Structured Electronic Design

Analysis and budgeting of biasing errors

Anton J.M. Montagne

Modeling and analysis of biasing errors

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$$(1+\delta)(1+\epsilon) \approx 1 + \delta + \epsilon;\ \delta \ll 1, \epsilon \ll 1$$

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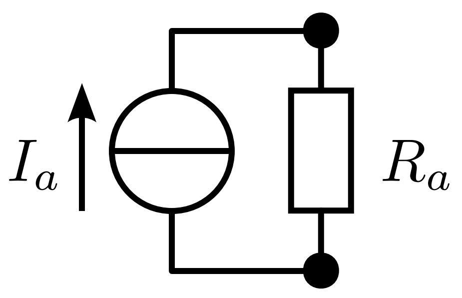

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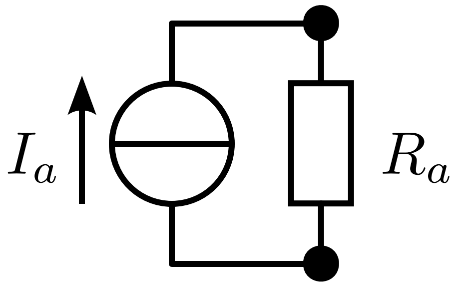

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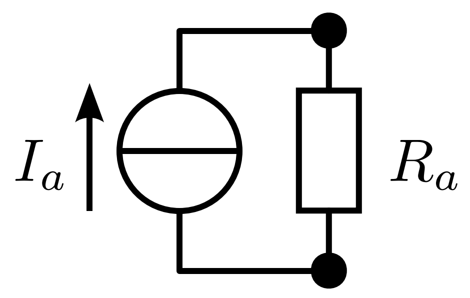

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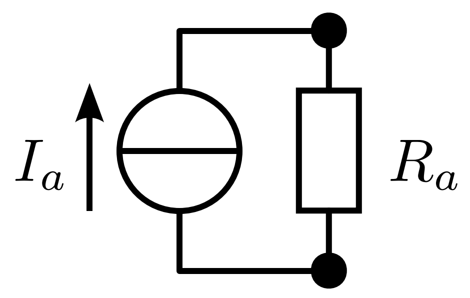

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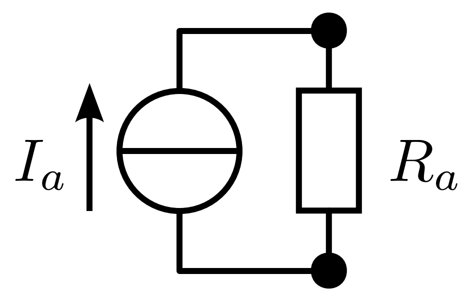

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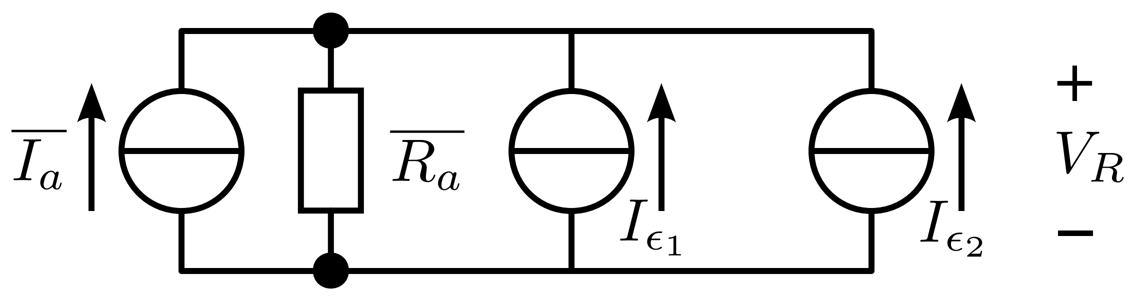

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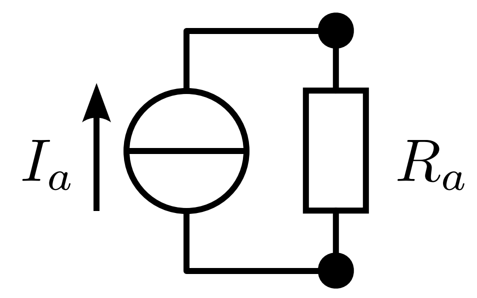

