Structured Electronic Design

Frequency stability of feedback amplifiers

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Introduction

1. A system is stable if all its poles have a negative real part
2. For a negative feedback system it can be formulated as:
   a. All poles of the ideal gain must have a negative real part
   b. The loop gain reference has been selected such that the asymptotic gain equals the ideal gain
   c. The poles of the servo function must have a negative real part
   d. The poles of the direct transfer must have a negative real part
3. We will assume a, b and d to be true, and present techniques to evaluate c:
   a. Routh-Hurwitz criterion
   b. Nyquist criterion
   c. Root locus technique

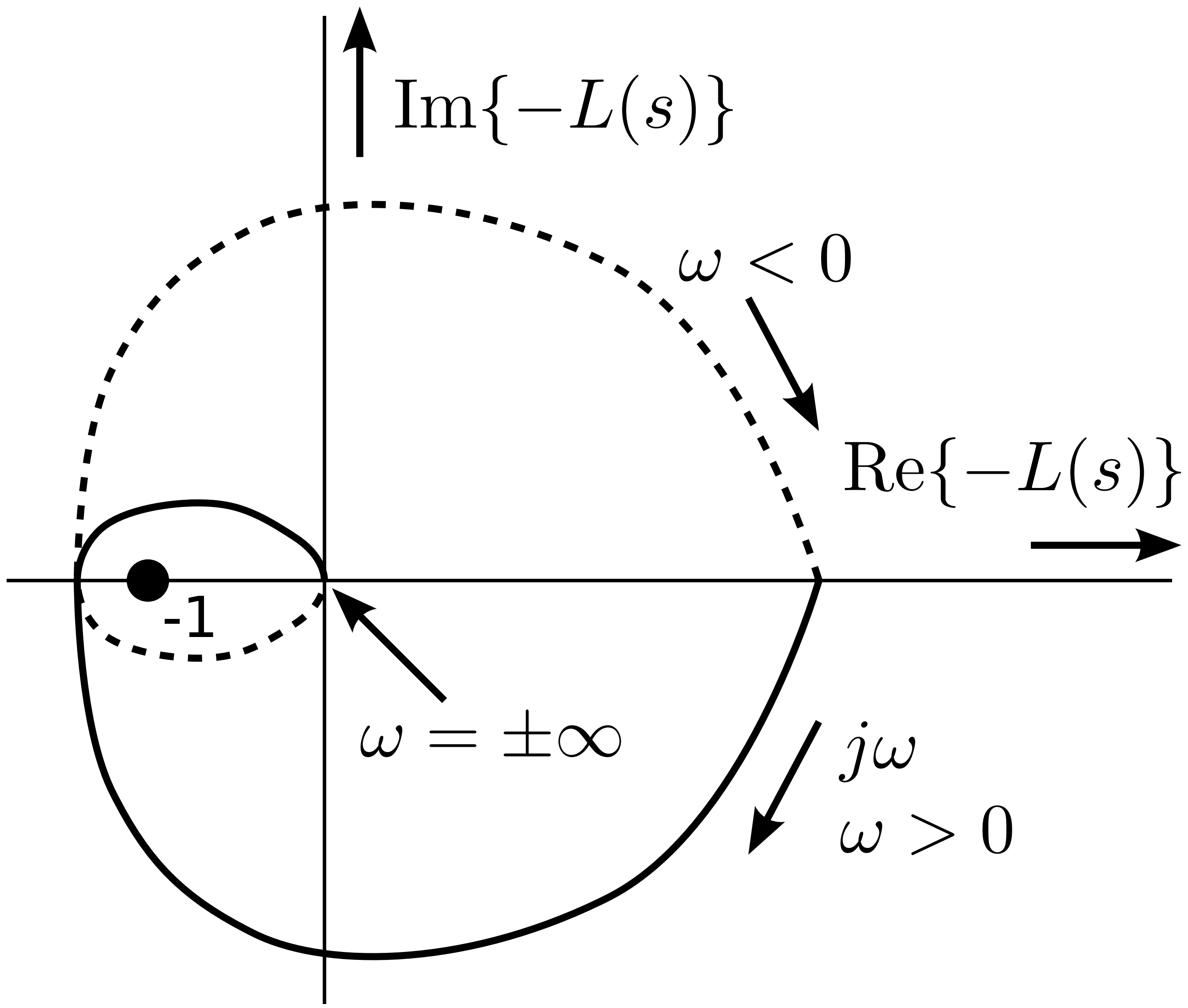
Routh-Hurwitz criterion

1. Routh test: 1876, Edward John Routh, 1895 Adolf Hurwitz
2. Coefficients of the characteristic polynomial taken as input for the Routh Array
3. The number of sign changes in the first column of this array is equal to the number of solutions of this polynomial that have a positive real part
4. For detailed description, refer to: Control Theory
5. Implemented in SLiCAP

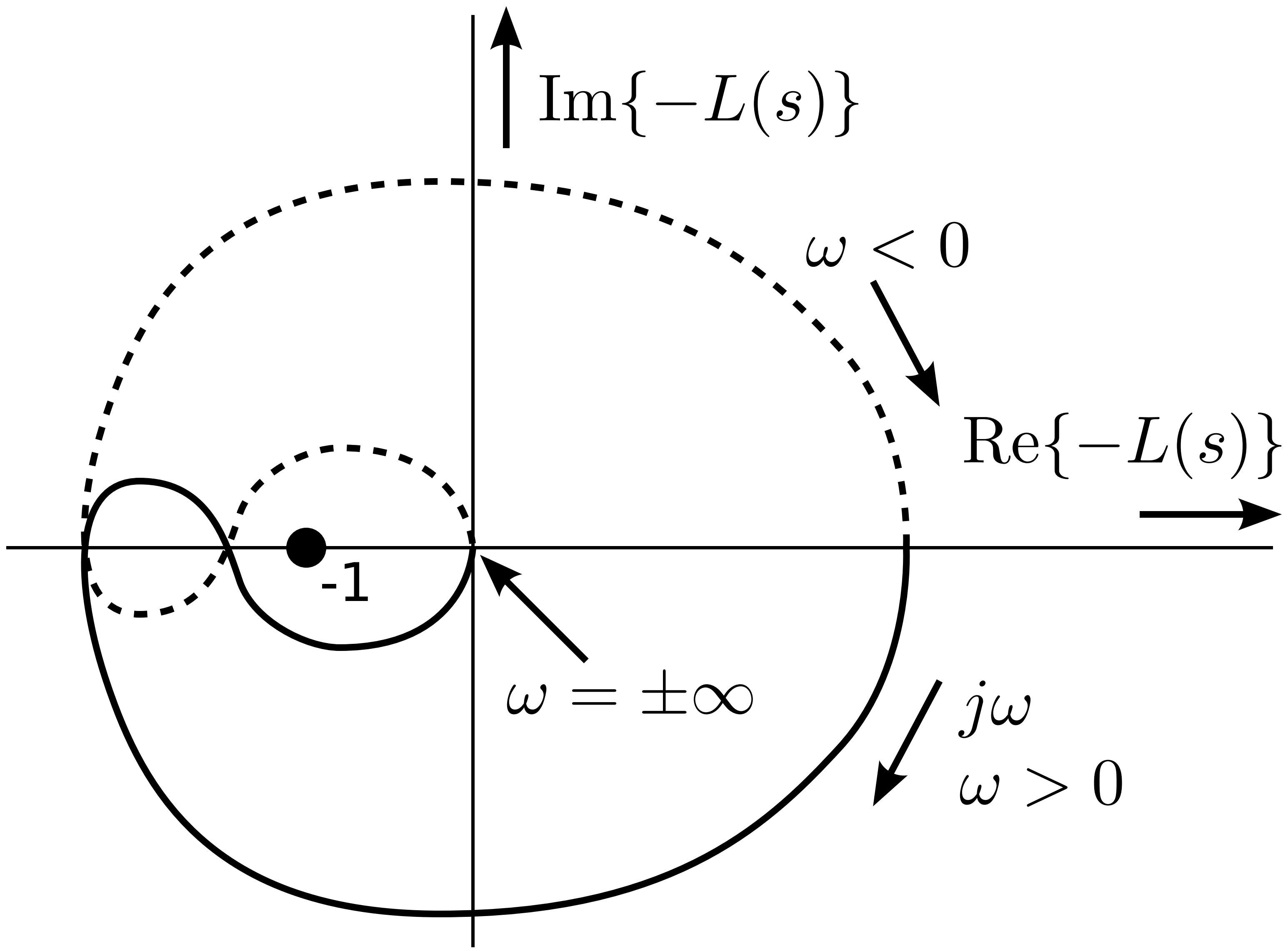
Nyquist criterion

1. Nyquist: 1932
2. Consider a contour plot of the loop gain (-L)
3. The number of poles with a positive real part equals
   The difference between the number of clockwise encirclements of the point (-1,0) by the contour, and the number of poles of L that have a positive real part (the latter one is usually zero)
4. For detailed description, refer to: Control Theory
5. Phase margin and amplitude margin are often taken as measure for stability
6. Implemented in SLiCAP

Nyquist plots

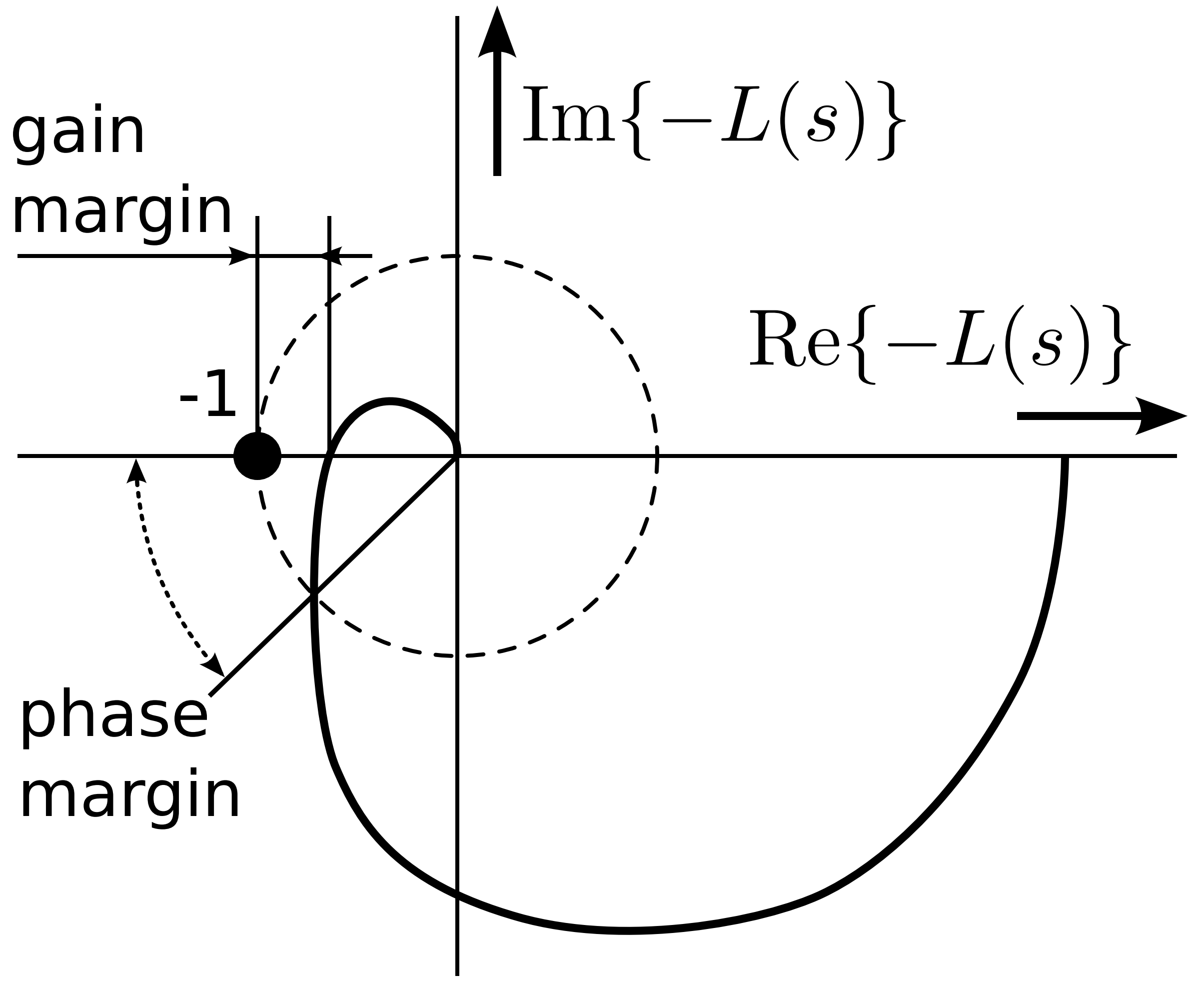


One clockwise encirclement



Zero clockwise encirclements

Gain margin and phase margin



Indicative measure for stability

How far is the system from instability ...

