Structured Electronic Design

Phantom Zero Compensation of 2nd-order Systems

Anton J.M. Montagne

Phantom compensation 2nd order system

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}}-\frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}}-\frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})}-\frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right)+\frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right) + \frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right) + \frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑ A negative real zero increases the coefficient of s

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}}-\frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})}-\frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right)+\frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z}-\frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right)+\frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑

A negative real zero increases the coefficient of s

Absolute value of the sum of the poles can only be increased!

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑

A negative real zero increases the coefficient of s
Absolute value of the sum of the poles can only be increased!

↓

Second-order MFM if:

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑

A negative real zero increases the coefficient of s

Absolute value of the sum of the poles can only be increased!

↓

Second-order MFM if: $S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{\omega_h} + \frac{s^2}{\omega_h^2}}$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right) + \frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right) + \frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑

A negative real zero increases the coefficient of s

Absolute value of the sum of the poles can only be increased!

↓

Second-order MFM if: $S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{\omega_h} + \frac{s^2}{\omega_h^2}} \rightarrow \left\{ z = -\frac{\omega_h^2}{\sqrt{2}\omega_h + p_1 + p_2} \right.$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right)+\frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right)+\frac{s^2}{p_1p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑

A negative real zero increases the coefficient of s
Absolute value of the sum of the poles can only be increased!

↓

Second-order MFM if: $S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{\omega_h}+\frac{s^2}{\omega_h^2}} \rightarrow \begin{cases} z = -\frac{\omega_h^2}{\sqrt{2}\omega_h+p_1+p_2} \\ |p_1+p_2| < \frac{\omega_h}{\sqrt{2}} \end{cases}$

Phantom compensation 2nd order system

$$L(s) = L_{DC}\frac{1-s/z}{(1-s/p_1)(1-s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1-L_{DC}}\frac{1-s/z}{\frac{(1-s/p_1)(1-s/p_2)}{1-L_{DC}} - \frac{L_{DC}}{1-L_{DC}}(1-s/z)}$$

$$A_f = A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(\frac{L_{DC}}{z(1-L_{DC})} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Phantom zero canceled by pole in the ideal gain (= asymptotic gain)

$$A_f \approx A_i(s)\frac{-L_{DC}}{1-L_{DC}}\frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1 p_2(1-L_{DC})}\right) + \frac{s^2}{p_1 p_2(1-L_{DC})}}$$

← Low-pass cut-off frequency designed with LP product

↑

A negative real zero increases the coefficient of s

Absolute value of the sum of the poles can only be increased!

↓

Second-order MFM if: $S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{\omega_h}+\frac{s^2}{\omega_h^2}}$ → $\begin{cases} z = -\frac{\omega_h^2}{\sqrt{2}\omega_h + p_1 + p_2} \\ |p_1+p_2| < \frac{\omega_h}{\sqrt{2}} \end{cases}$

