Structured Electronic Design

Phantom Zero Compensation of 2nd-order Systems

Anton J.M. Montagne

$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

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$$S(s) = \frac{-L_{DC}}{1 - L_{DC}} \frac{1 - s/z}{\frac{(1 - s/p_1)(1 - s/p_2)}{1 - L_{DC}} - \frac{L_{DC}}{1 - L_{DC}}(1 - s/z)}}$$

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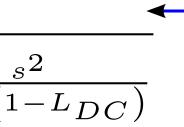
$$A_f = A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(\frac{L_{DC}}{z(1 - L_{DC})} - \frac{p_1 + p_2}{p_1 p_2(1 - L_{DC})}\right) + \frac{1}{p_1 p_2(z_1)}}$$

 $\frac{s^2}{\left(1-L_{DC}\right)}$

$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

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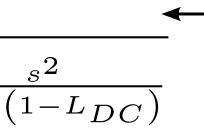
 Phantom zero canceled $\frac{s^2}{(1-L_{DC})}$ by pole in the ideal gain (= asymptotic gain)

$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

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$$A_f = A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(\frac{L_{DC}}{z(1 - L_{DC})} - \frac{p_1 + p_2}{p_1 p_2(1 - L_{DC})}\right) + \frac{1}{p_1 p_2(z_1)}}$$

$$A_f \approx A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(-\frac{1}{z} - \frac{p_1 + p_2}{p_1 p_2 (1 - L_{DC})} \right) + \frac{s^2}{p_1 p_2 (1 - L_{DC})}}$$



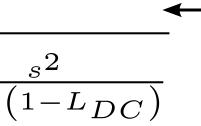
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$$A_f = A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(\frac{L_{DC}}{z(1 - L_{DC})} - \frac{p_1 + p_2}{p_1 p_2(1 - L_{DC})}\right) + \frac{1}{p_1 p_2(z_1)}}$$

$$A_f \approx A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(-\frac{1}{z} - \frac{p_1 + p_2}{p_1 p_2 (1 - L_{DC})} \right) + \frac{s^2}{p_1 p_2 (1 - L_{DC})}}$$



Phantom zero canceled
 by pole in the ideal gain
 (= asymptotic gain)

Low-pass cut-off frequency designed with LP product

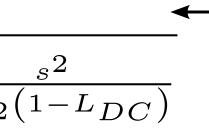
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$$A_{f} \approx A_{i}(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(-\frac{1}{z} - \frac{p_{1} + p_{2}}{p_{1}p_{2}(1 - L_{DC})} \right) + \frac{s^{2}}{p_{1}p_{2}(1 - L_{DC})}}$$

A negative real zero increases the coefficient of s



Phantom zero canceled
 by pole in the ideal gain
 (= asymptotic gain)

) - Low-pass cut-off frequency designed with LP product

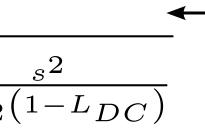
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A negative real zero increases the coefficient of s Absolute value of the sum of the poles can only be increased!



Phantom zero canceled
 by pole in the ideal gain
 (= asymptotic gain)

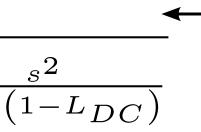
) - Low-pass cut-off frequency designed with LP product

$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

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$$A_f = A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(\frac{L_{DC}}{z(1 - L_{DC})} - \frac{p_1 + p_2}{p_1 p_2(1 - L_{DC})}\right) + \frac{1}{p_1 p_2(z_1)}}$$

Second-order MFM if:



Phantom zero canceled
 by pole in the ideal gain
 (= asymptotic gain)

Low-pass cut-off frequency designed with LP product

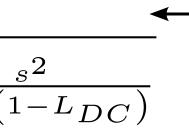
ncreased!