Gain margin: how much in dB the loop gain must be increased to cause instability

Phase margin: how much in degrees the phase lag of the loop gain must be increased to cause instability

Rules of thumb:
1. Gain margin better than 6 dB
2. Phase margin better than 30 degrees

Warnings:
1. No one-to-one mapping on frequency response
2. Property of loop gain

Root locus technique

1. Introduced by Evans in 1948
2. Graphical method to find poles from the servo function from the poles of the loop gain
3. 10 construction rules (see book)
4. For proof refer to control theory
5. Here: examples with finite, nonzero DC loop gain
6. Implemented in SLiCAP

Root locus technique

$$L(s) = L_{\mathrm{DC}}\frac{N(s)}{D(s)}$$

$L_{\mathrm{DC}} =$ DC value of the loop gain

$N(s) = 0 \Longrightarrow$ Zeros of the loop gain

$D(s) = 0 \Longrightarrow$ Poles of the loop gain

$$S(s) = \frac{-L(s)}{1-L(s)} = \frac{-L_{\mathrm{DC}}N(s)}{D(s)-L_{\mathrm{DC}}N(s)}$$

$D(s) - L_{\mathrm{DC}}N(s) = 0 \Longrightarrow$ Poles of servo function

Root locus:

Paths traced out by the poles of the servo function while varying the DC loop gain

Paths depend on:

1. Poles of the loop gain
2. Zeros of the loop gain
3. DC loop gain

Starting point: DC loop gain equals zero
End point: DC loop gain equals infinity
Actual servo poles: DC loop gain $= L_{\mathrm{DC}}$

1. Number of branches equals number of poles of the loop gain
2. Symmetrical with respect to the real axis
3. Branches start at poles of the loop gain
4. Branches end at zeros of the loop gain or at infinity
5. Parts of the real axis left from odd number of poles + zeros belong to a branch
6. n poles and m zeros, then n - m asymptotes
7. Real axis intersection point asymptotes

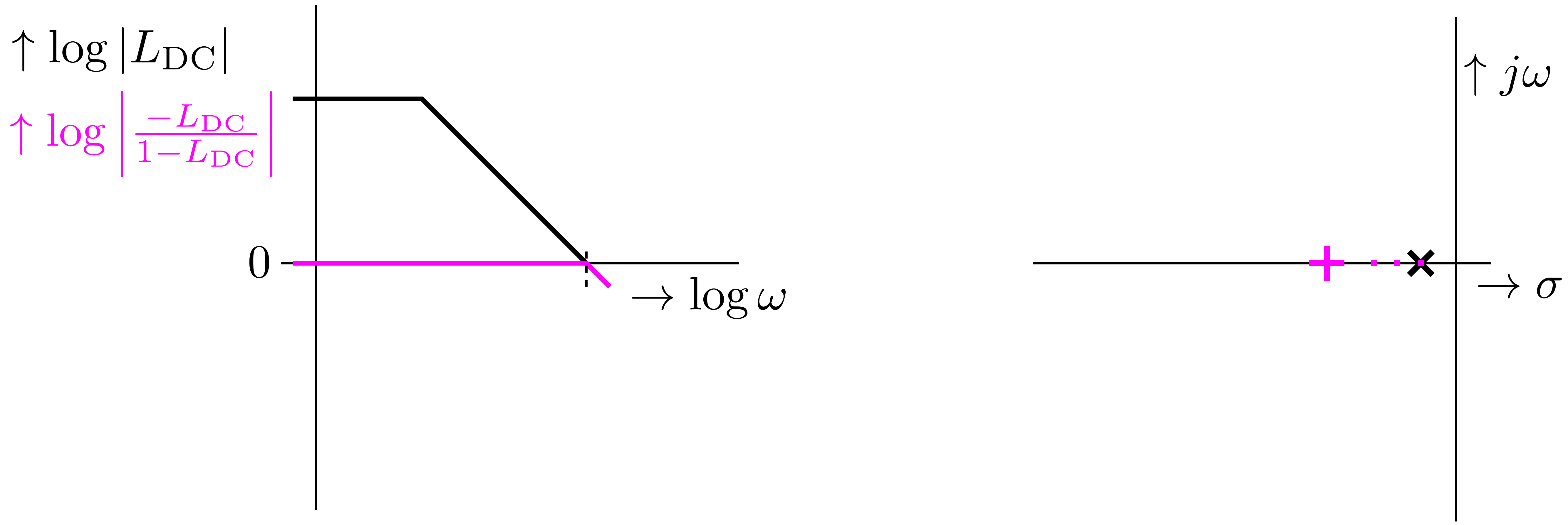
$$\sigma = \frac{\sum_{k=1}^{n} p_k - \sum_{i=1}^{m} z_i}{n-m}$$

8. Angle asymptotes equally spaced
9. Break away (and arrival) points

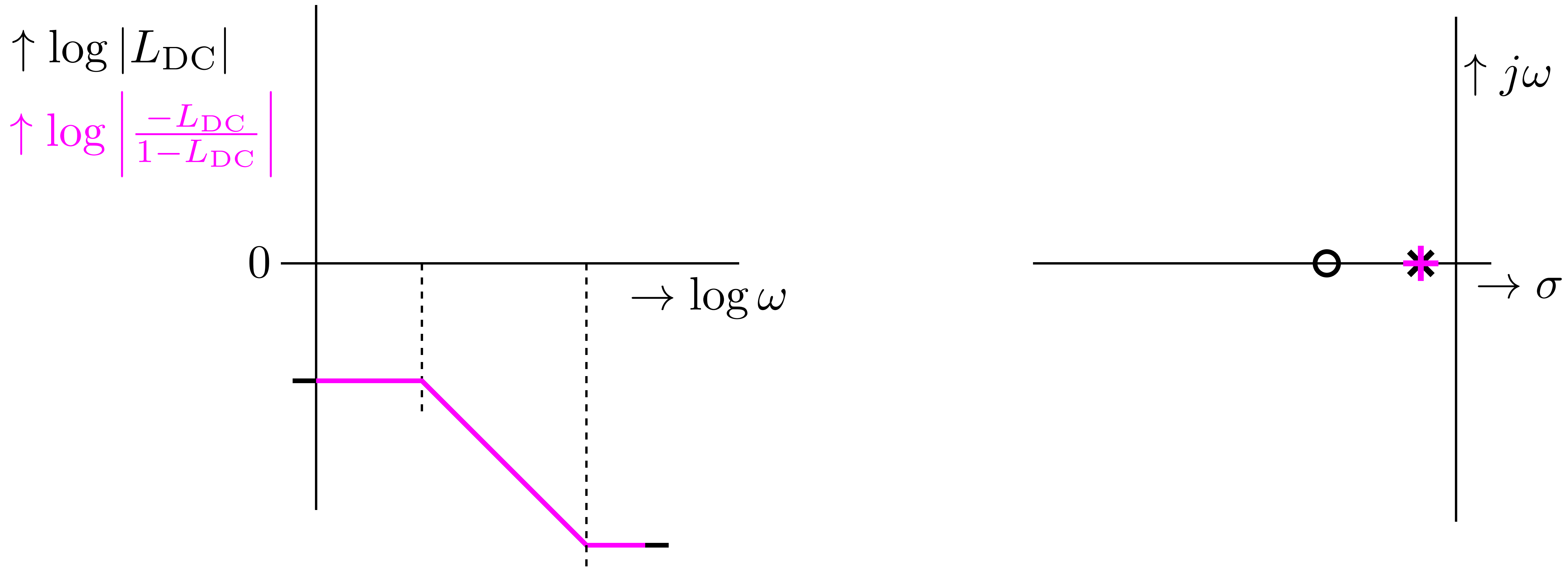
$$\frac{d}{ds}L(s) = 0$$

10. Break away angles equally spaced

Root locus first order



Root locus first order with zero left



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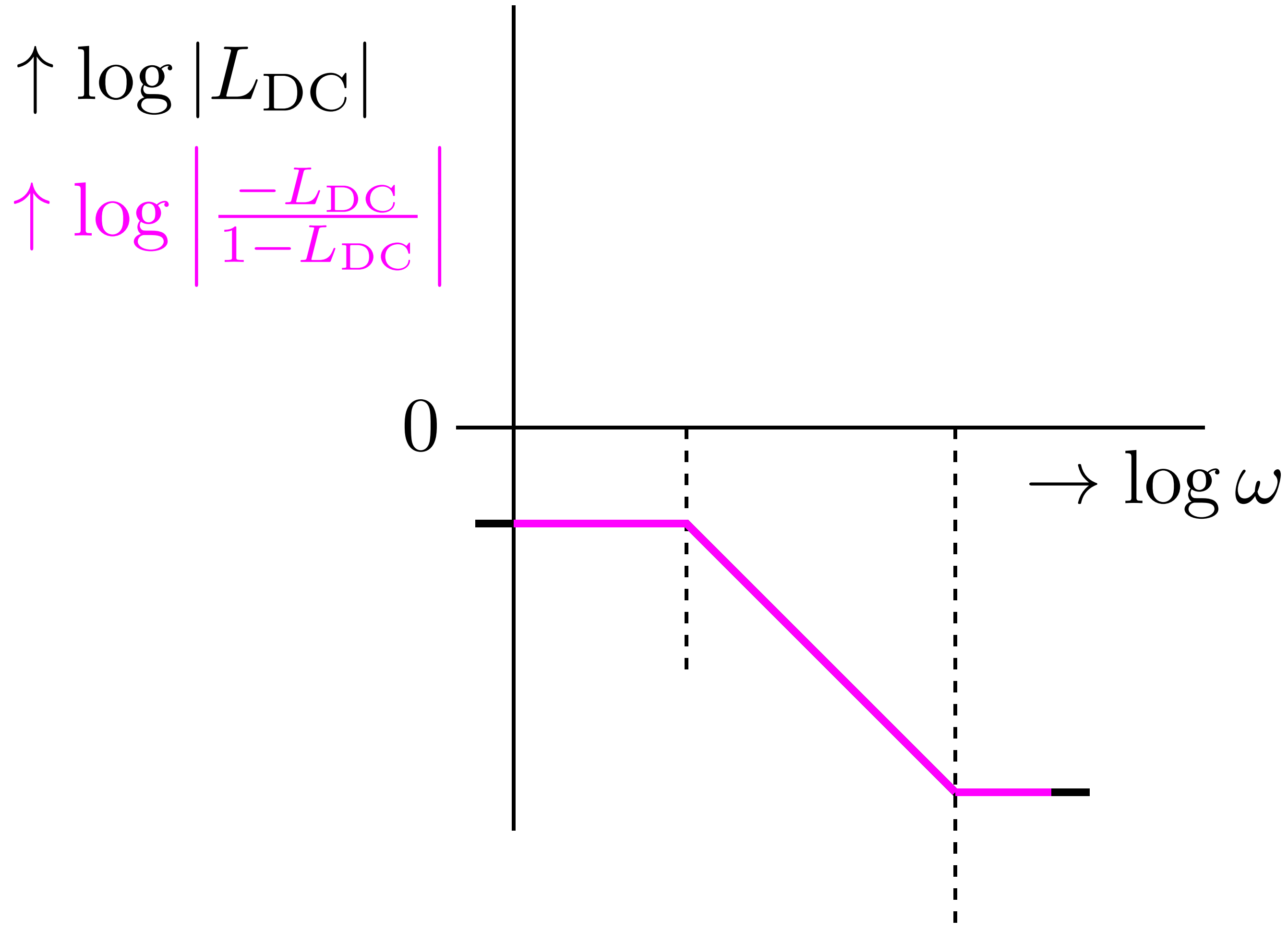

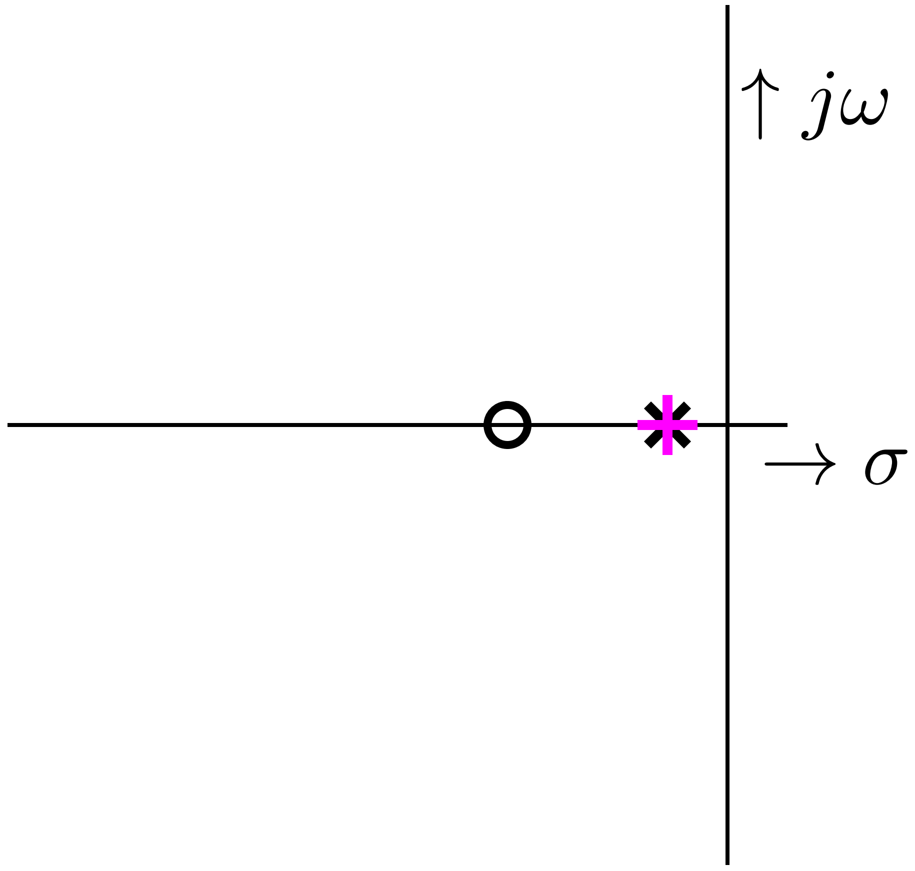

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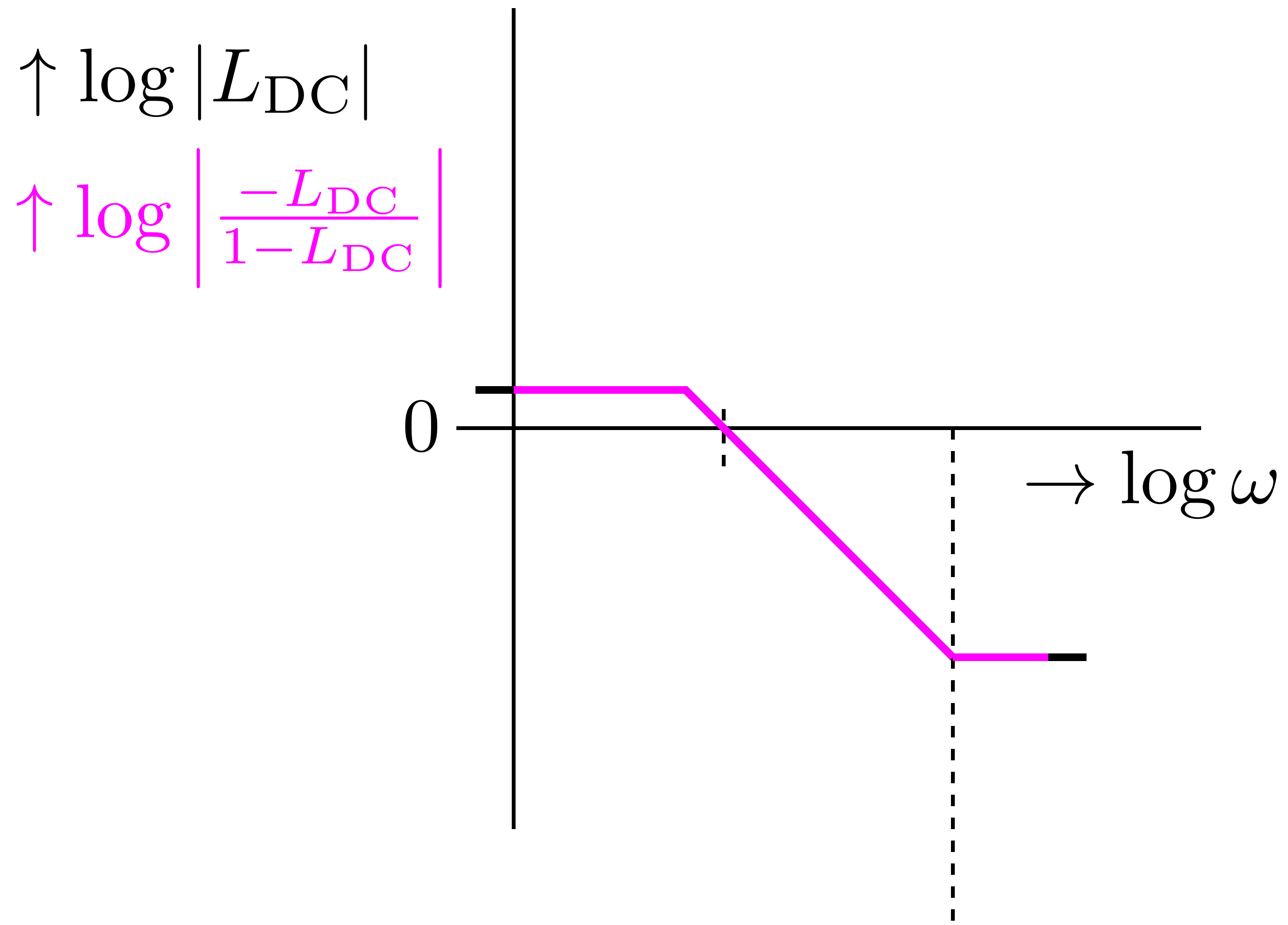