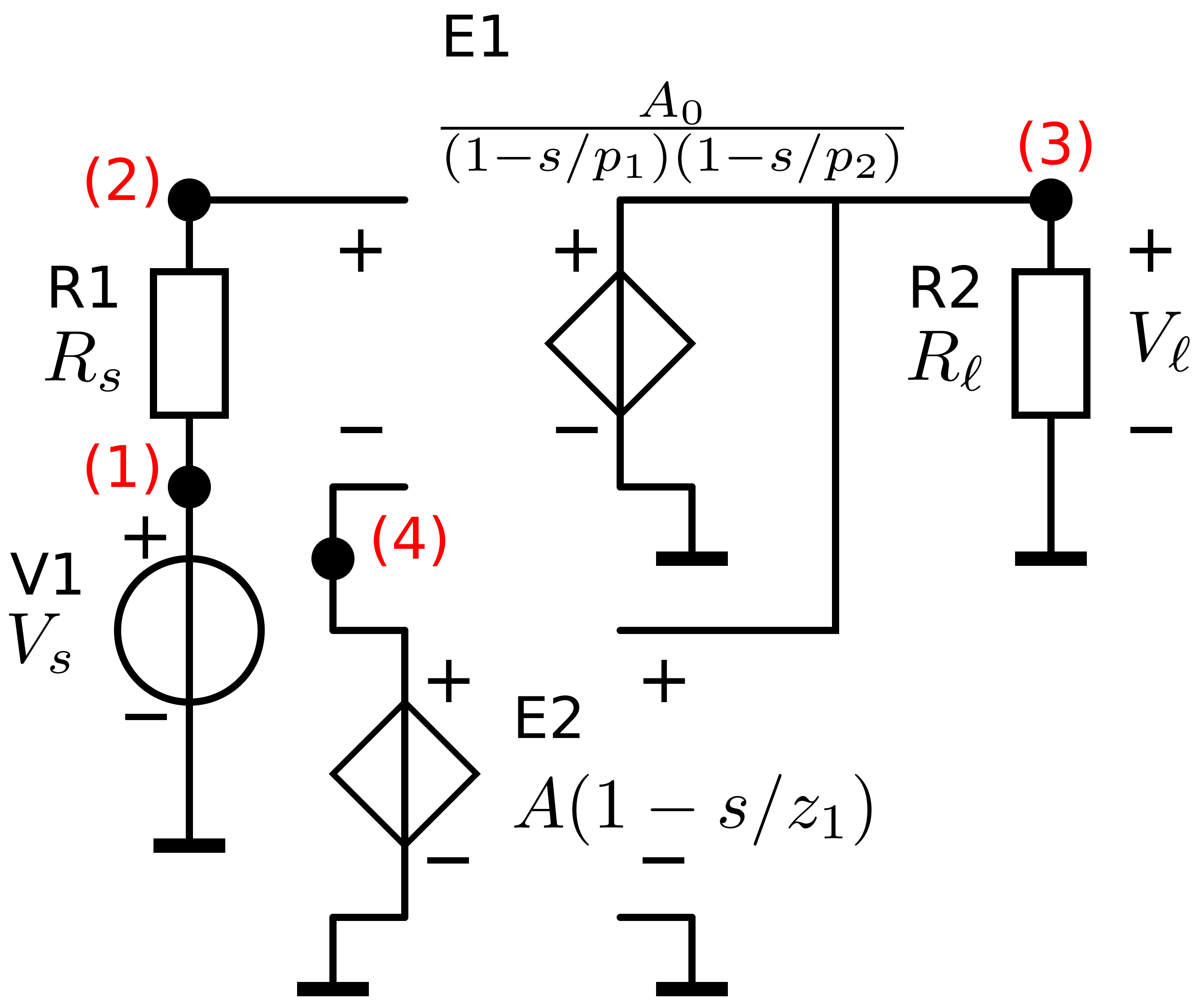
Phantom compensation 2nd order system

Phantom compensation 2nd order system

Example 12.2

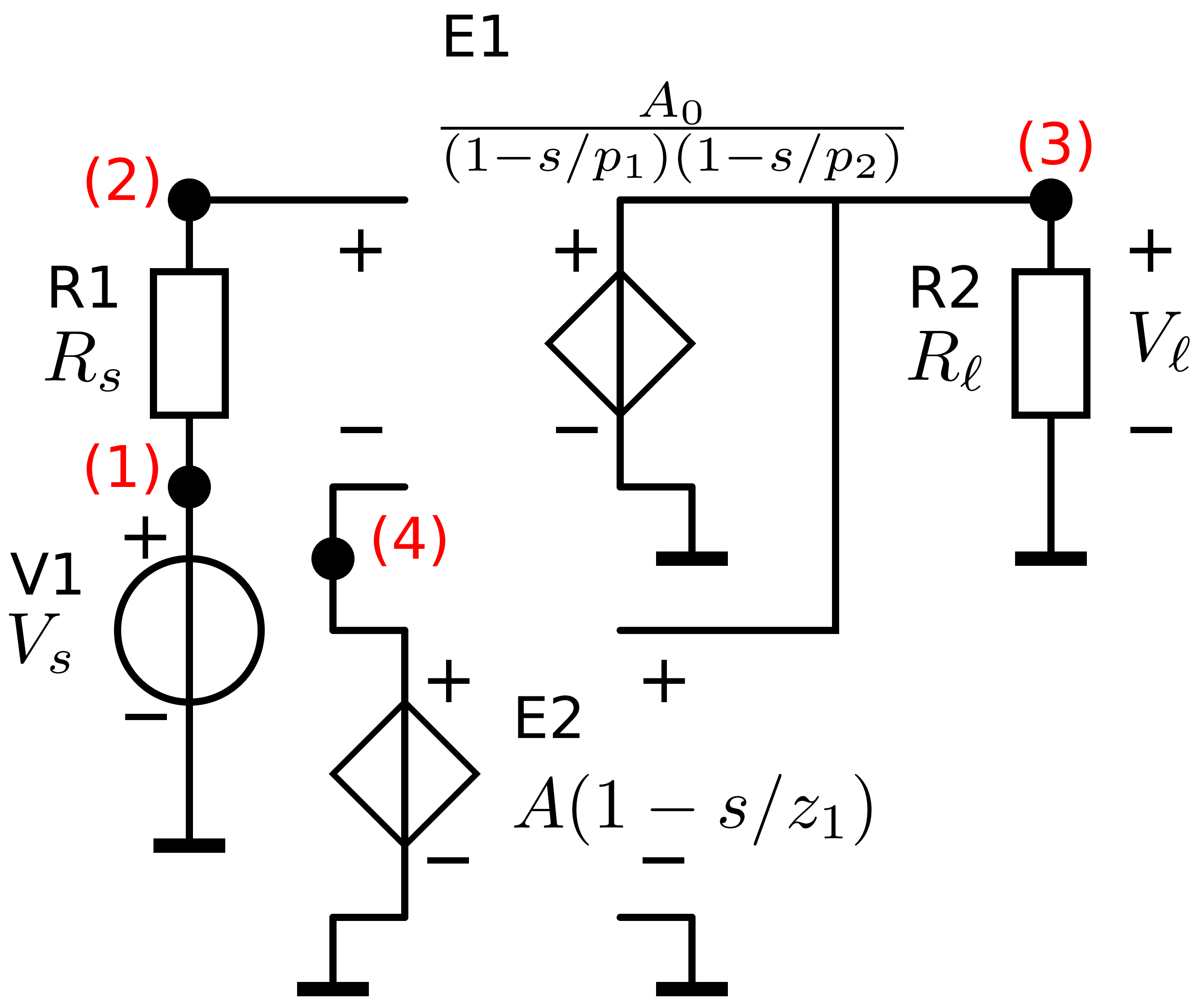
Phantom compensation 2nd order system

Example 12.2