$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

$$S(s) = \frac{-L_{DC}}{1 - L_{DC}} \frac{1 - s/z}{\frac{(1 - s/p_1)(1 - s/p_2)}{1 - L_{DC}} - \frac{L_{DC}}{1 - L_{DC}}(1 - s/z)}}$$

$$A_f = A_i(s) \frac{-L_{DC}}{1 - L_{DC}} \frac{1}{1 + s \left(\frac{L_{DC}}{z(1 - L_{DC})} - \frac{p_1 + p_2}{p_1 p_2(1 - L_{DC})}\right) + \frac{1}{p_1 p_2(z_1)}}$$

$$\begin{split} A_f \approx A_i(s) \frac{-L_{DC}}{1-L_{DC}} \frac{1}{1+s\left(-\frac{1}{z}-\frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right) + \frac{s^2}{p_1p_2(1-L_{DC})}} \\ \uparrow \\ \text{A negative real zero increases the coefficient of s} \\ \text{Absolute value of the sum of the poles can only be in} \\ \downarrow \\ \text{Second-order MFM if:} \quad S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{\omega_h} + \frac{s^2}{\omega_h^2}} \end{split}$$



 Phantom zero canceled by pole in the ideal gain $\frac{s^2}{(1-L_{DC})}$ by pole in the ideal gives (= asymptotic gain)

 $\overline{(f)}$ \leftarrow Low-pass cut-off frequency designed with LP product

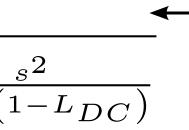
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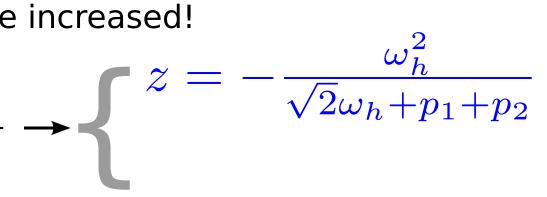
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 Phantom zero canceled $\frac{s^2}{(1-L_{DC})}$ by pole in the ideal gain (= asymptotic gain)

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$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

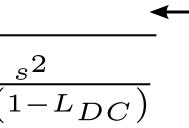
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$$A_f \approx A_i(s) \frac{-L_{DC}}{1-L_{DC}} \frac{1}{1+s\left(-\frac{1}{z} - \frac{p_1+p_2}{p_1p_2(1-L_{DC})}\right) + \frac{s^2}{p_1p_2(1-L_{DC})}}$$

$$\uparrow$$
A negative real zero increases the coefficient of s
Absolute value of the sum of the poles can only be in

$$\downarrow$$
Second-order MFM if: $S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{z}+\frac{s^2}{z}}$



 Phantom zero canceled $\frac{1}{s^2}$ by pole in the ideal gain (= asymptotic gain)

 $\overline{()}$ \leftarrow Low-pass cut-off frequency designed with LP product

ncreased! $\frac{1}{1+s\frac{\sqrt{2}}{\omega_h}+\frac{s^2}{\omega_h^2}} \rightarrow \begin{bmatrix} z = -\frac{\omega_h^2}{\sqrt{2}\omega_h+p_1+p_2} \\ |p_1+p_2| < \frac{\omega_h}{\sqrt{2}} \end{bmatrix}$

$$L(s) = L_{DC} \frac{1 - s/z}{(1 - s/p_1)(1 - s/p_2)}$$

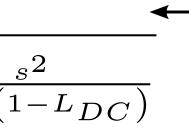
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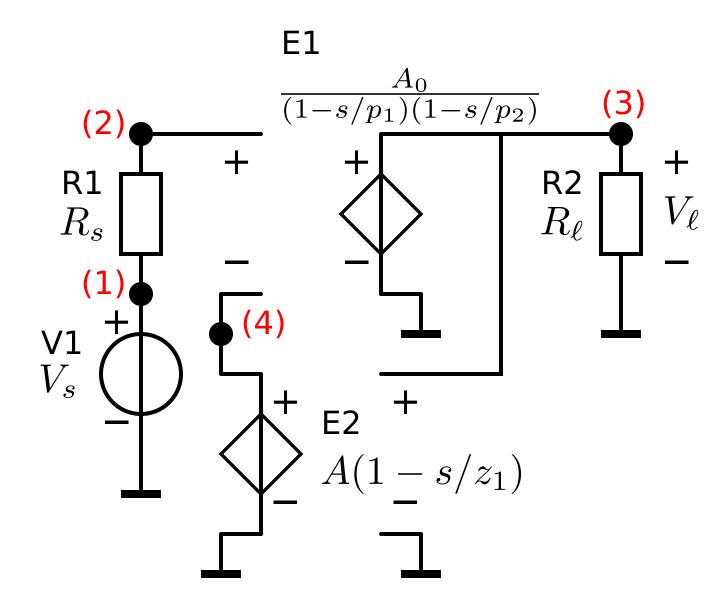
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Second-order MFM if: $S'(s) = \frac{1}{1+s\frac{\sqrt{2}}{z}+\frac{s^2}{z}}$