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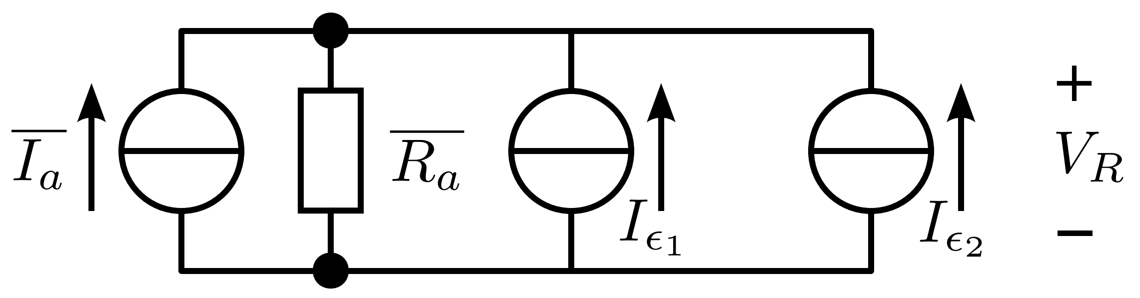

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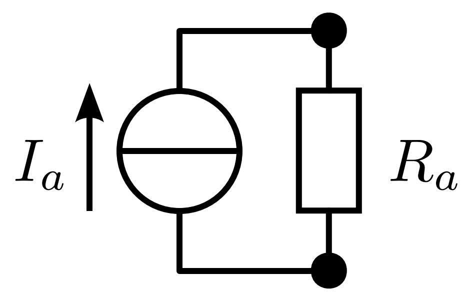

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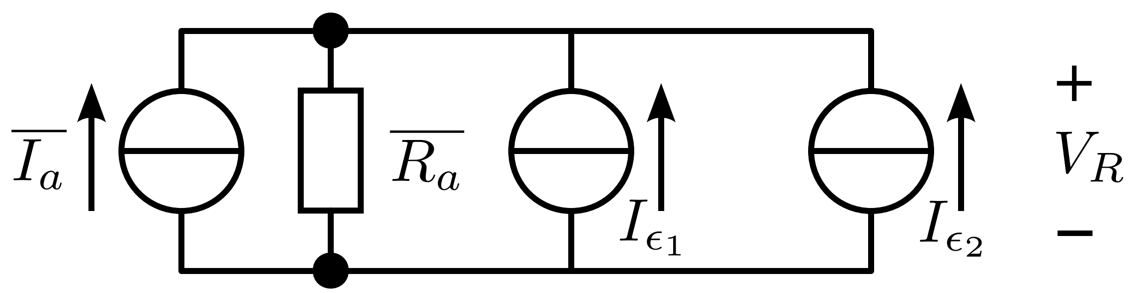

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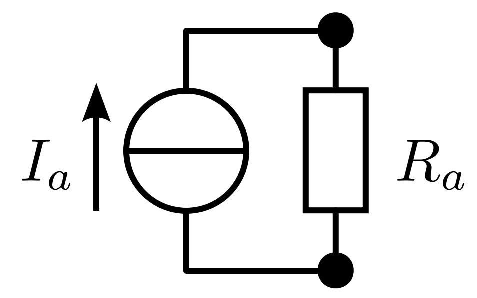

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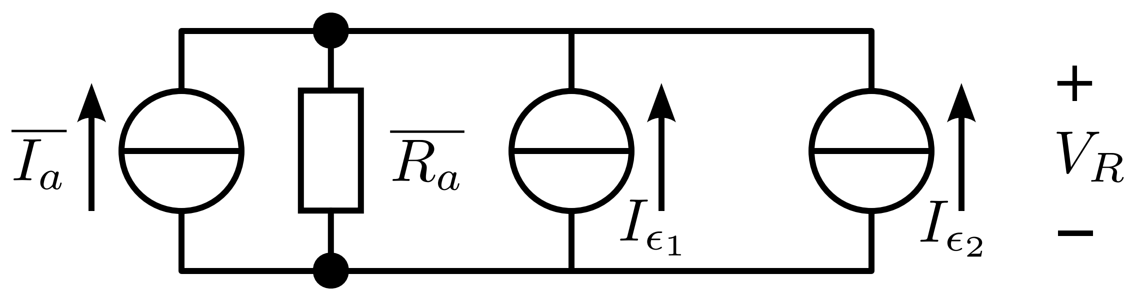

