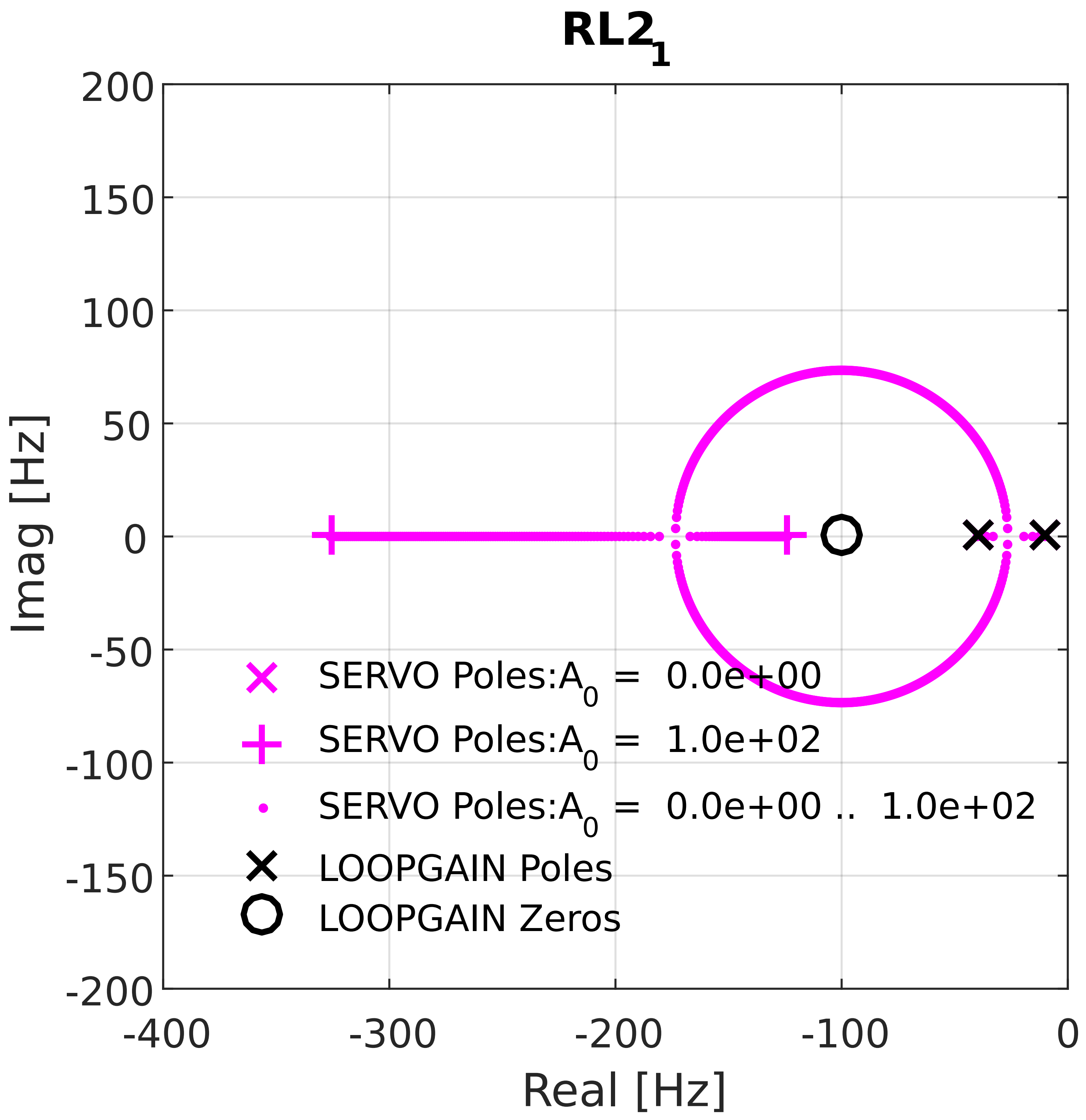
See section 11.5.3

Root locus SLiCAP second order

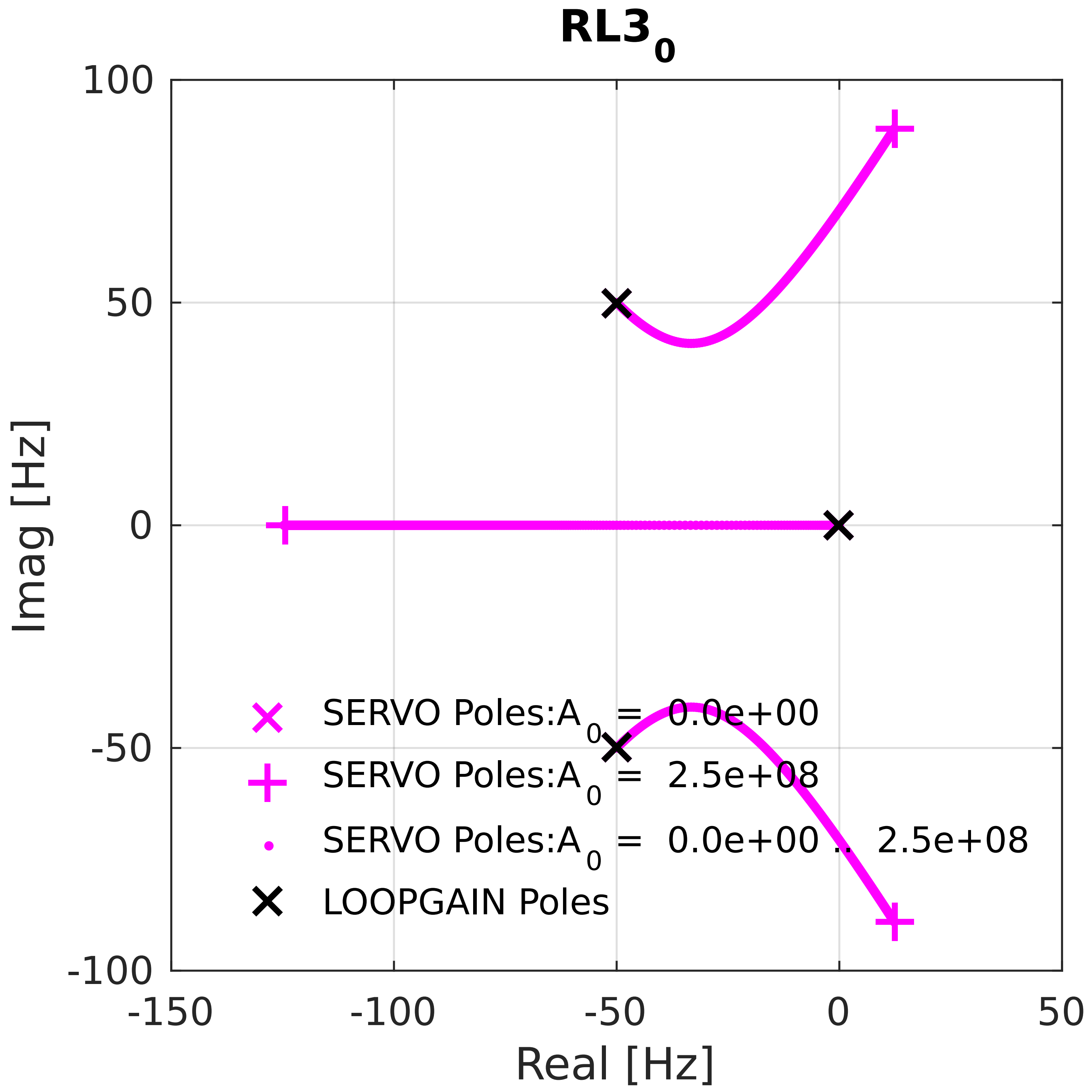


Root locus SLiCAP second order





Root locus SLiCAP third order



Root locus SLiCAP third order