Phantom compensation 2nd order system

Example 12.2



$$p_1 = -1 \text{ Hz},$$

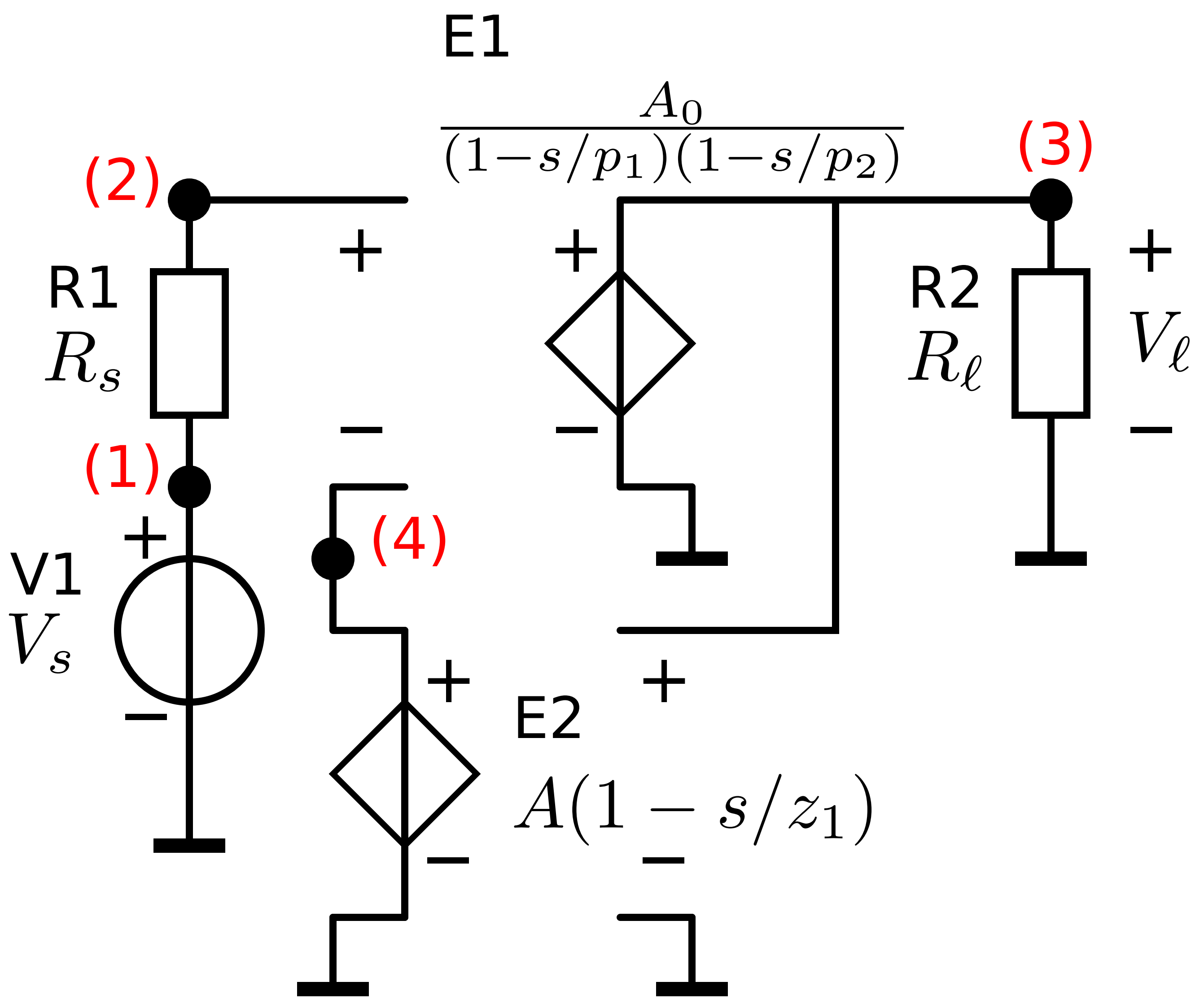
$$p_2 = -100 \text{ Hz},$$

$$A_0 = 10^6 \text{ [-]},$$

$$A = \frac{1}{100} \text{ [-]}.$$

Phantom compensation 2nd order system