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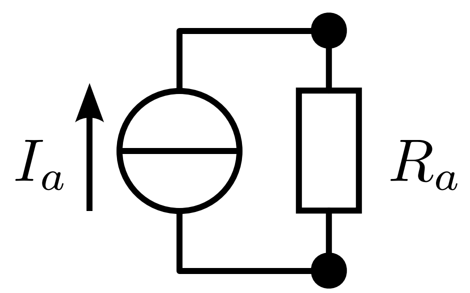

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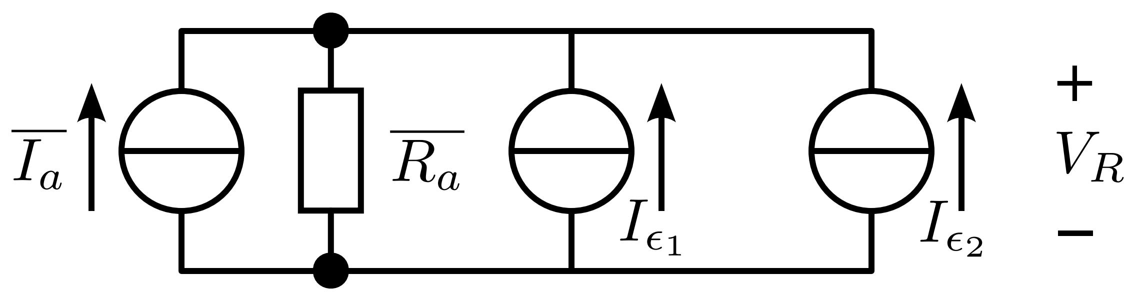

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Calculation of the variance of the voltage across the resistor:

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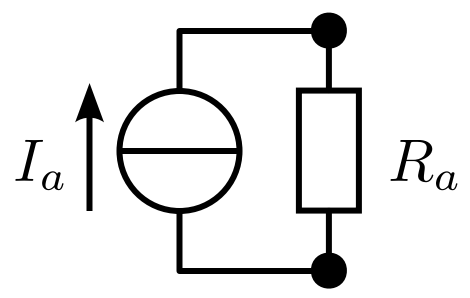

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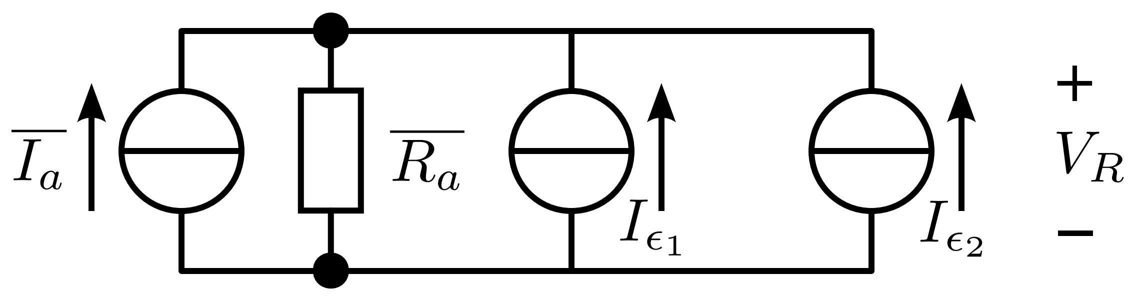

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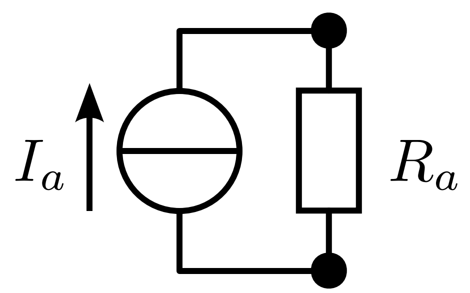

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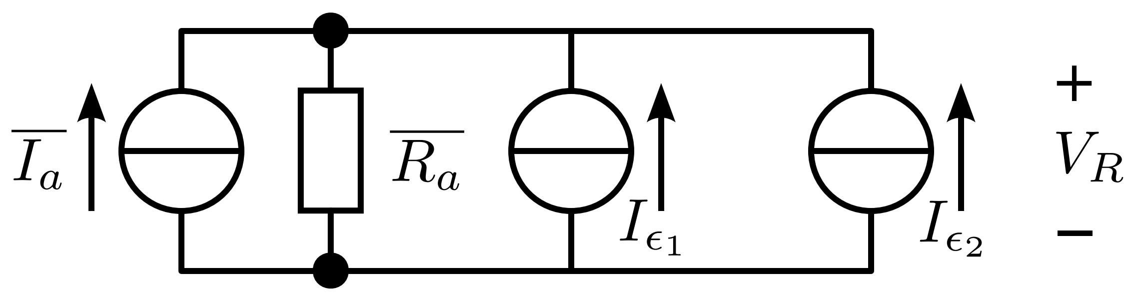