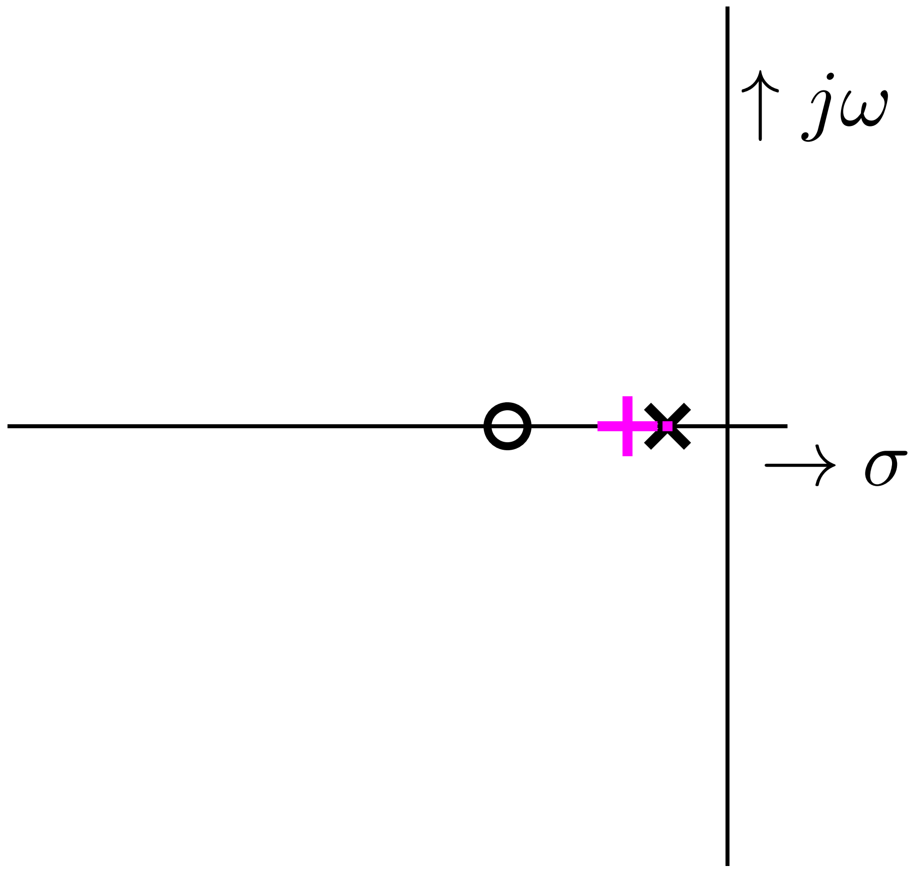

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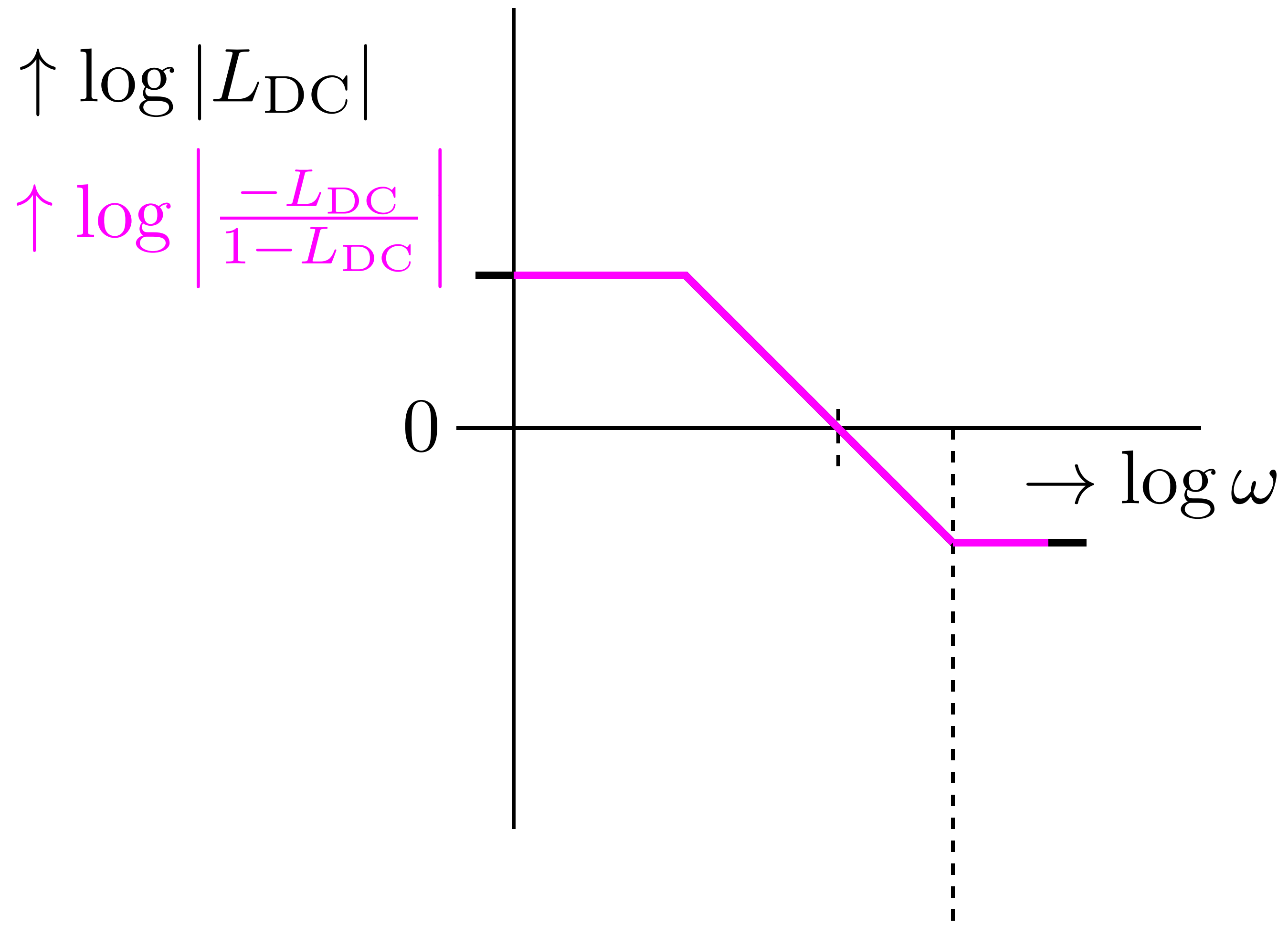

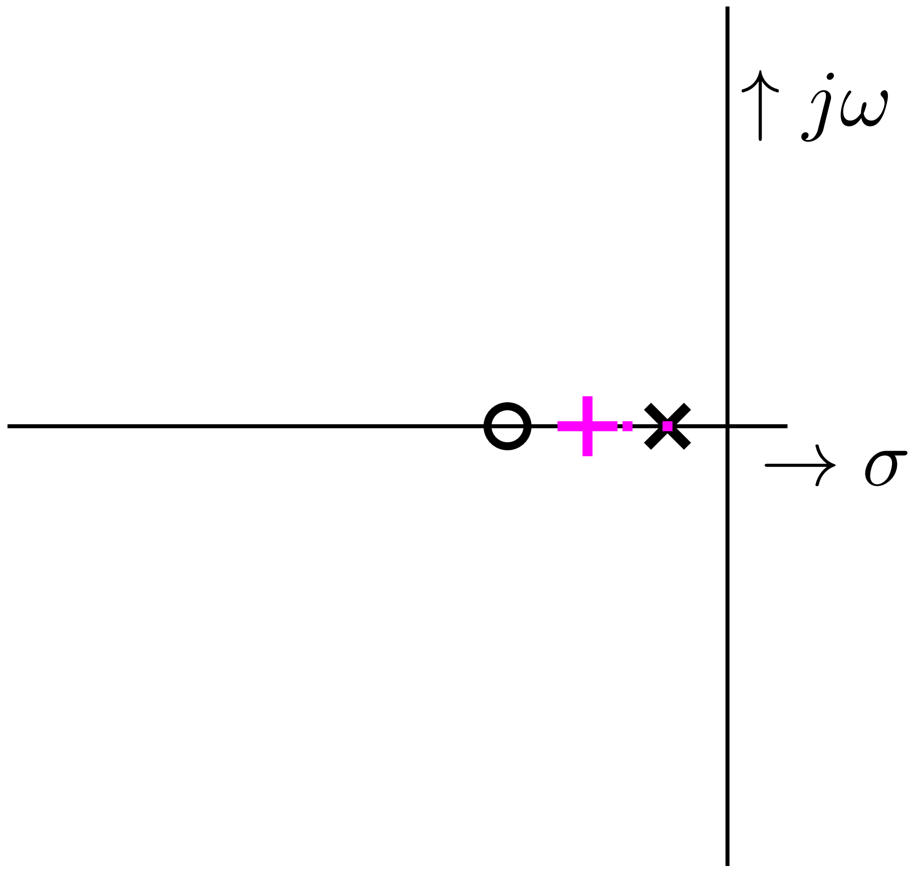

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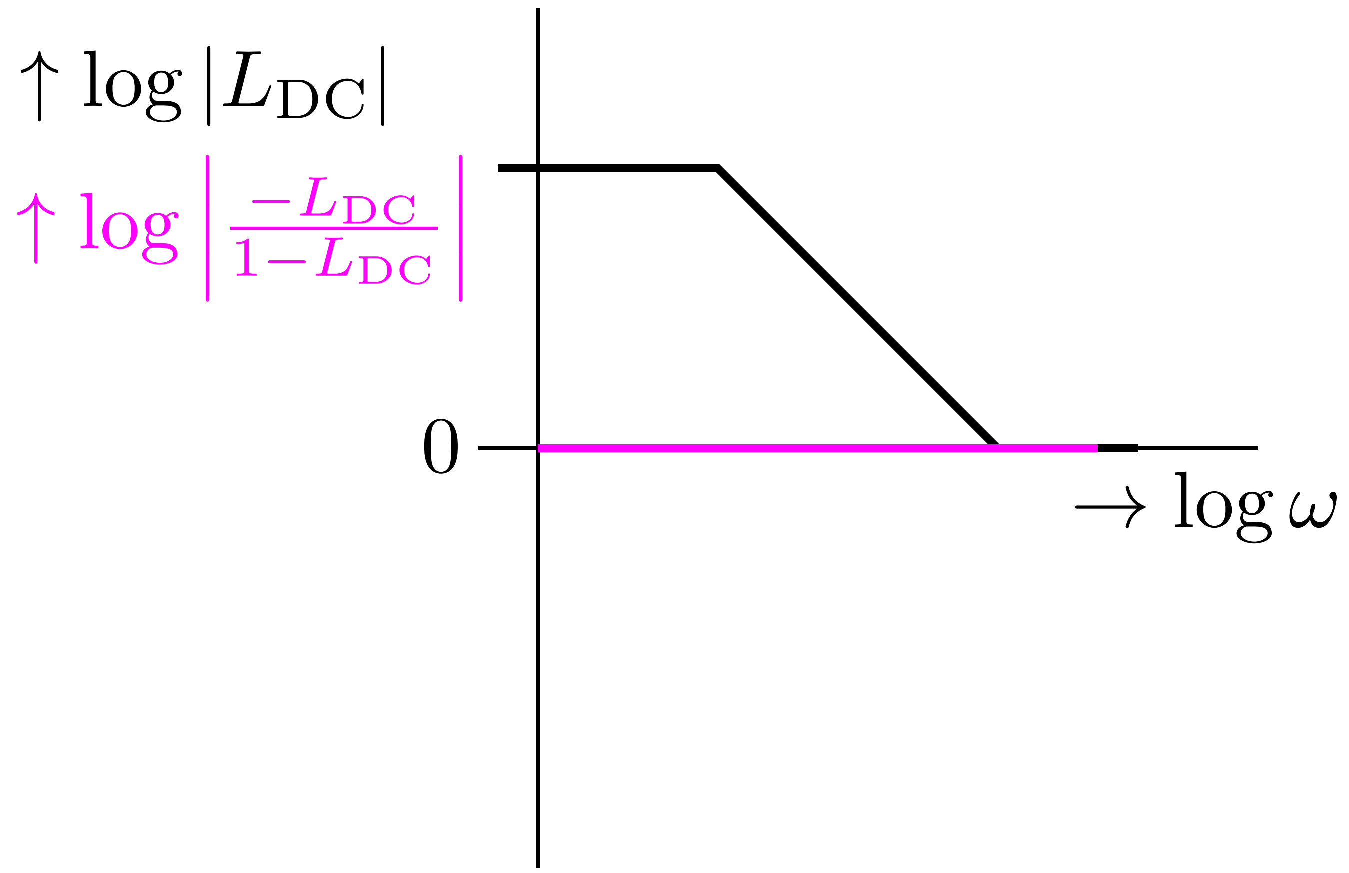