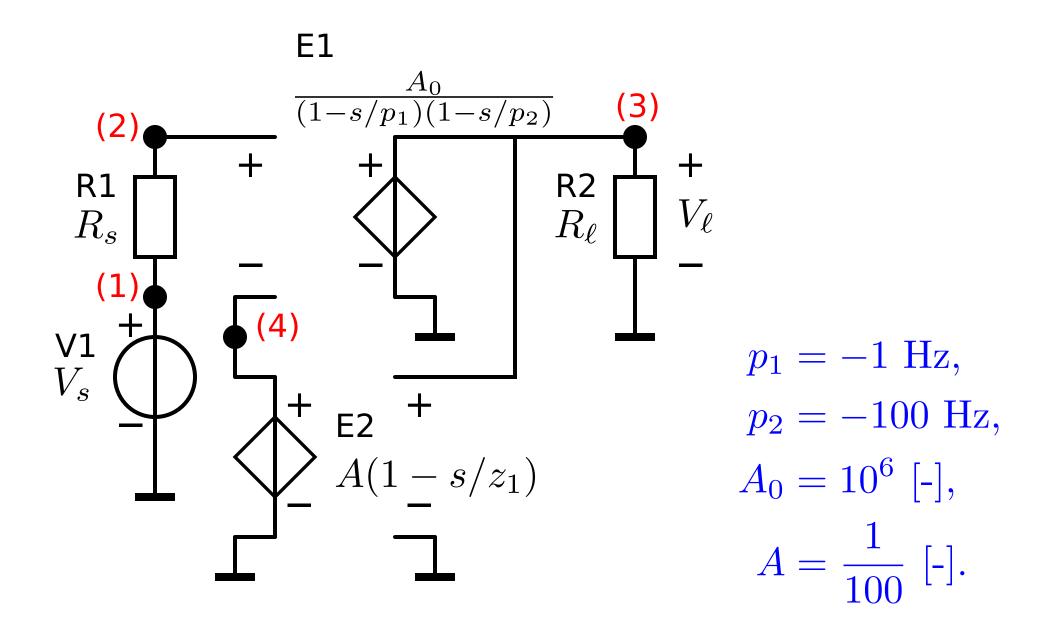


 Phantom zero canceled $\frac{1}{s^2}$ by pole in the ideal gain (= asymptotic gain)