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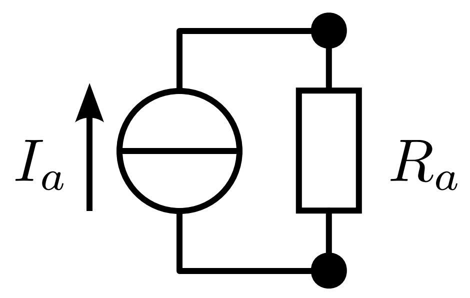

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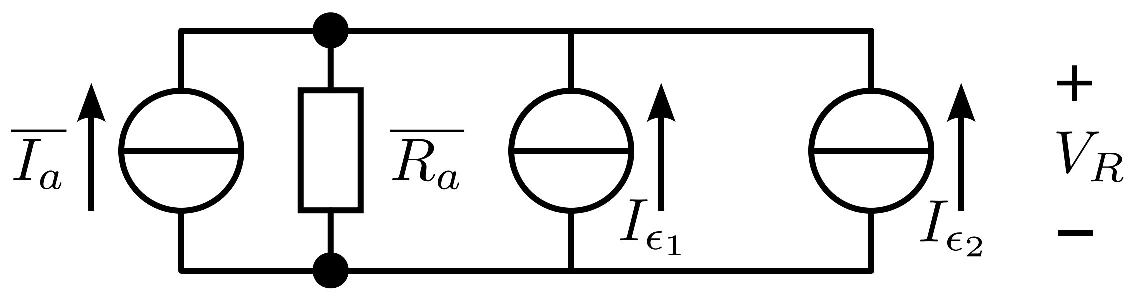

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Standard deviation of the voltage across the resistor:

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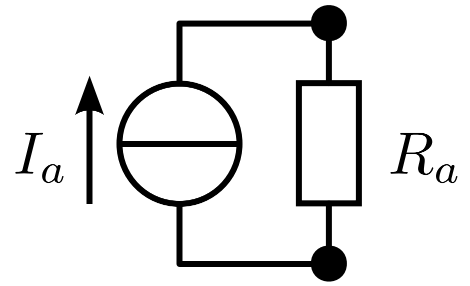

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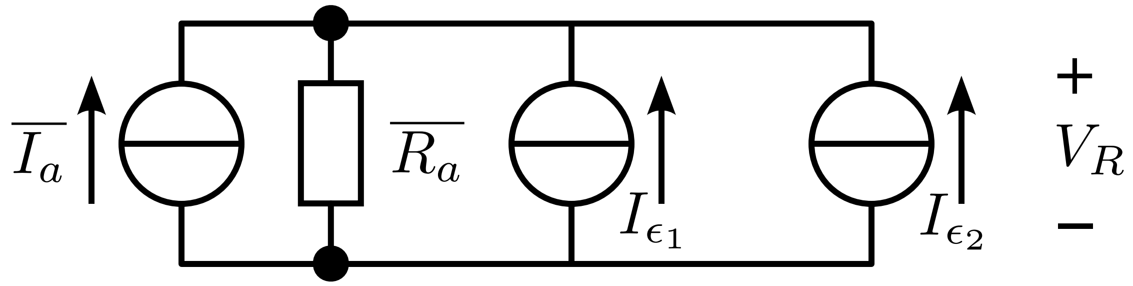

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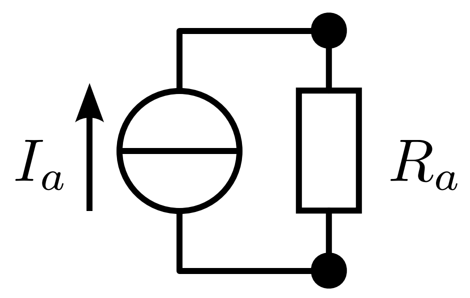