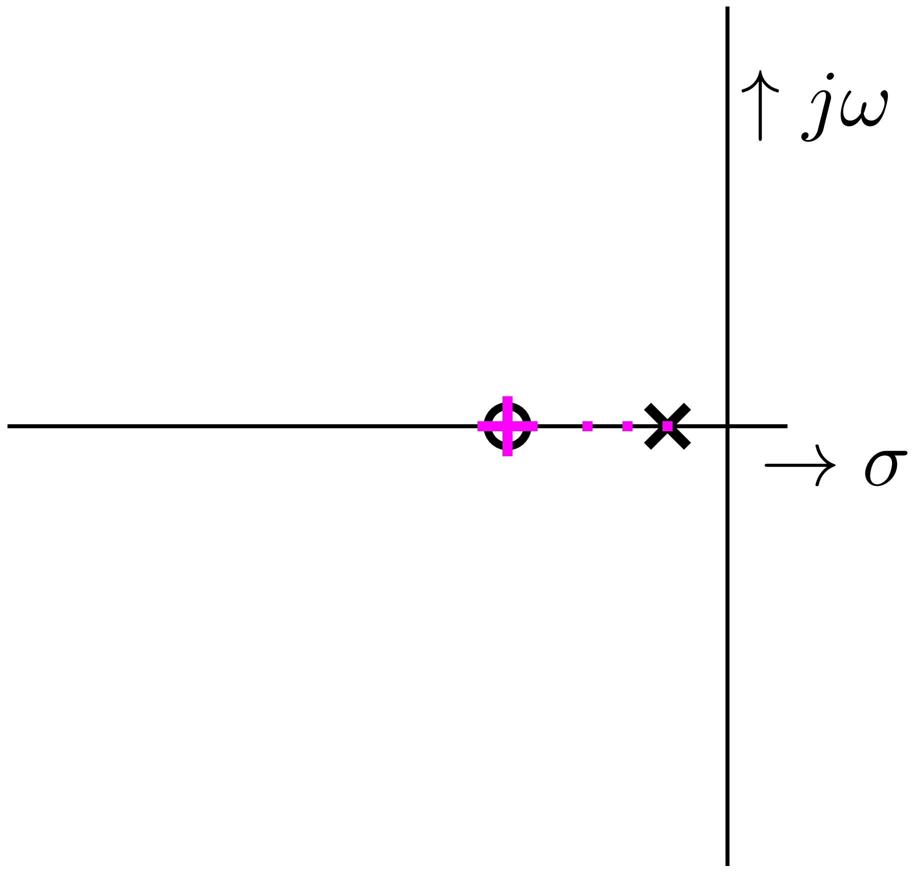

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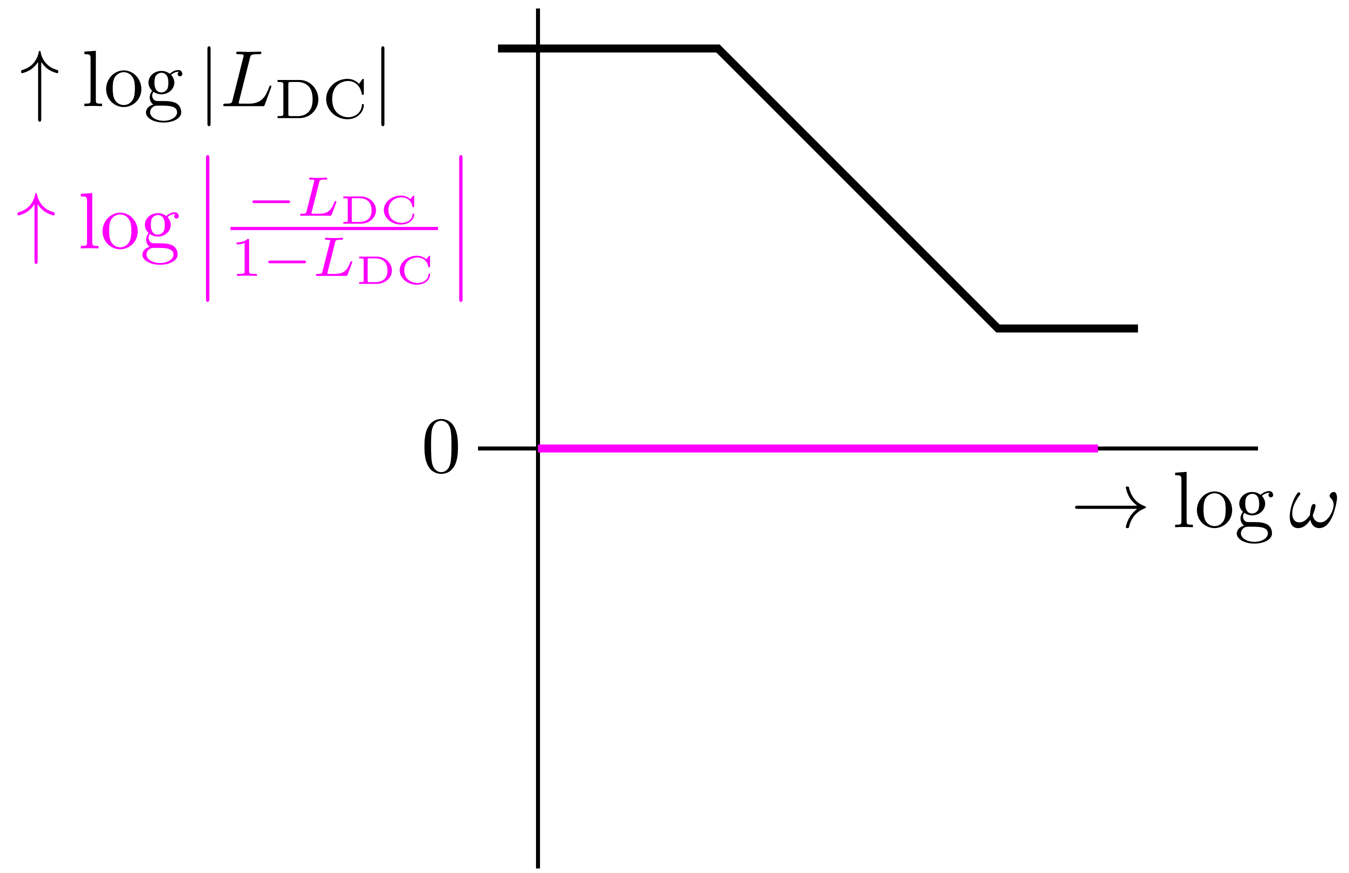

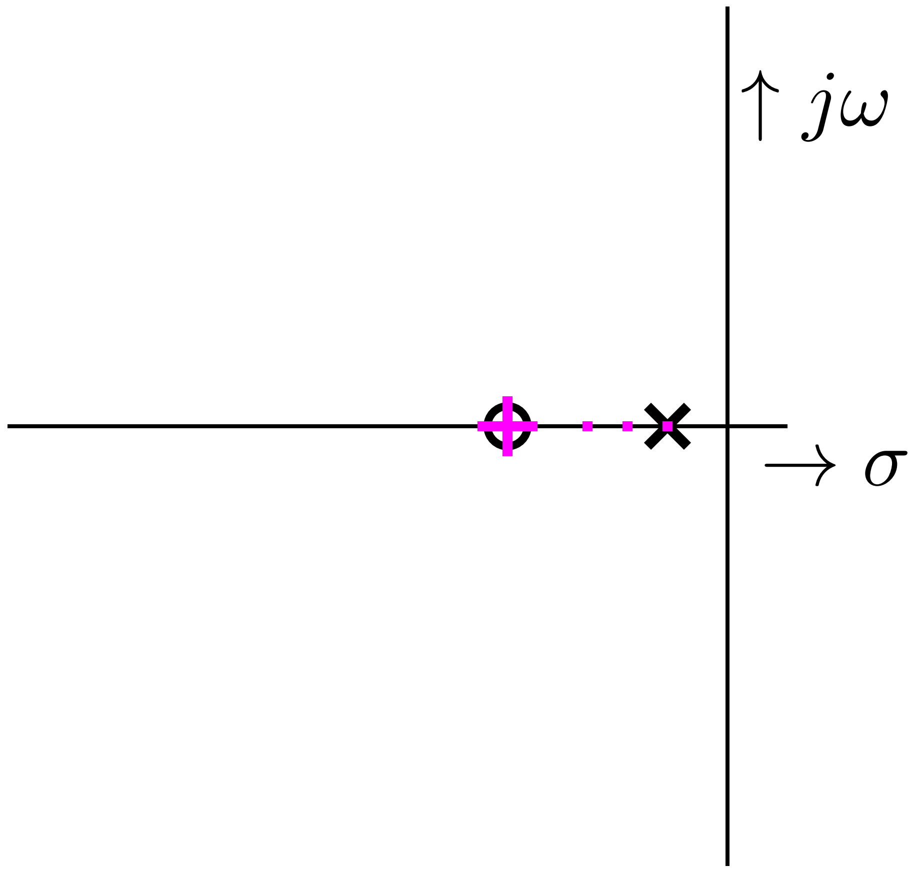

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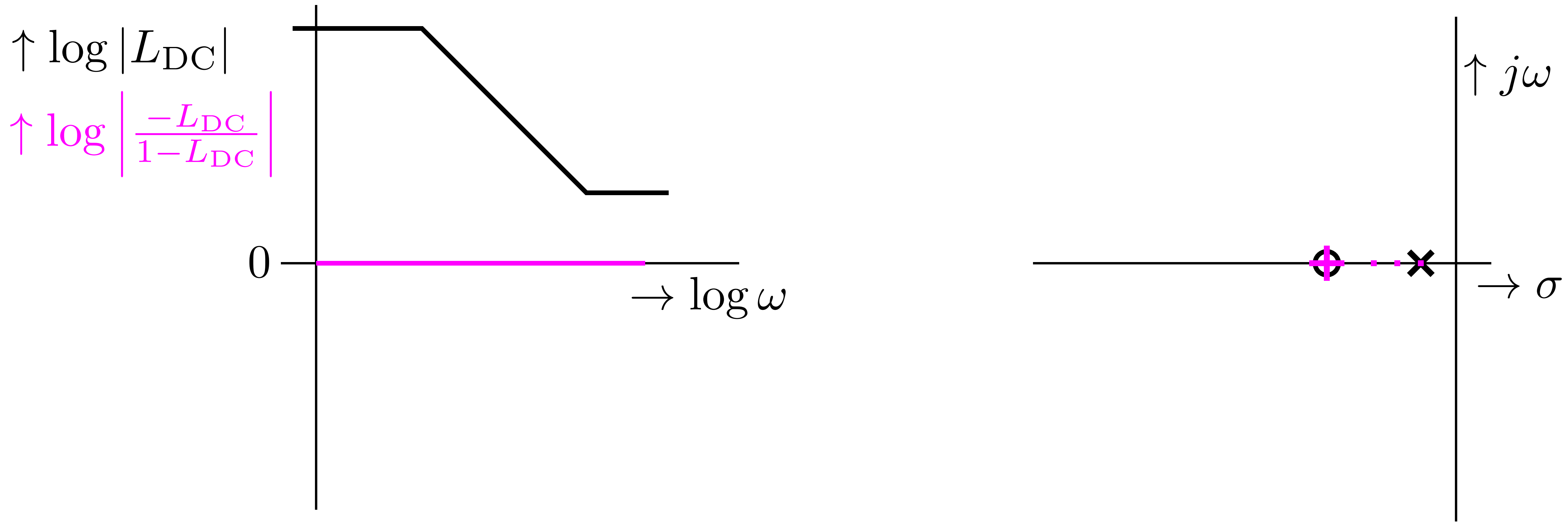
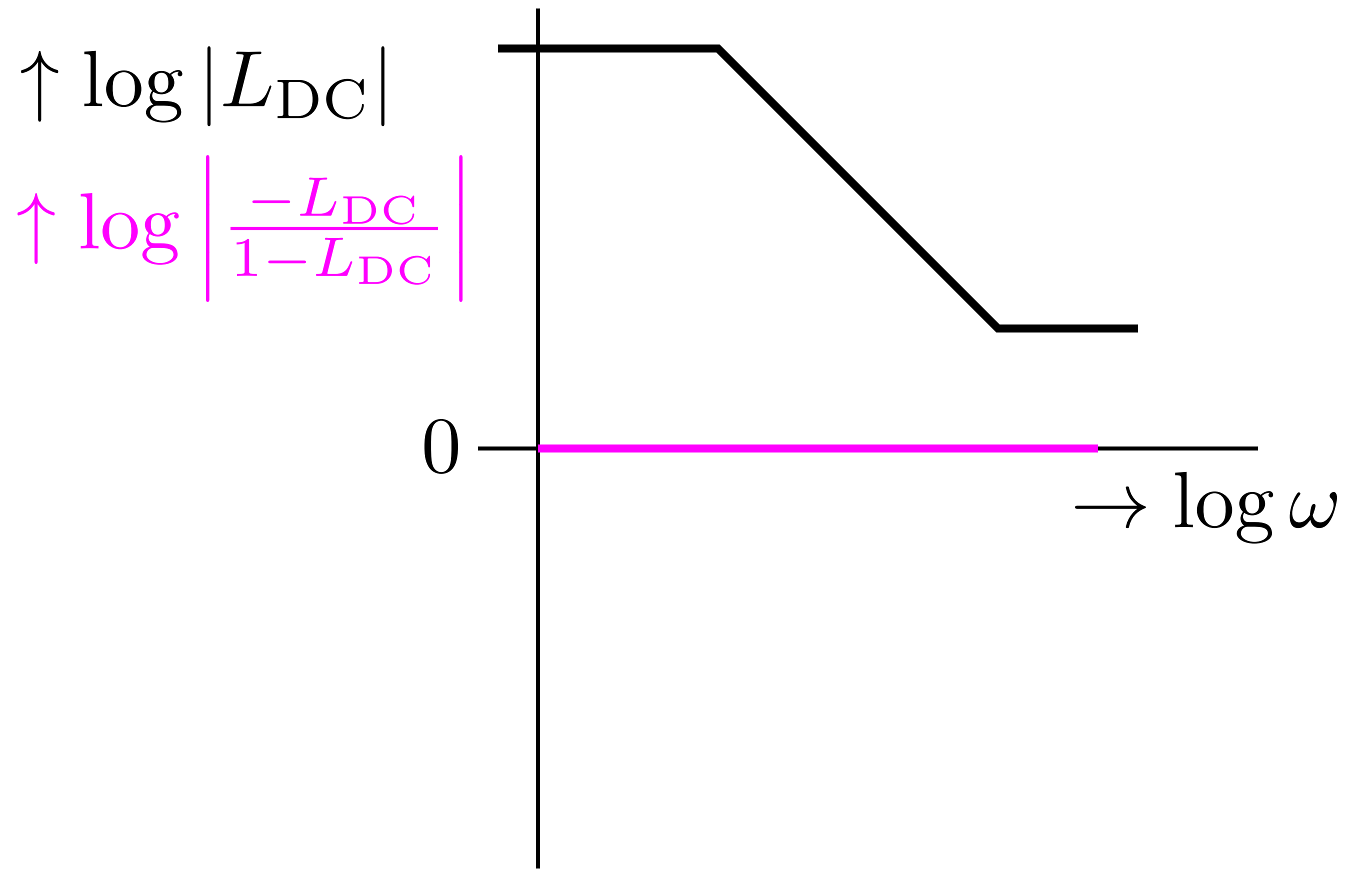
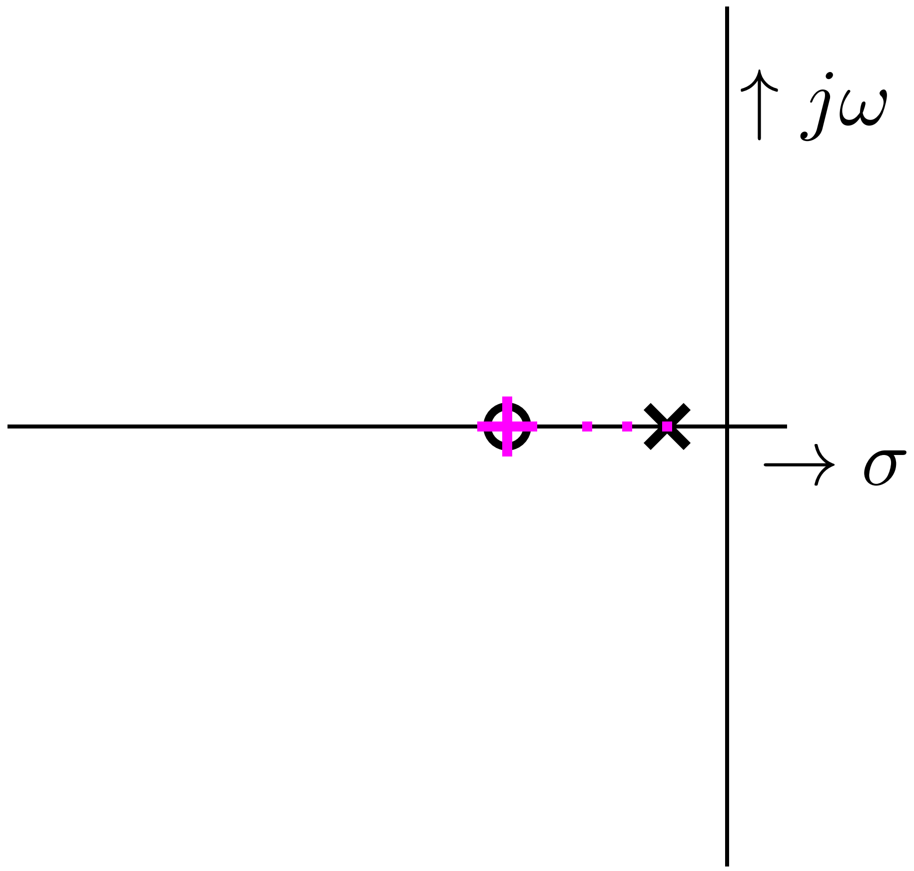