Example 12.2

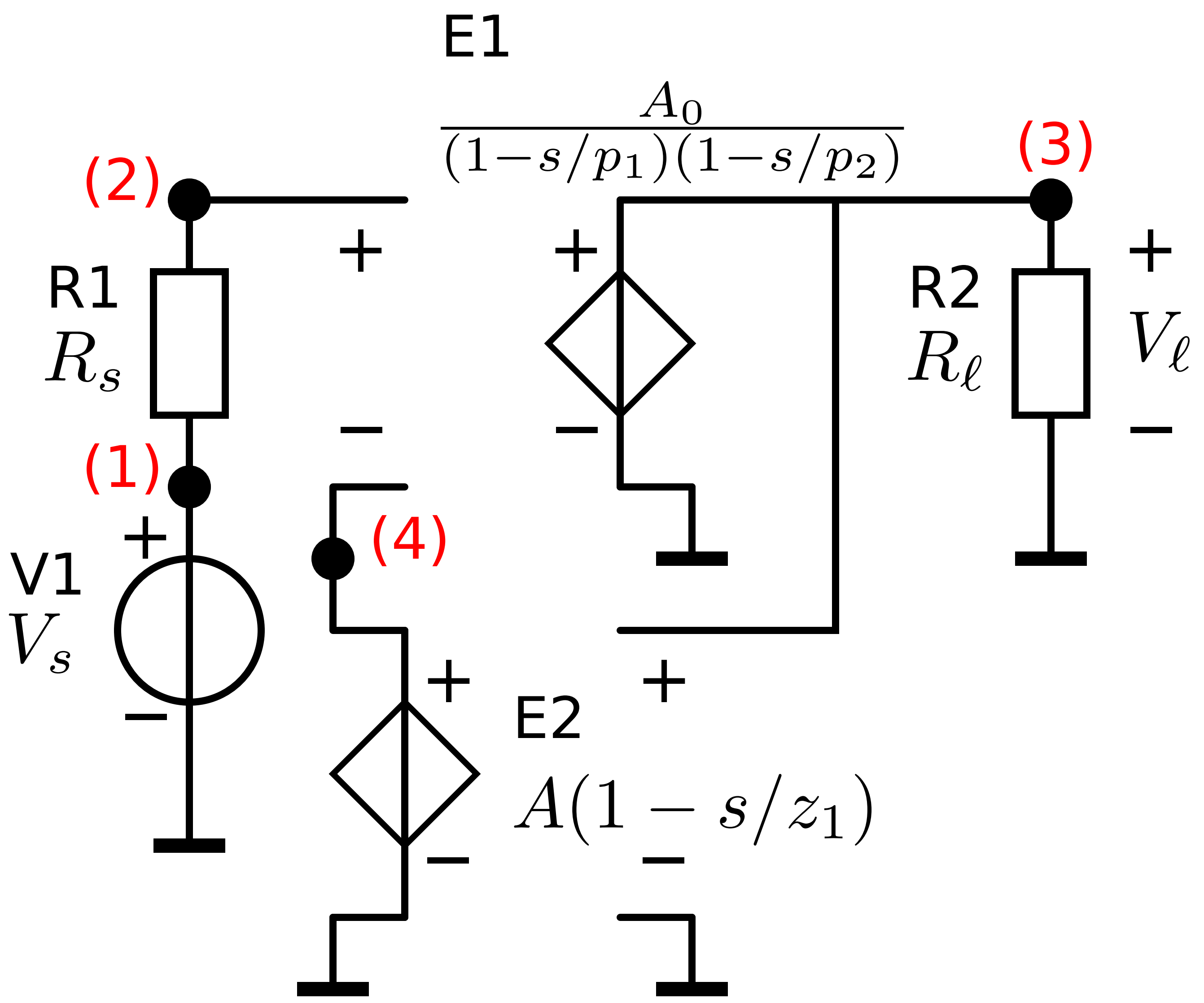


$$p_1 = -1 \text{ Hz},$$
$$p_2 = -100 \text{ Hz},$$
$$A_0 = 10^6 \text{ [-]},$$
$$A = \frac{1}{100} \text{ [-]}.$$

$$f_h = \sqrt{1 \cdot 100 \cdot \frac{1}{100} \cdot 10^6} = 1000 \text{ [Hz]}$$