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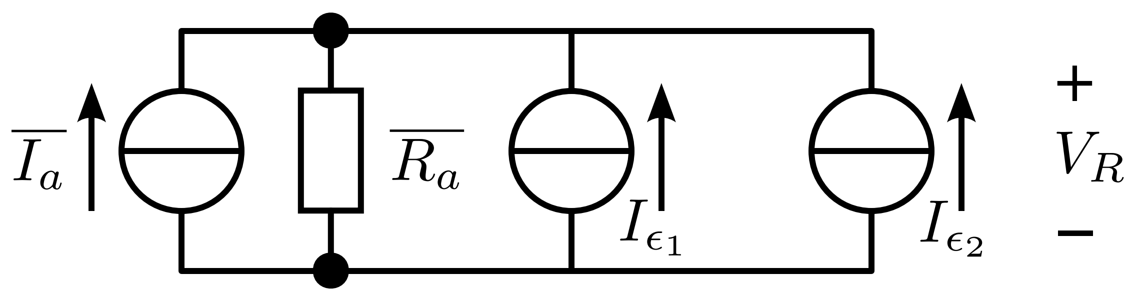

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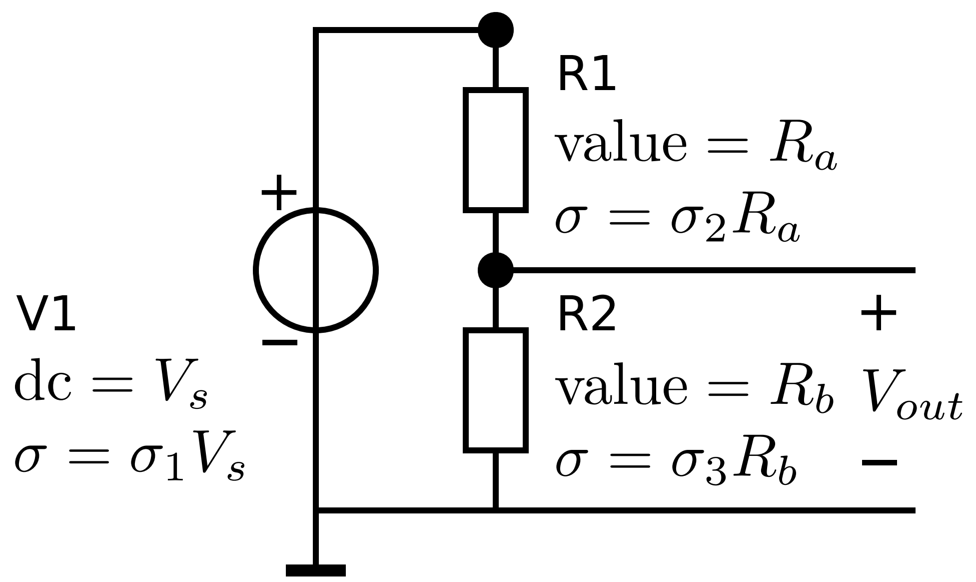
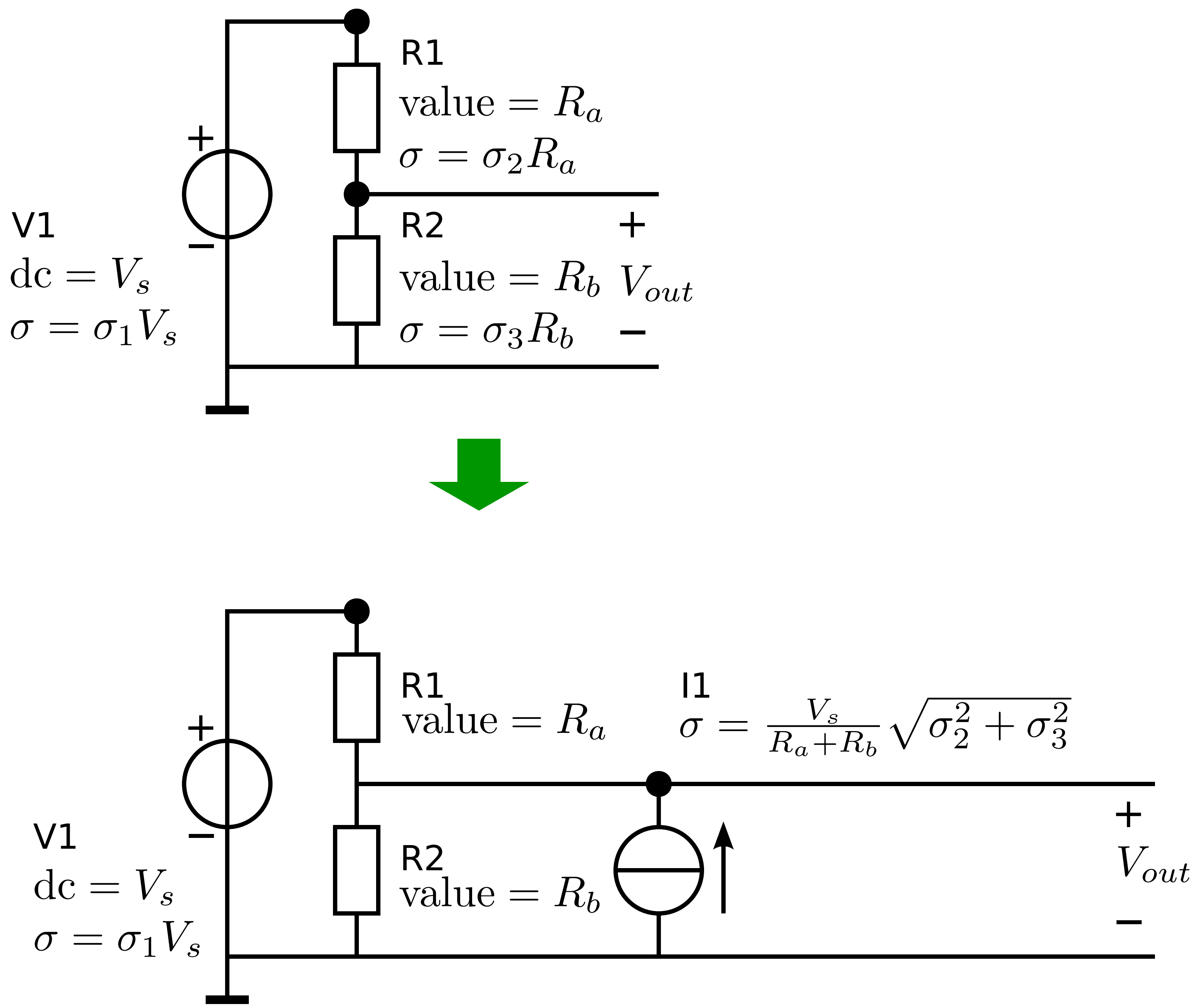
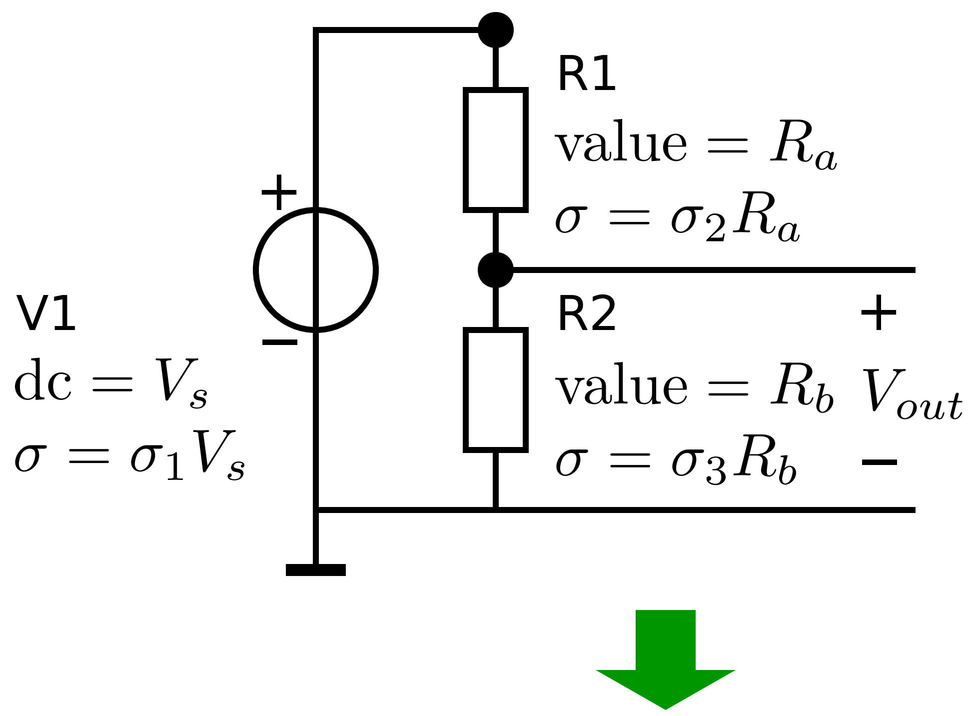
Influence of supply and resistor tolerances