Note: pole only drops on the zero if DC loop gain is infinite!

Root locus first order with zero left





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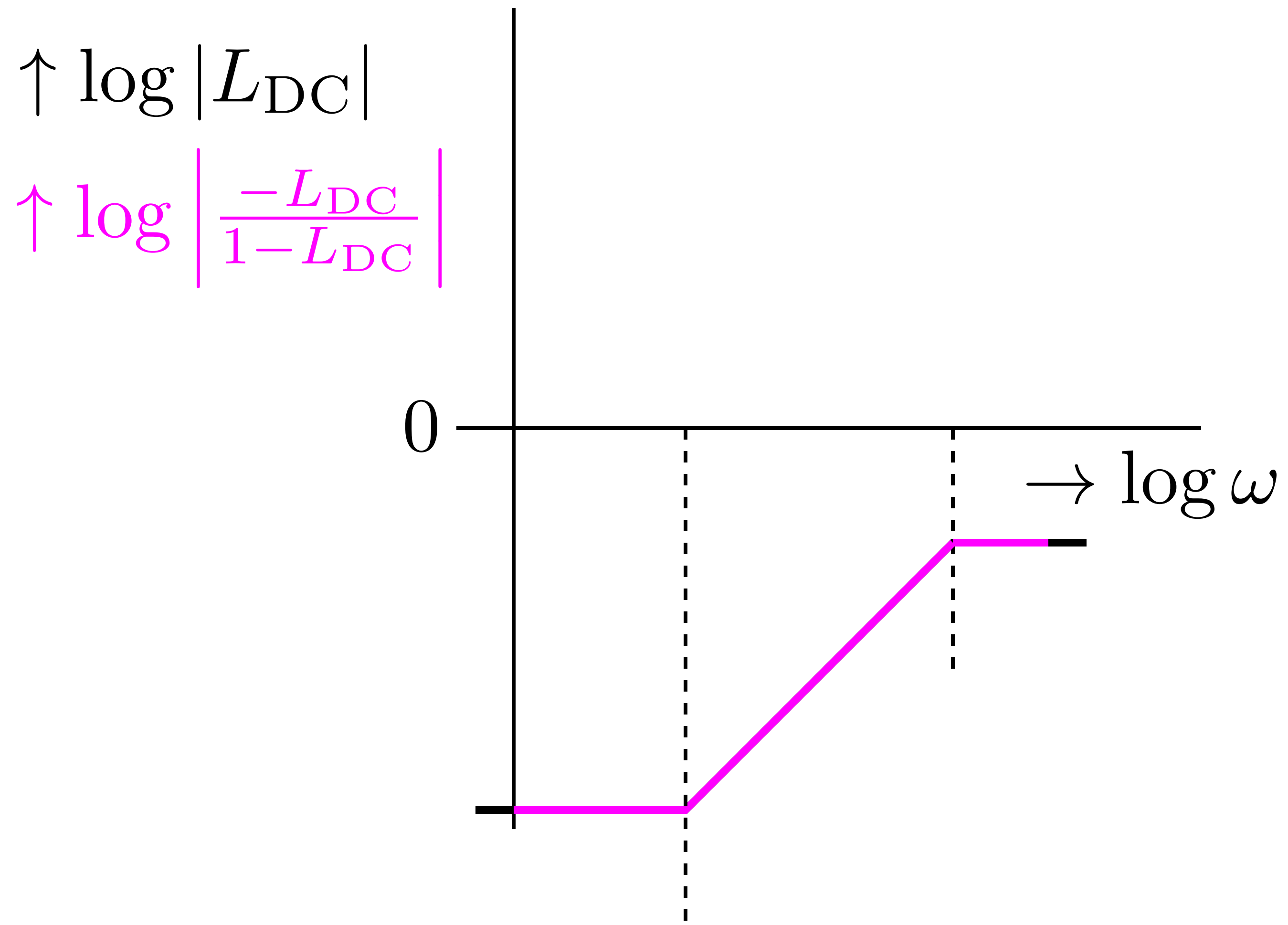
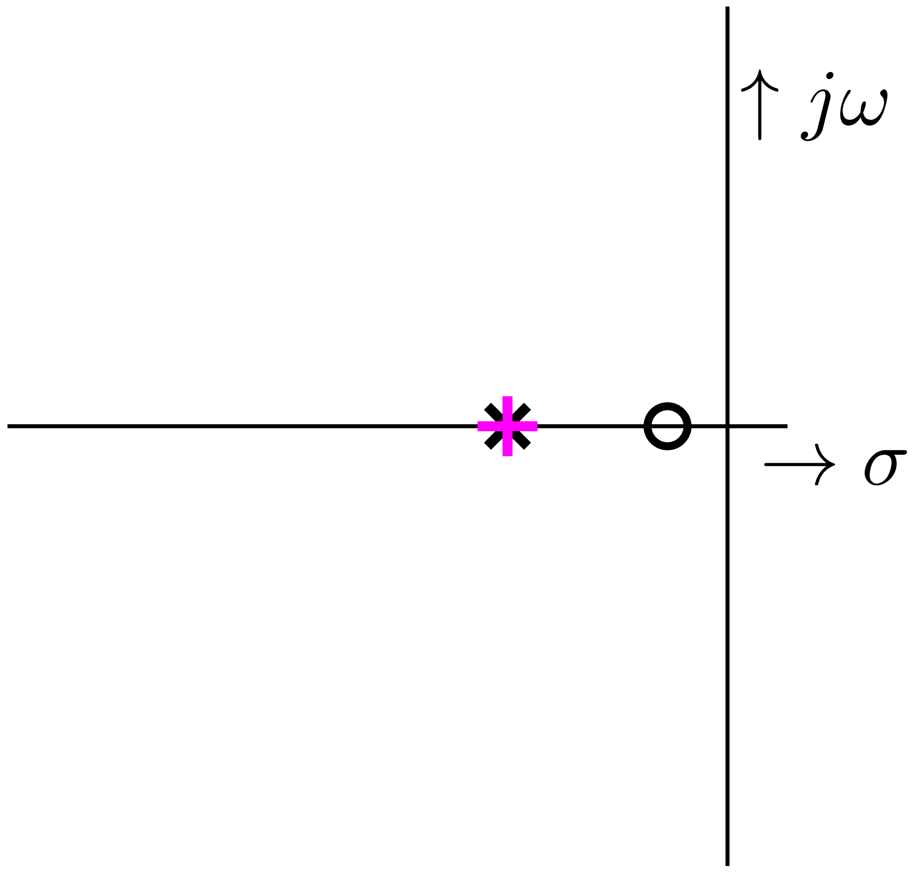
Root locus first order with zero right





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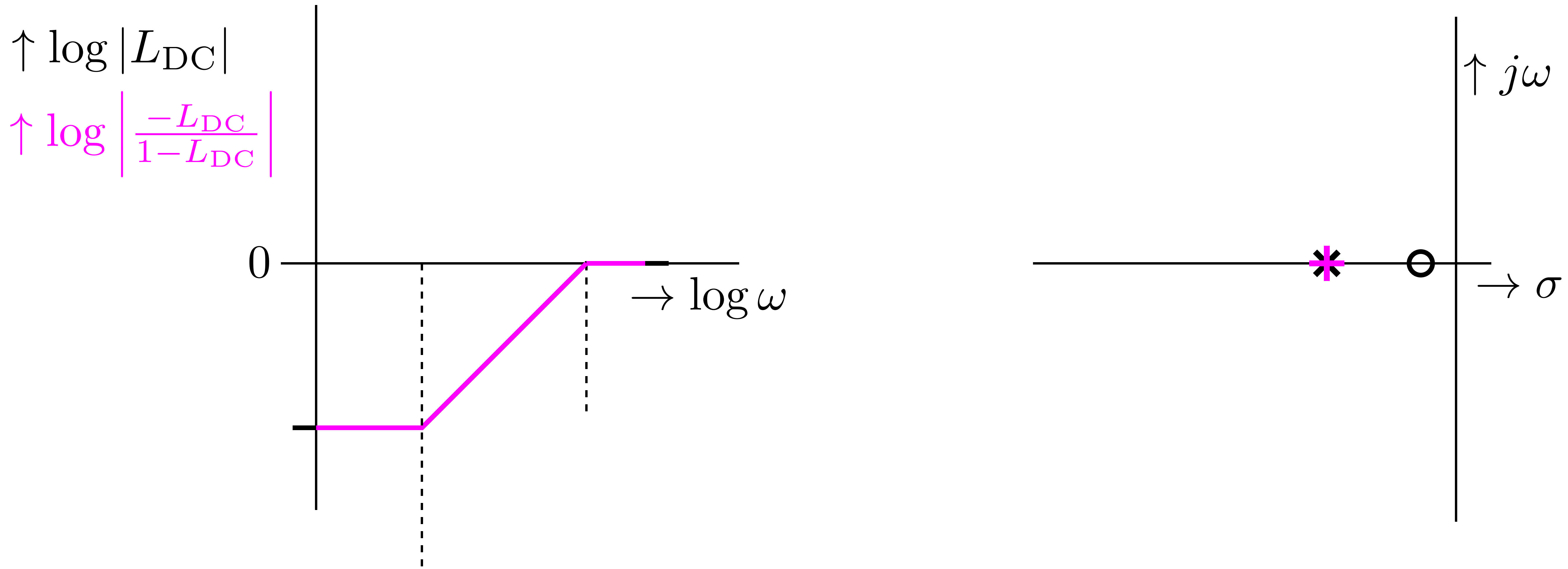