Phantom compensation 2nd order system

Example 12.2



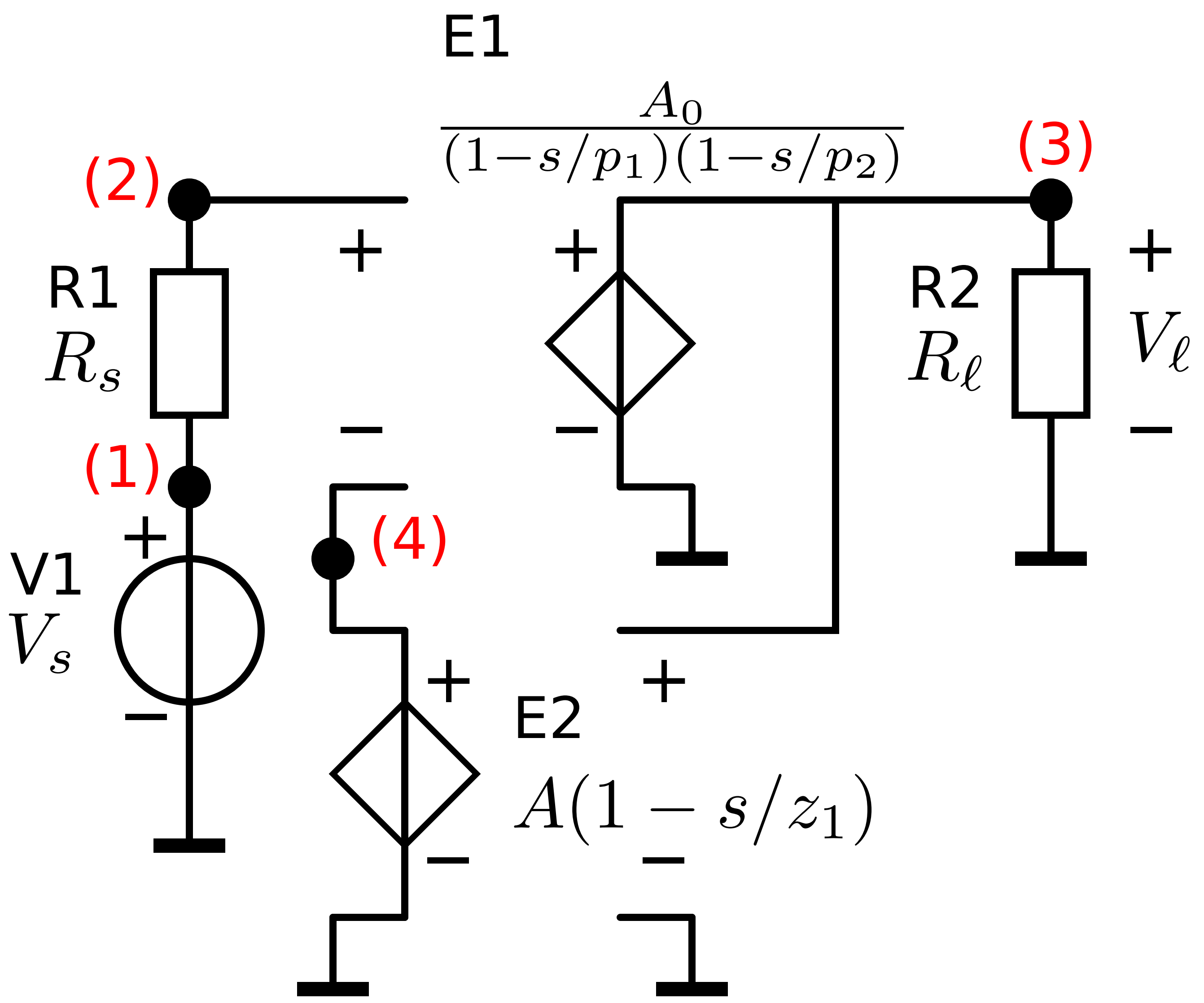
$$p_1 = -1 \text{ Hz},$$
$$p_2 = -100 \text{ Hz},$$
$$A_0 = 10^6 \text{ [-]},$$
$$A = \frac{1}{100} \text{ [-]}.$$