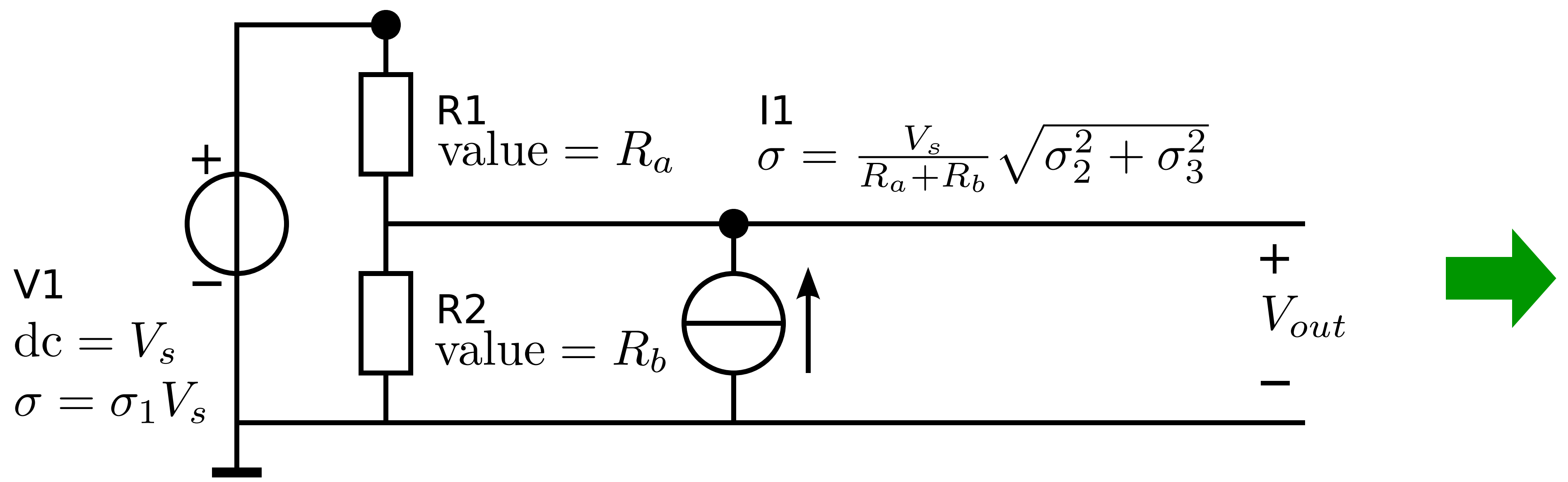


Influence of supply and resistor tolerances



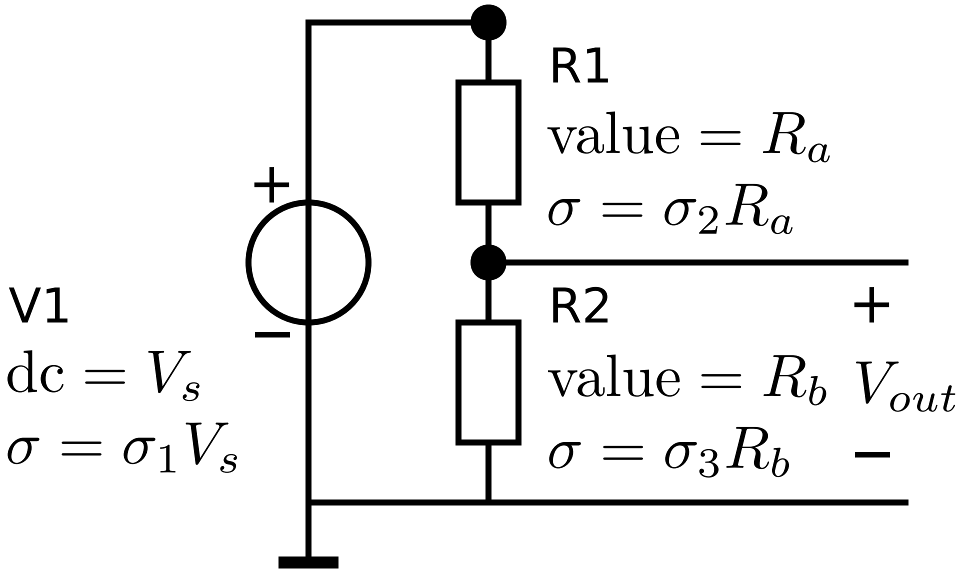
Influence of supply and resistor tolerances