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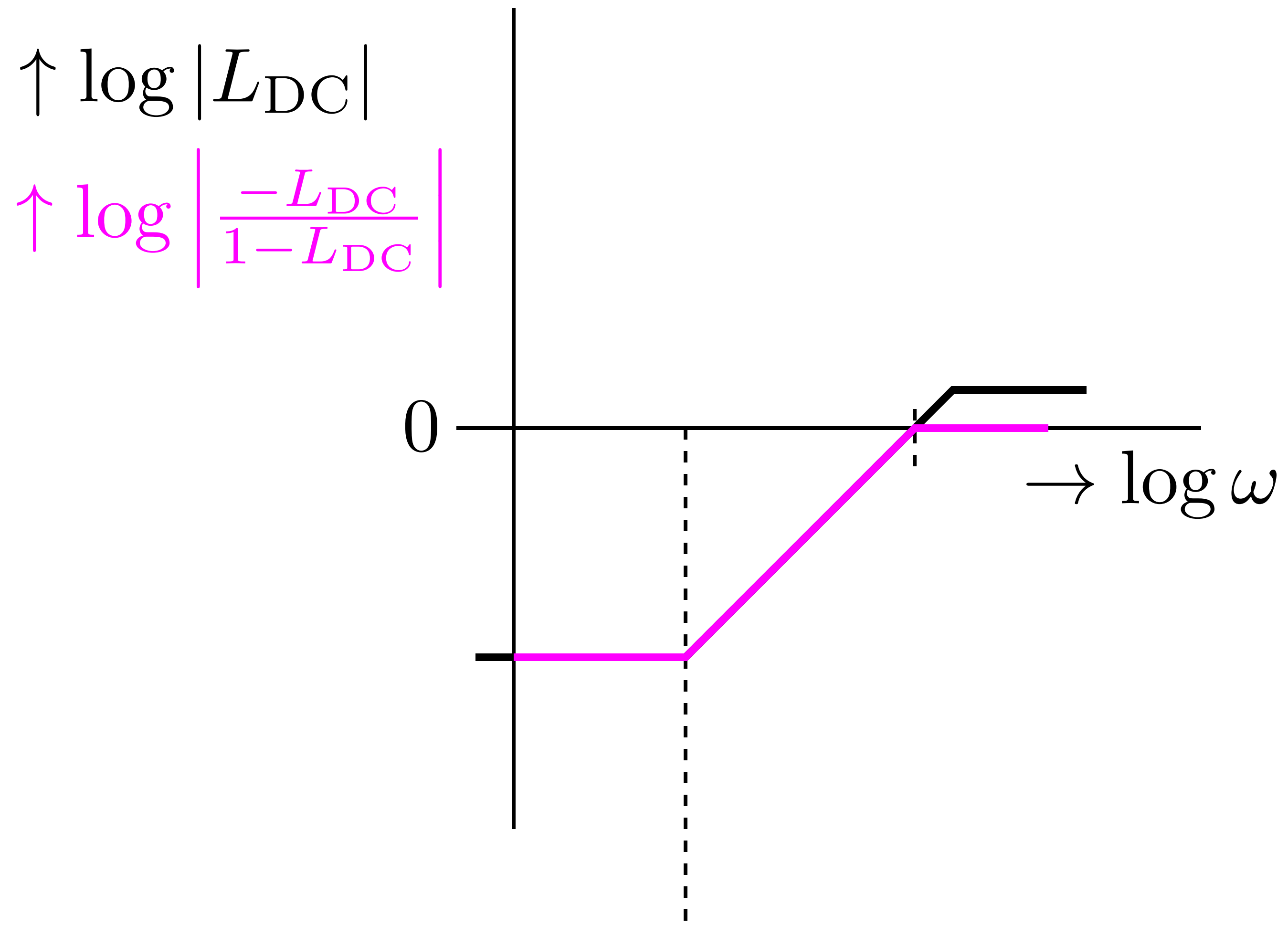

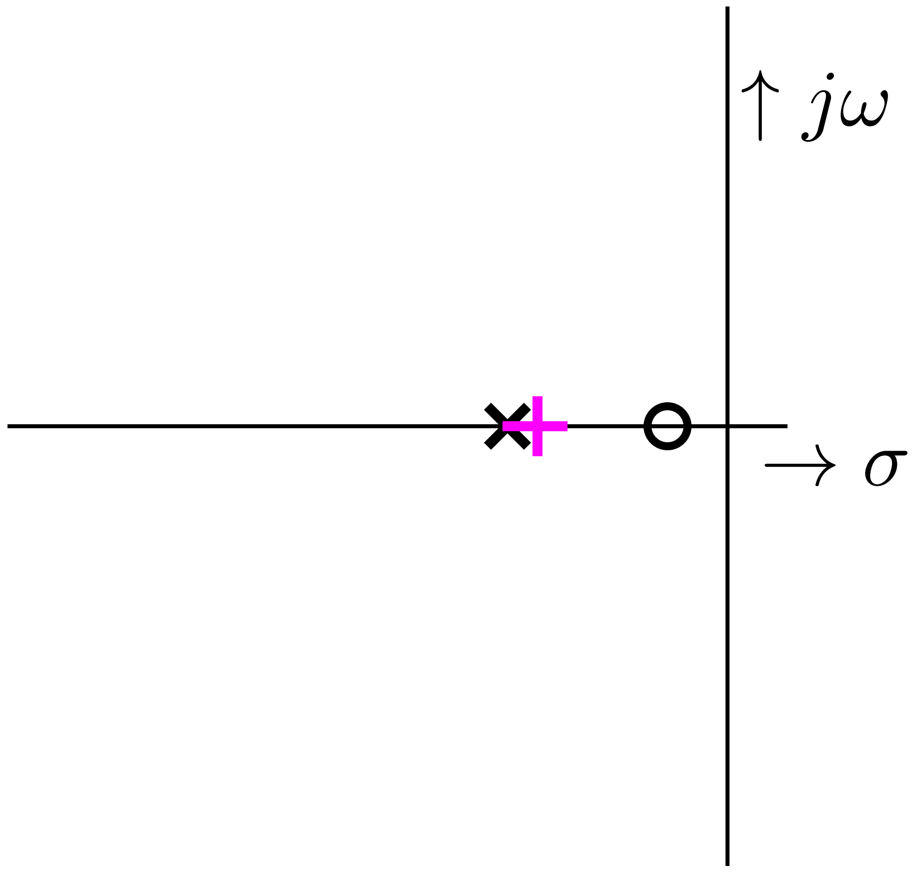

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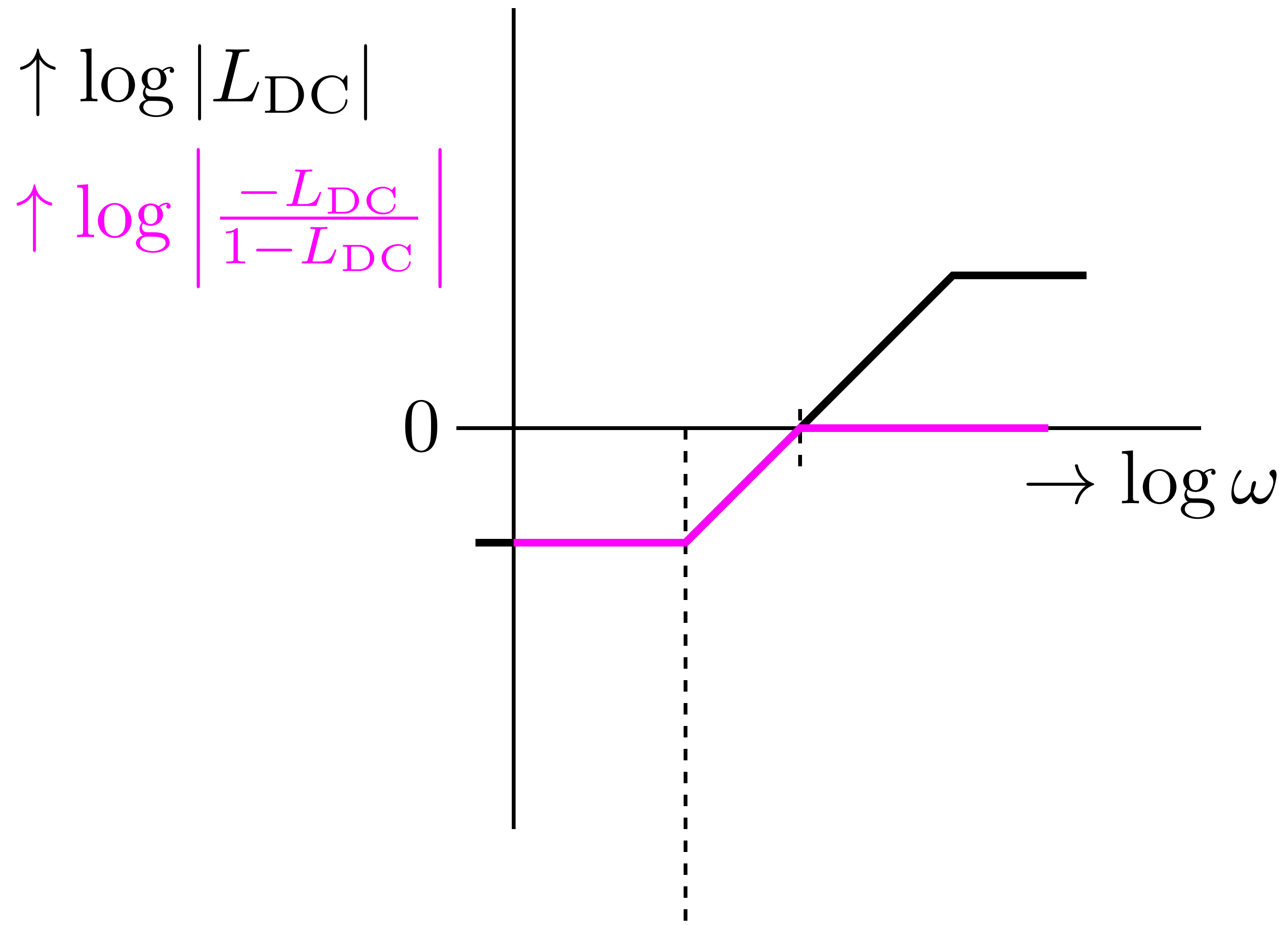

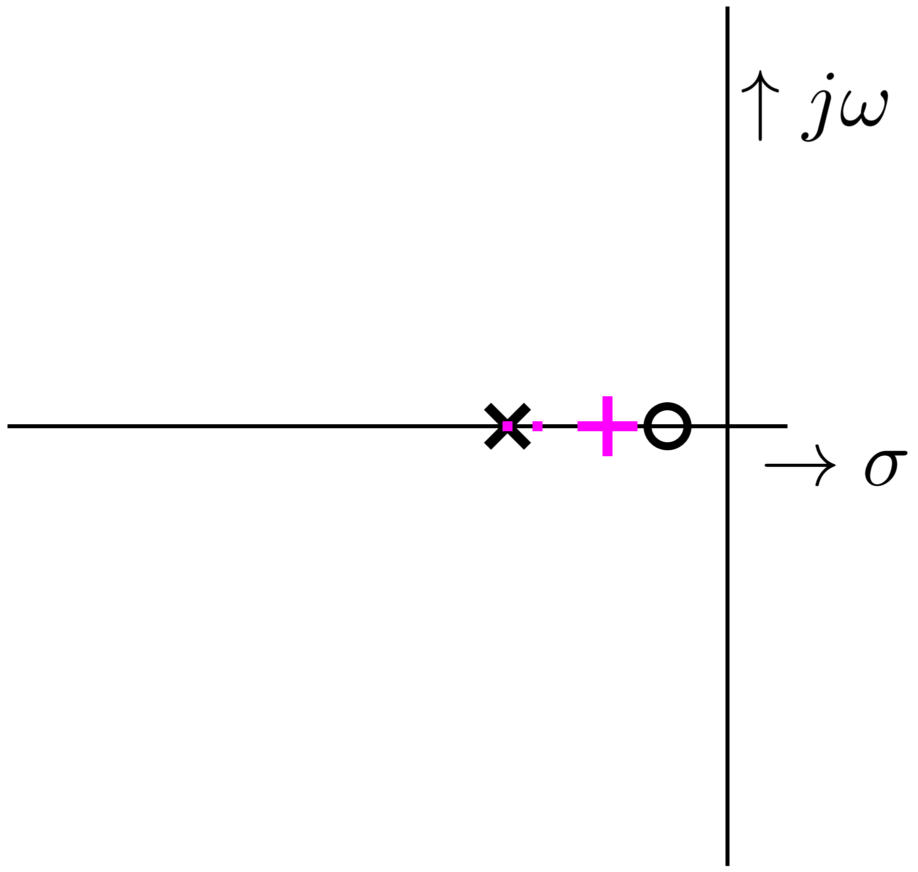

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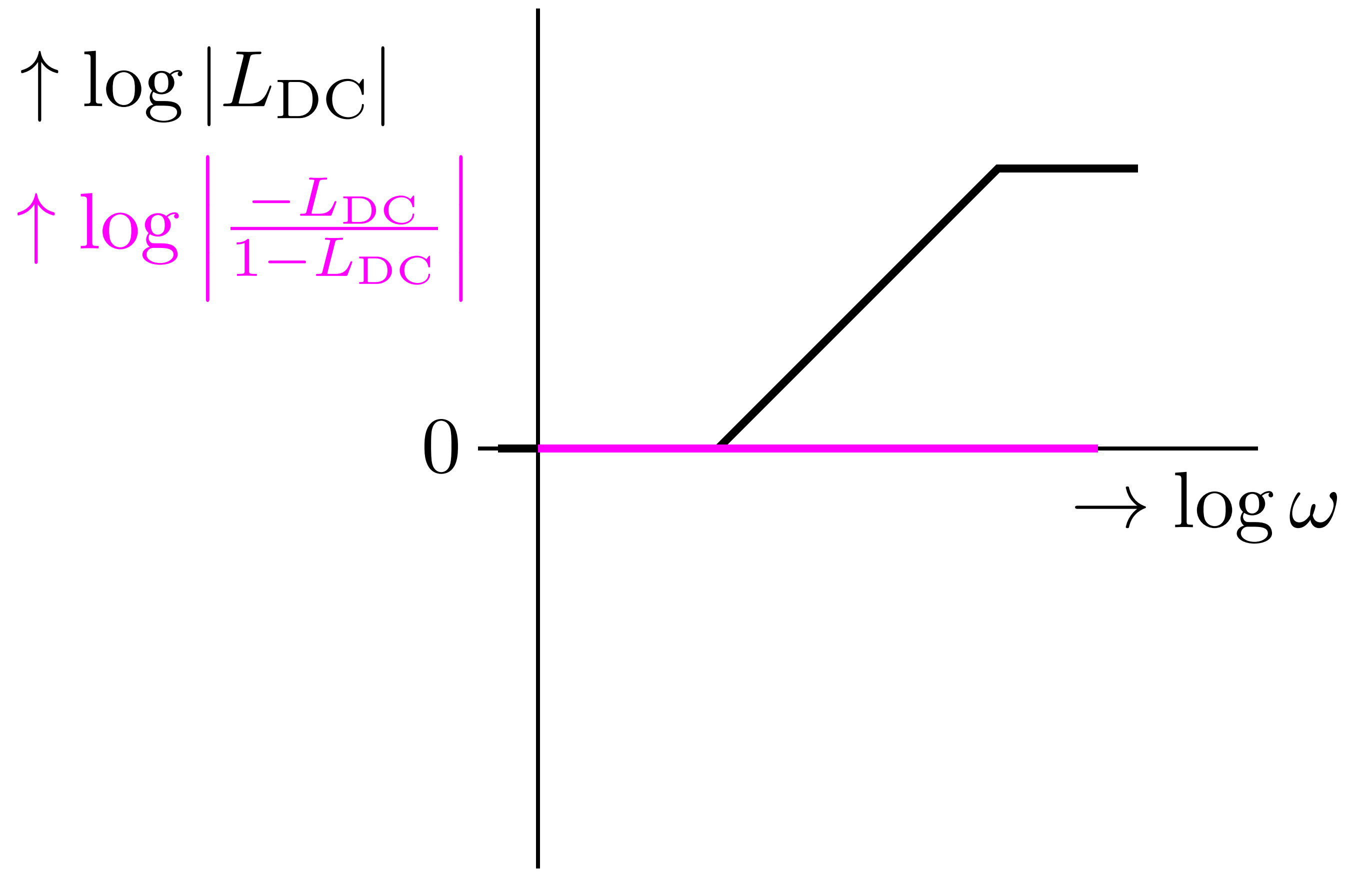