$$f_h = \sqrt{1 \cdot 100 \cdot \frac{1}{100} \cdot 10^6} = 1000 \text{ [Hz]}$$

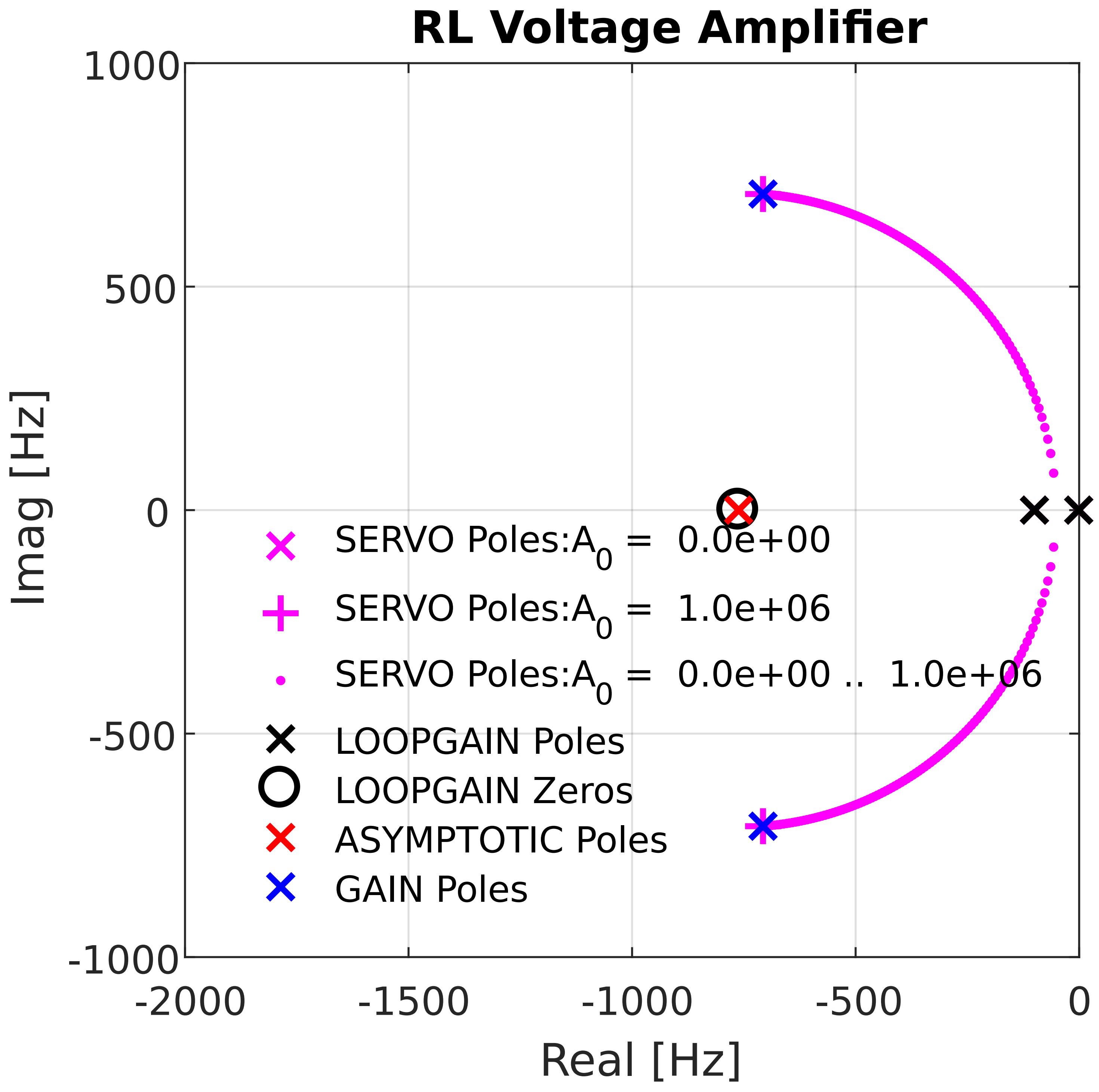
$$z = -\frac{10^6}{1000\sqrt{2}-101} = -761.49 \text{ [Hz]}$$