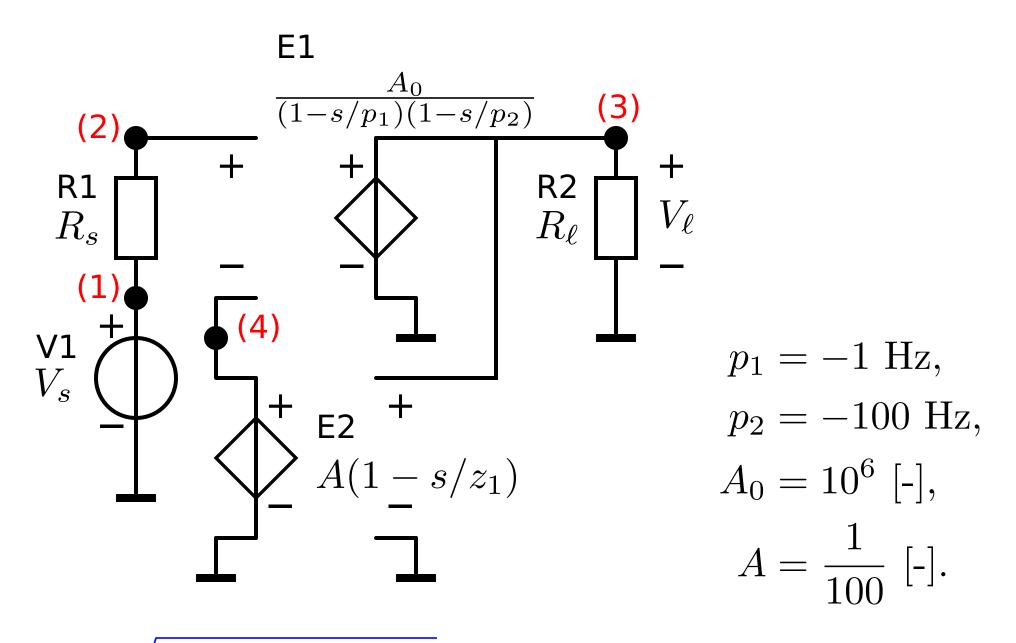
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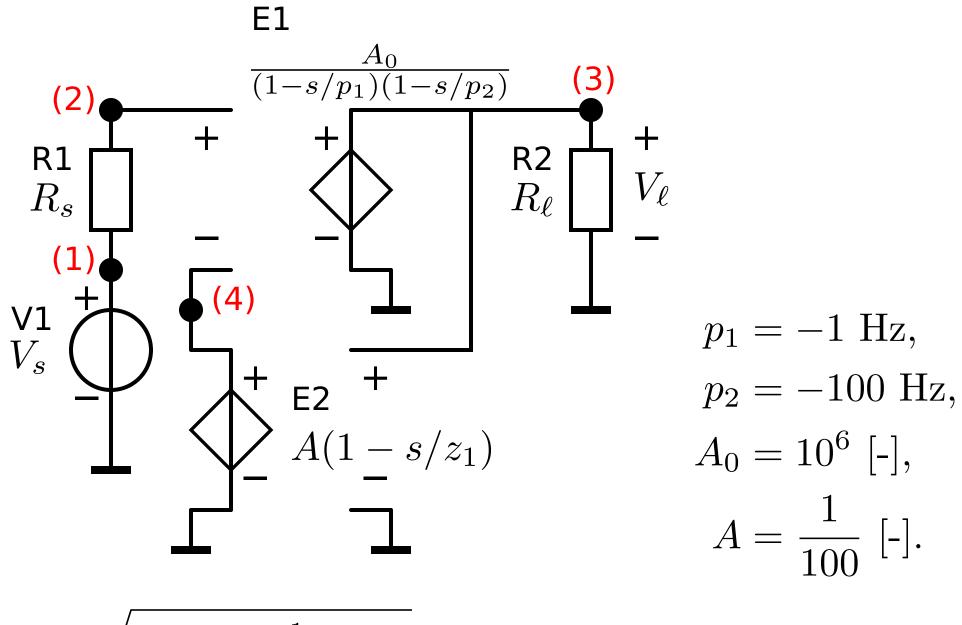