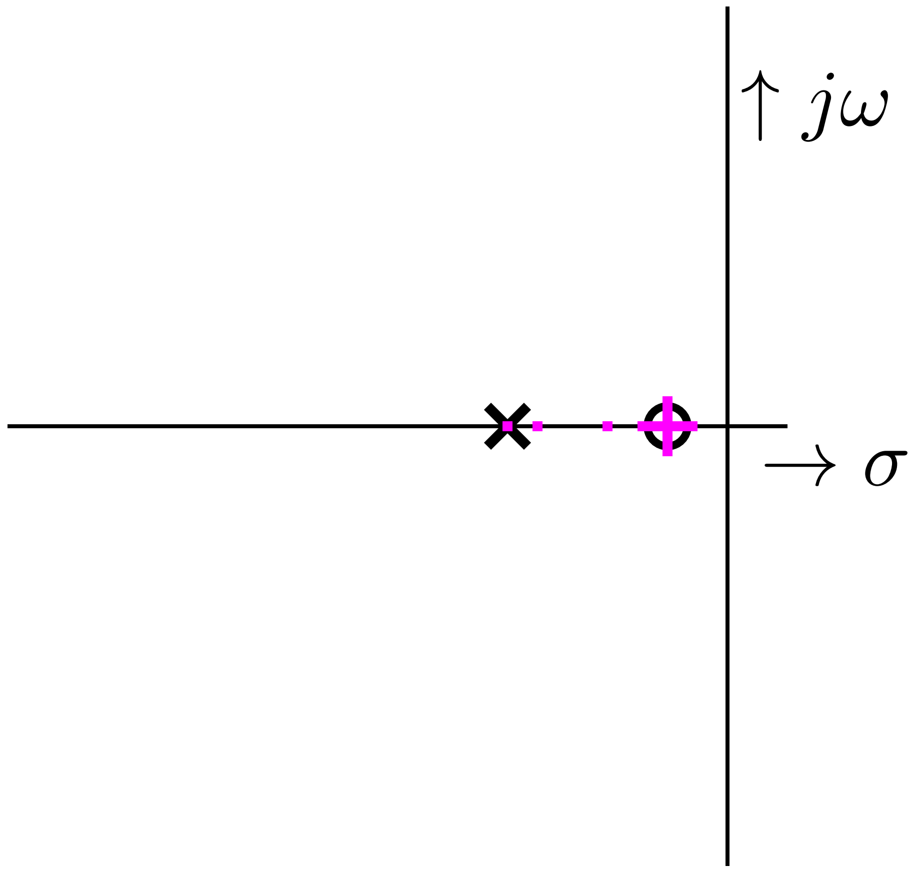

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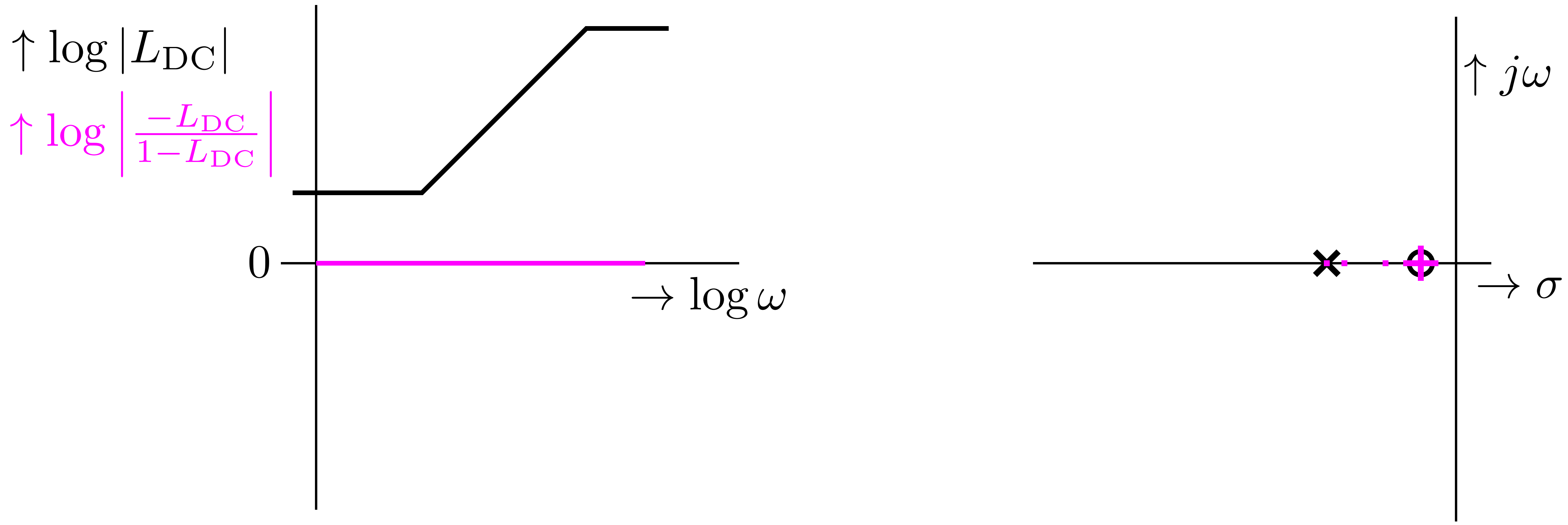

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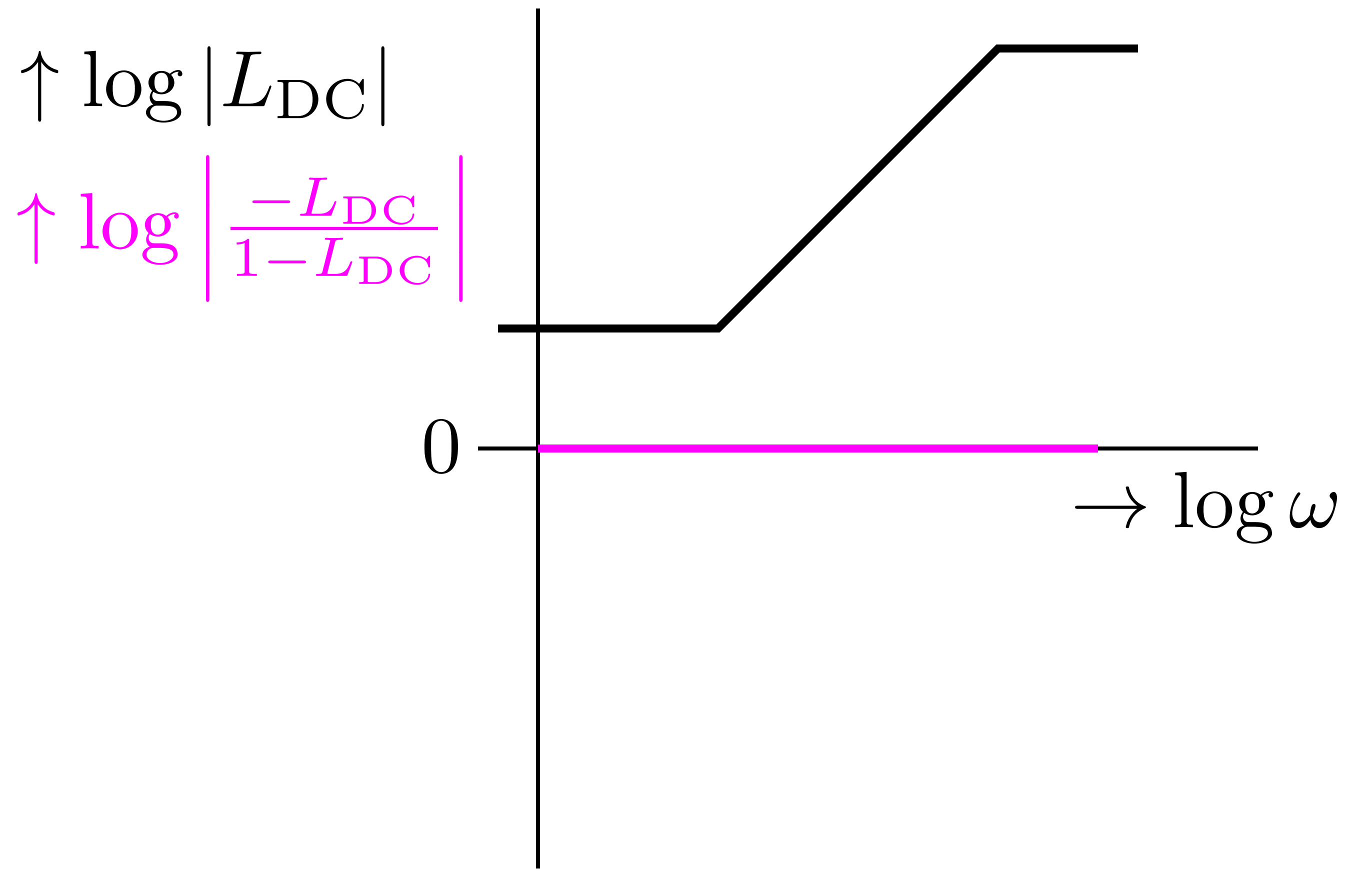
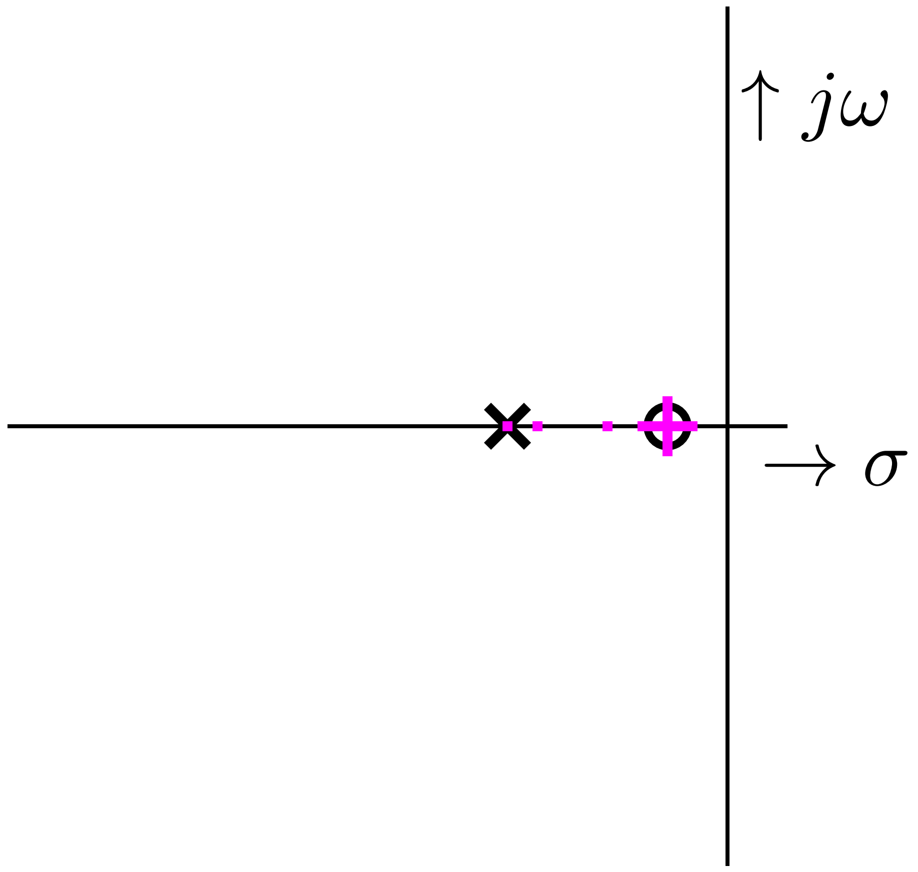
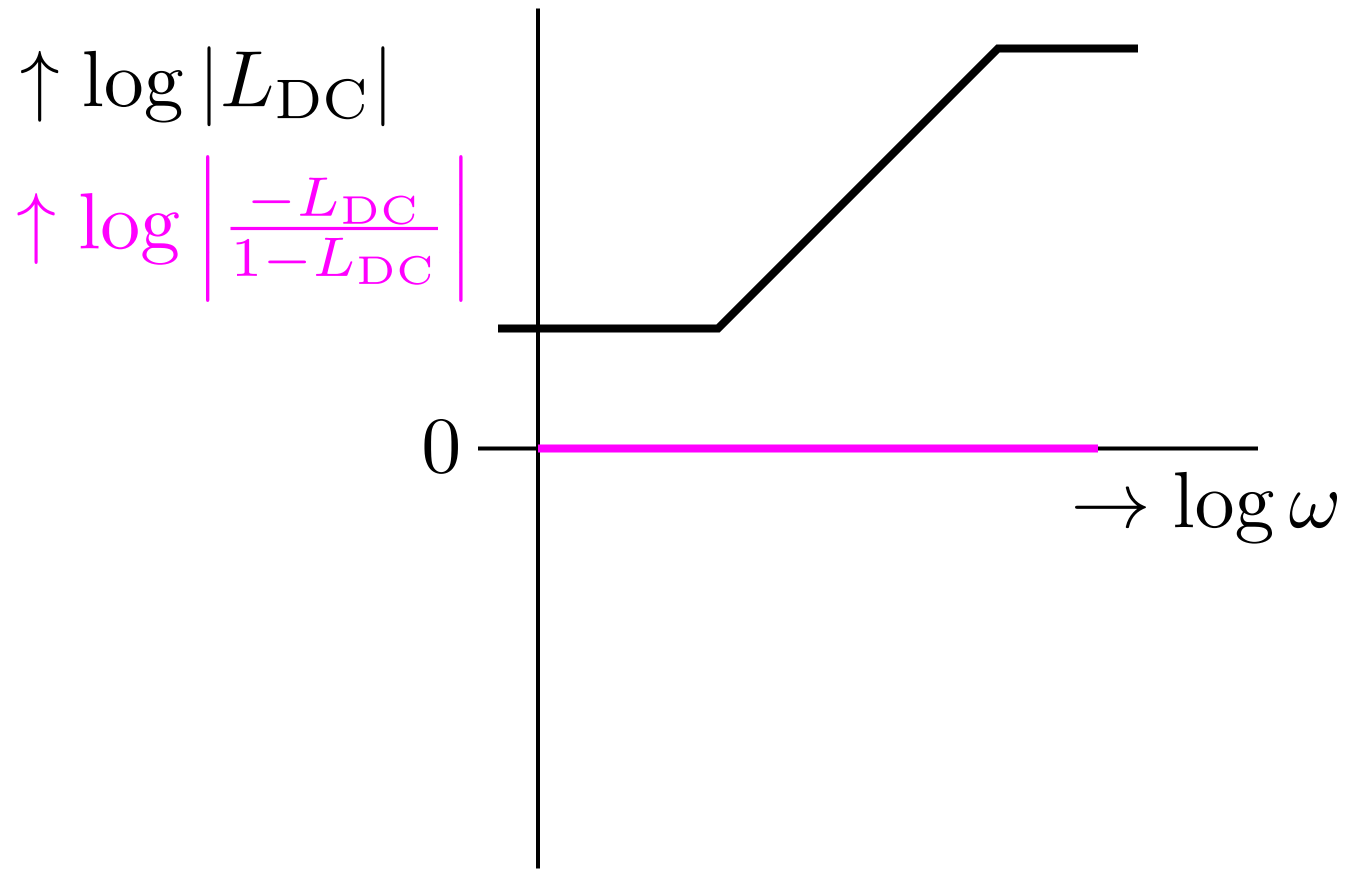
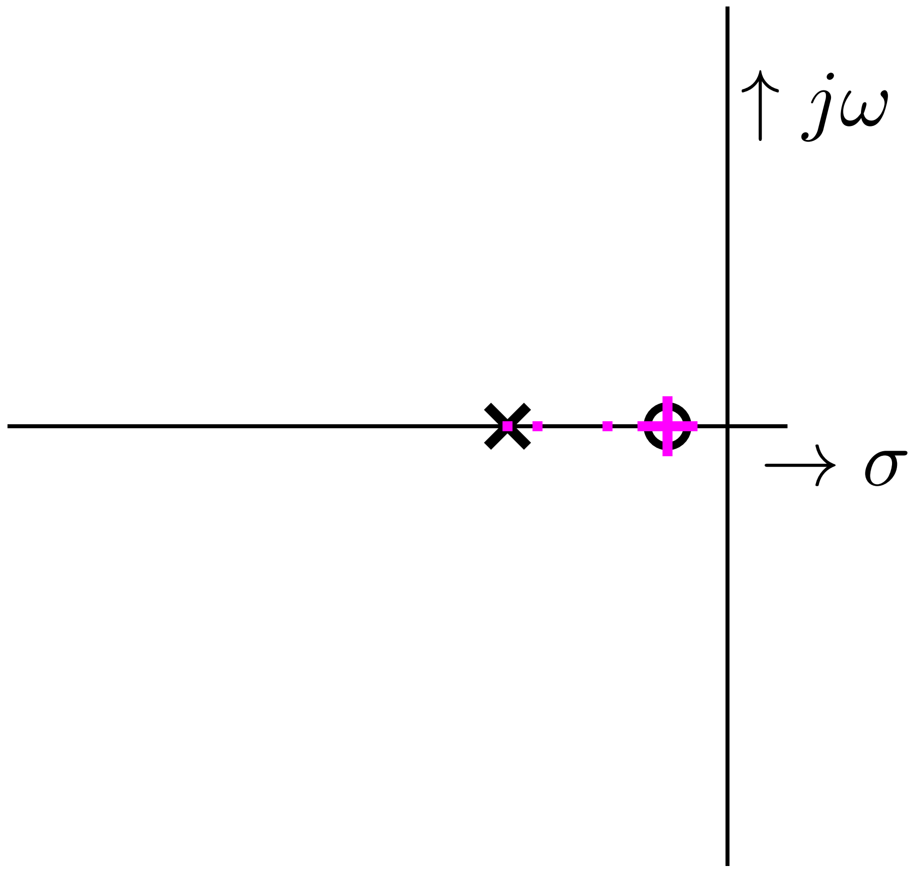