Phantom compensation 2nd order system

Example 12.2



$$p_1 = -1 \text{ Hz},$$
$$p_2 = -100 \text{ Hz},$$
$$A_0 = 10^6 \text{ [-]},$$
$$A = \frac{1}{100} \text{ [-]}.$$

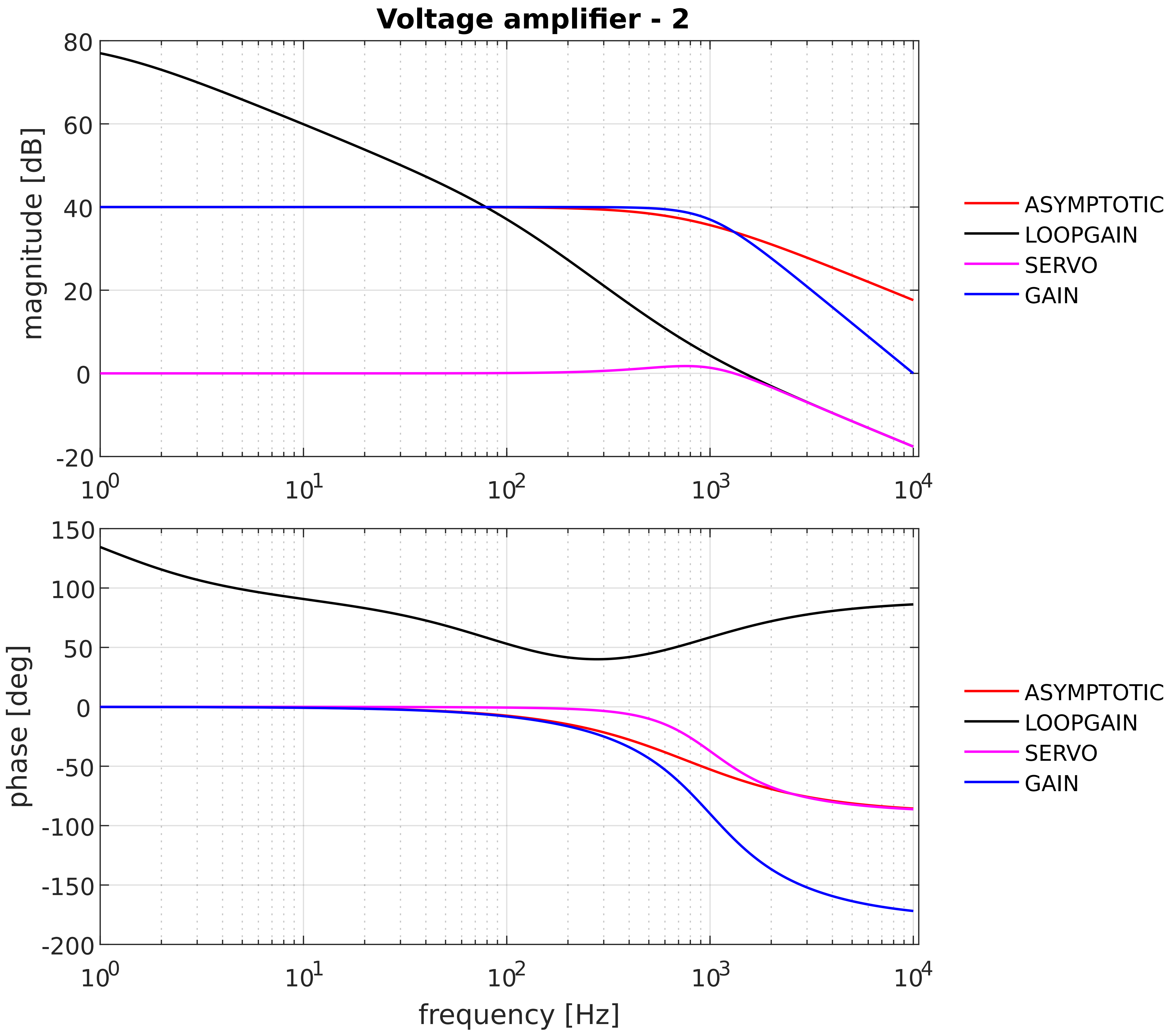


$$f_h = \sqrt{1 \cdot 100 \cdot \frac{1}{100} \cdot 10^6} = 1000 \text{ [Hz]}$$