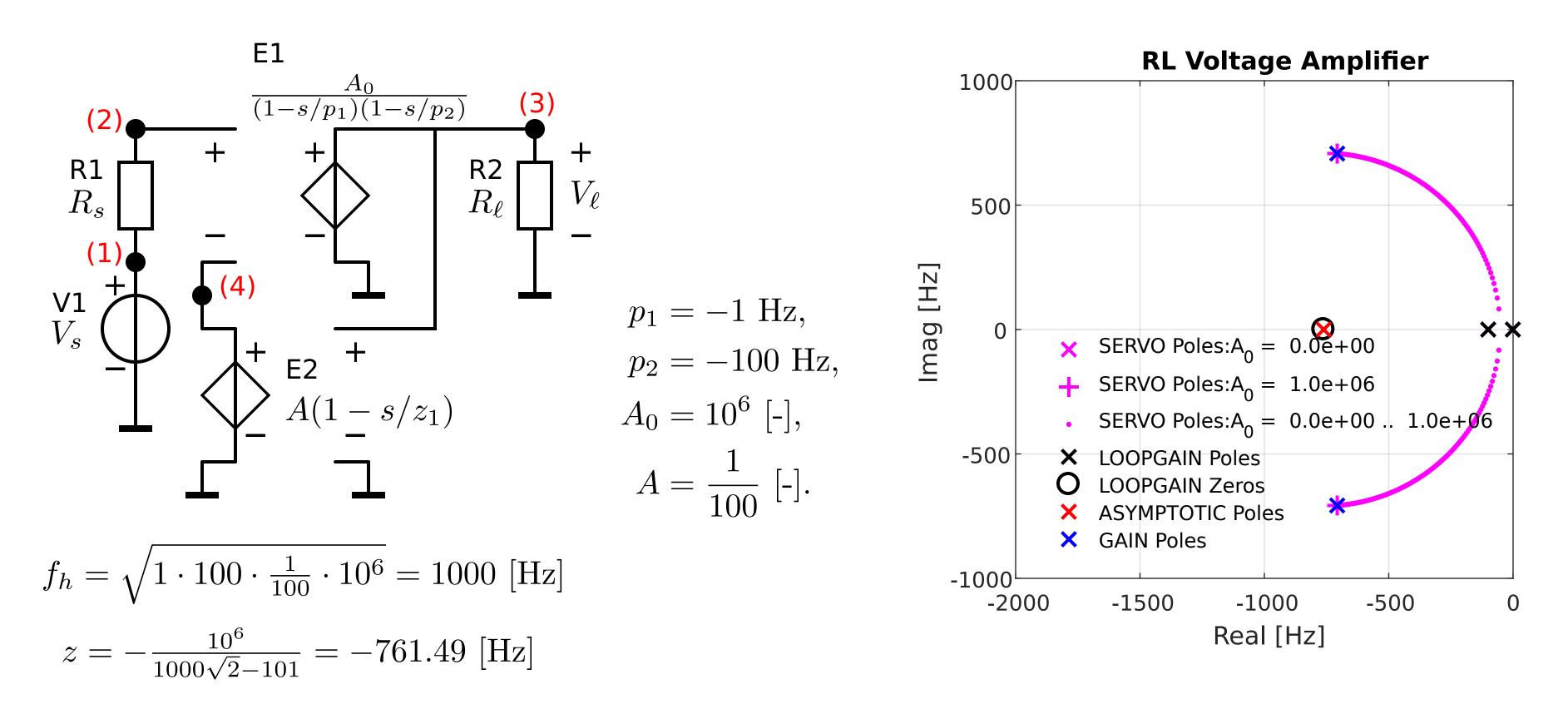
Example 12.2



 $f_h = \sqrt{1 \cdot 100 \cdot \frac{1}{100} \cdot 10^6} = 1000 \text{ [Hz]}$

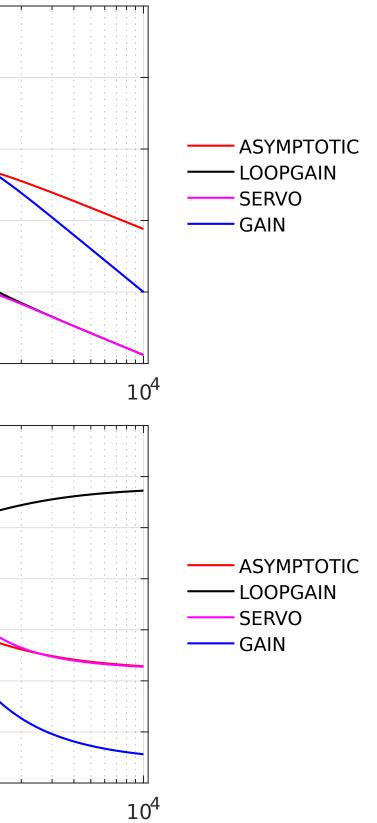


$$f_h = \sqrt{1 \cdot 100 \cdot \frac{1}{100} \cdot 10^6} = 1000 \text{ [Hz]}$$
$$z = -\frac{10^6}{1000\sqrt{2} - 101} = -761.49 \text{ [Hz]}$$