Note: pole only drops on the zero if DC loop gain is infinite!

Root locus first order with zero right

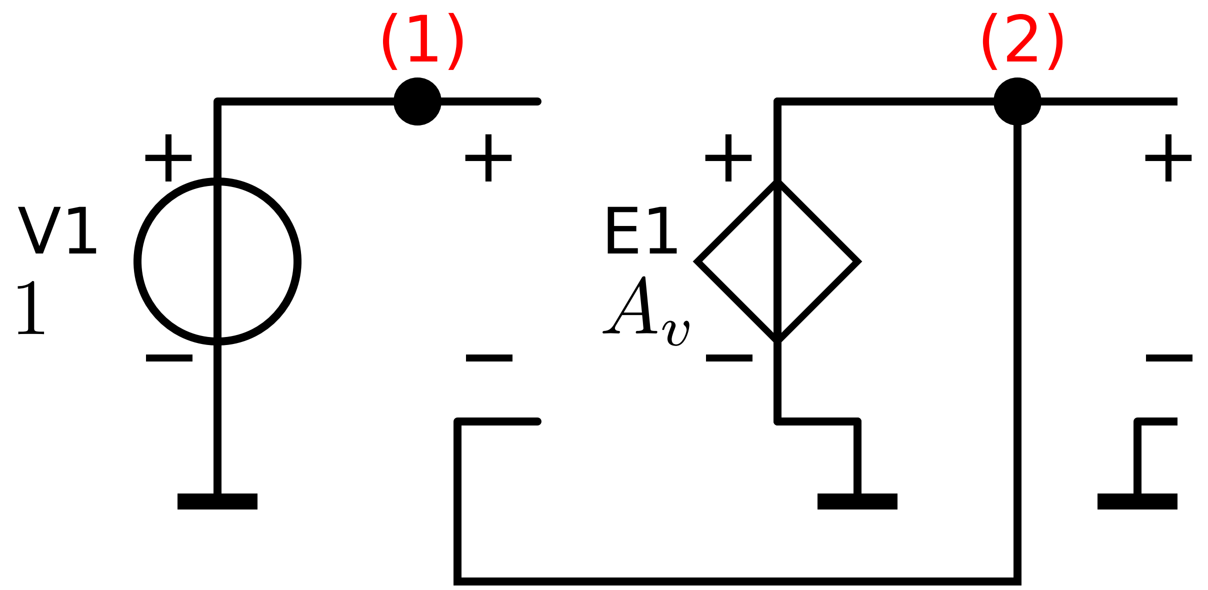




Note: pole only drops on the zero if DC loop gain is infinite!

Root locus SLiCAP

Circuit for plotting root locus



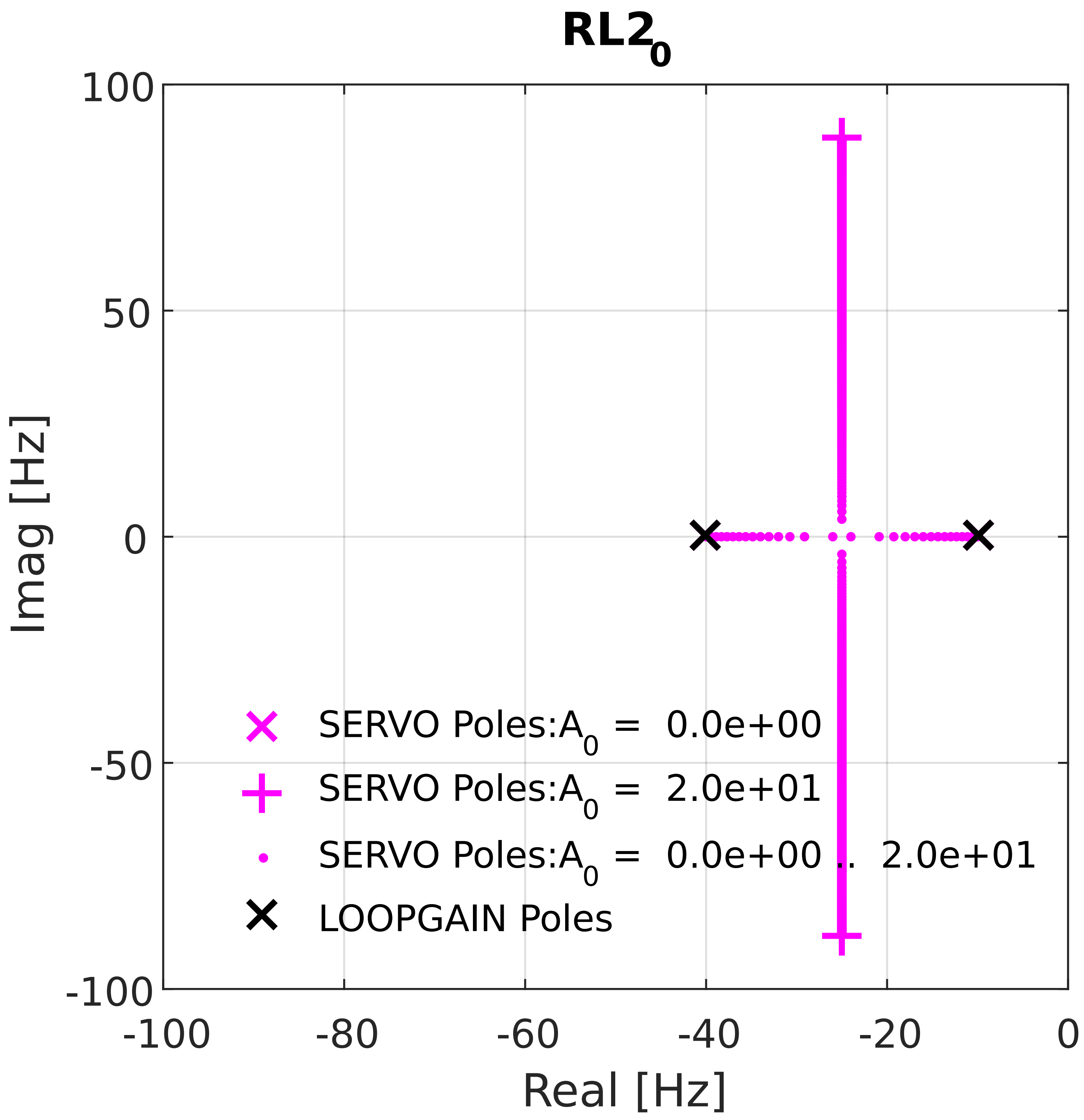
E1 = loop gain reference
Loop gain equals voltage gain of E1
Transfer of E1 has DC gain, poles and zeros

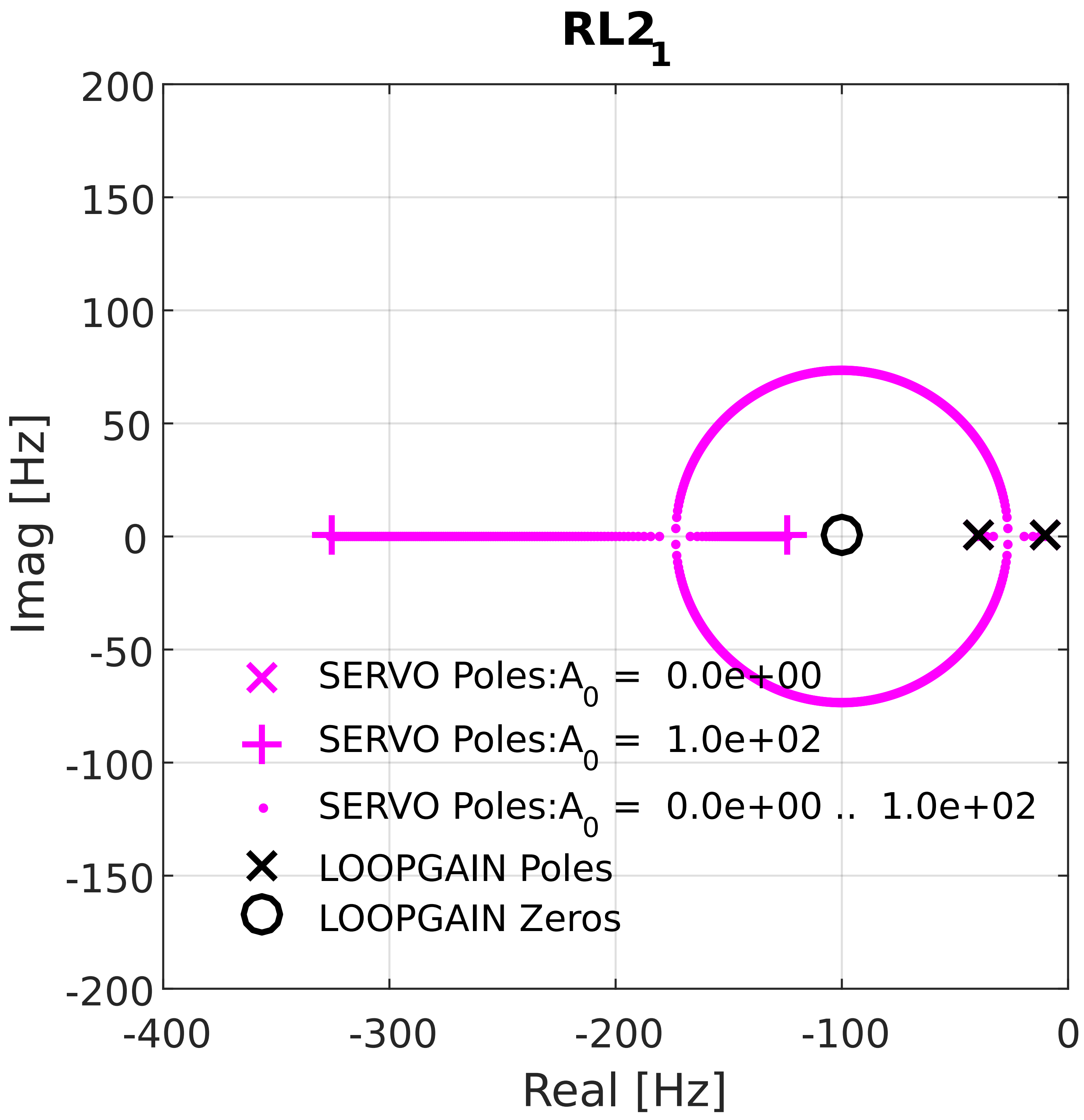
Root locus plot:

1. Poles of the loop gain
2. Zeros of the loop gain
3. Poles of the servo function while stepping the DC gain of E1

See section 11.5.3

Root locus SLiCAP second order





Root locus SLiCAP third order