$$z = -\frac{10^6}{1000\sqrt{2}-101} = -761.49 \text{ [Hz]}$$

Phantom compensation 2nd order system

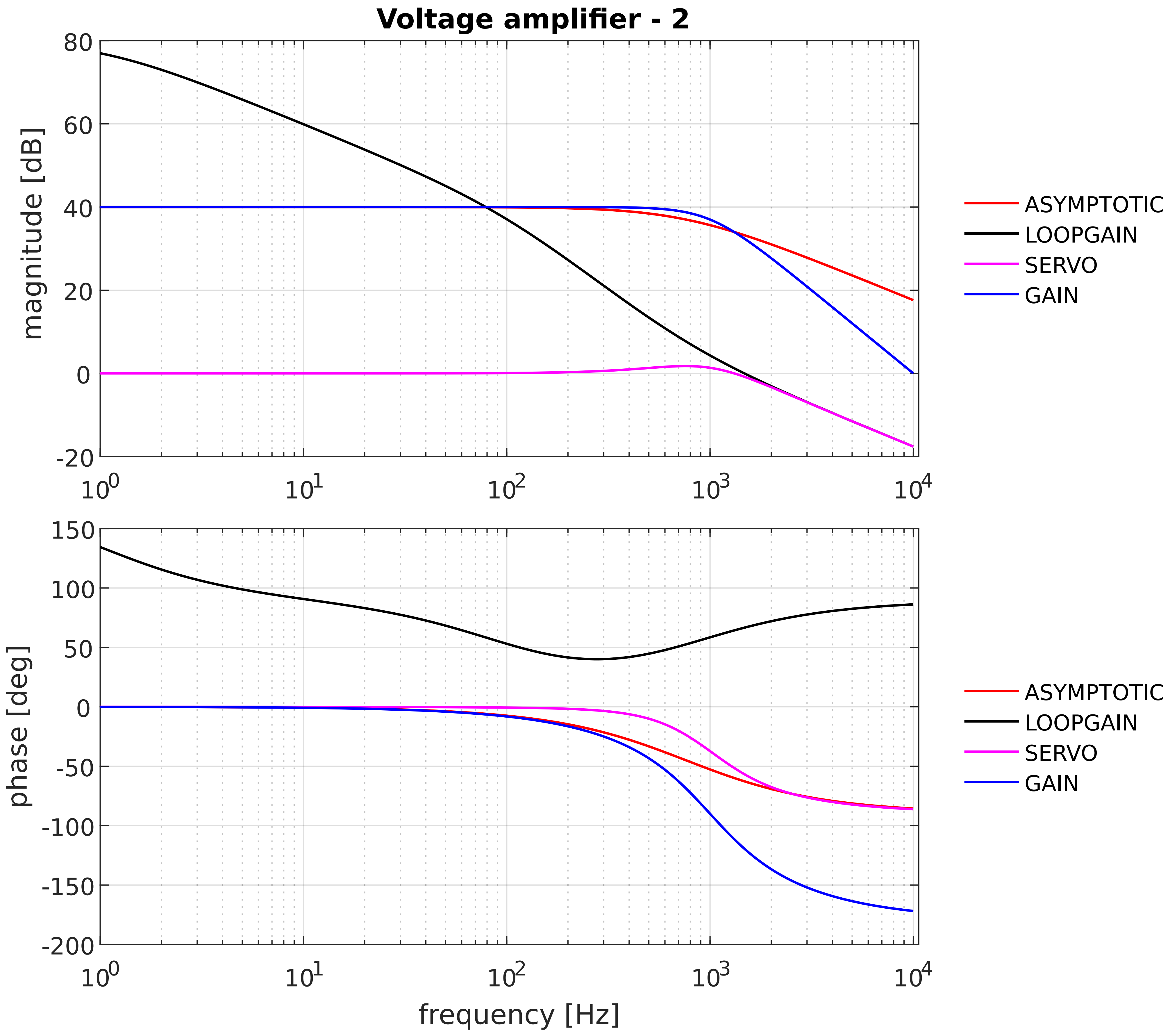
Example 12.2



Phantom compensation 2nd order system

Example 12.2

Phase margin:
67 degrees
at 1.5kHz



Phantom compensation 2nd order system

Example 12.2

Phase margin:
67 degrees
at 1.5kHz