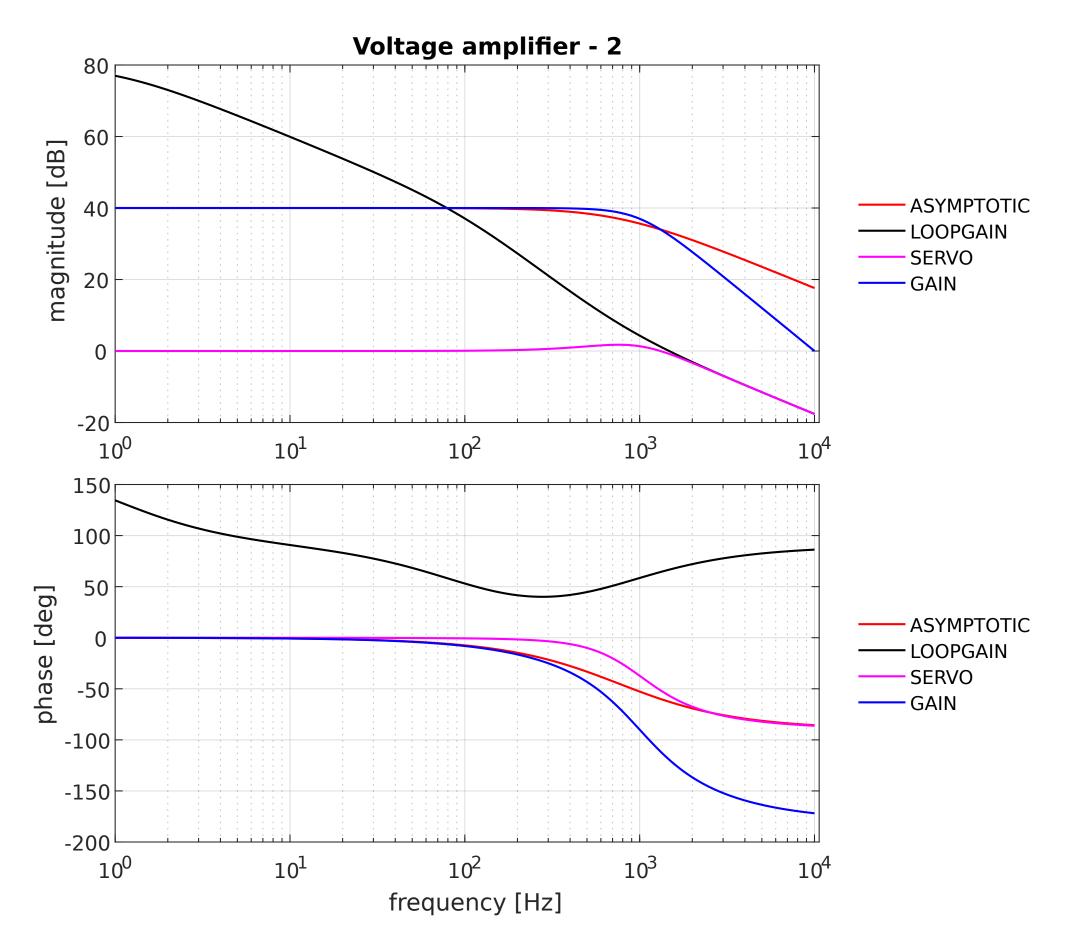
Example 12.2 Voltage amplifier - 2 80 60 magnitude [dB] 40 20 0 -20 10² 10³ 10^{1} 10^{0} 150 100 50 phase [deg] 0 -50 -100 -150 -200 10³ 10² 10^{0} 10^{1}

frequency [Hz]



Example 12.2

Phase margin: 67 degrees at 1.5kHz



Example 12.2

Phase margin: 67 degrees at 1.5kHz