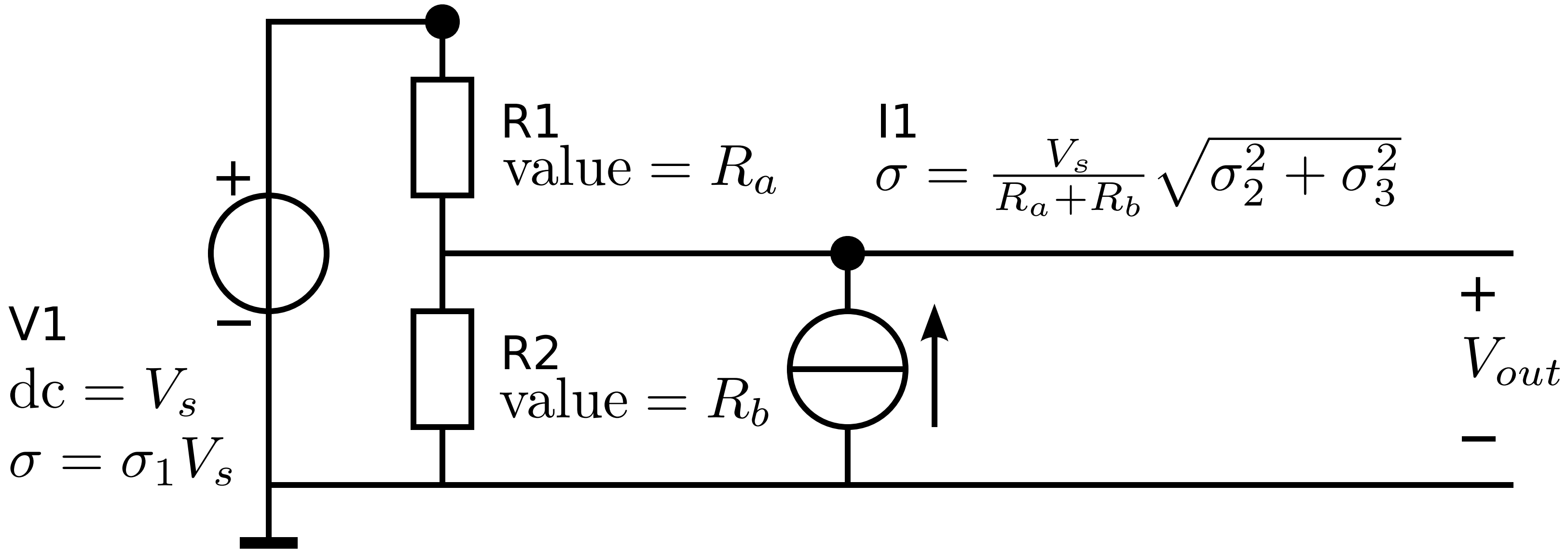




$$\overline{V_{out}} = V_s \frac{R_b}{R_a+R_b}$$

Influence of supply and resistor tolerances

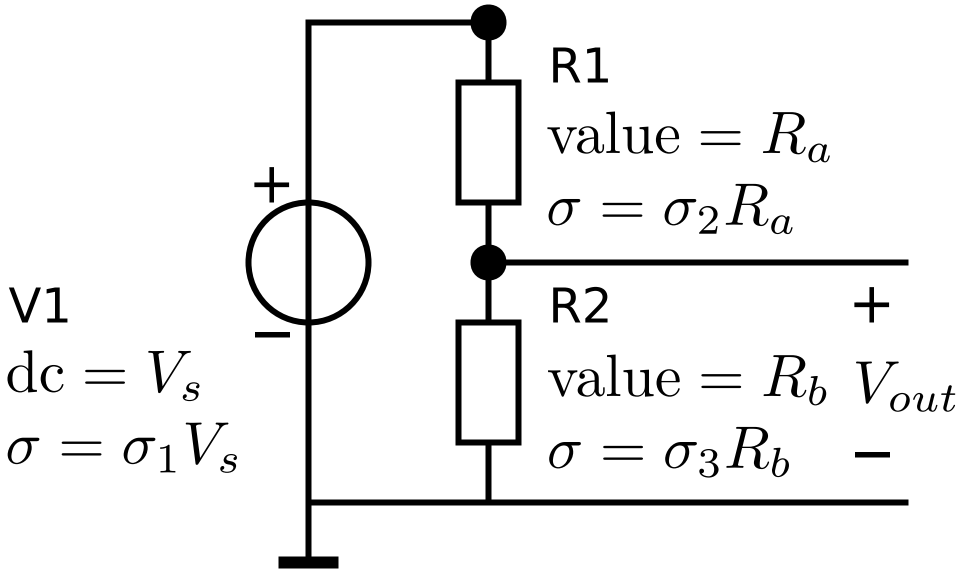


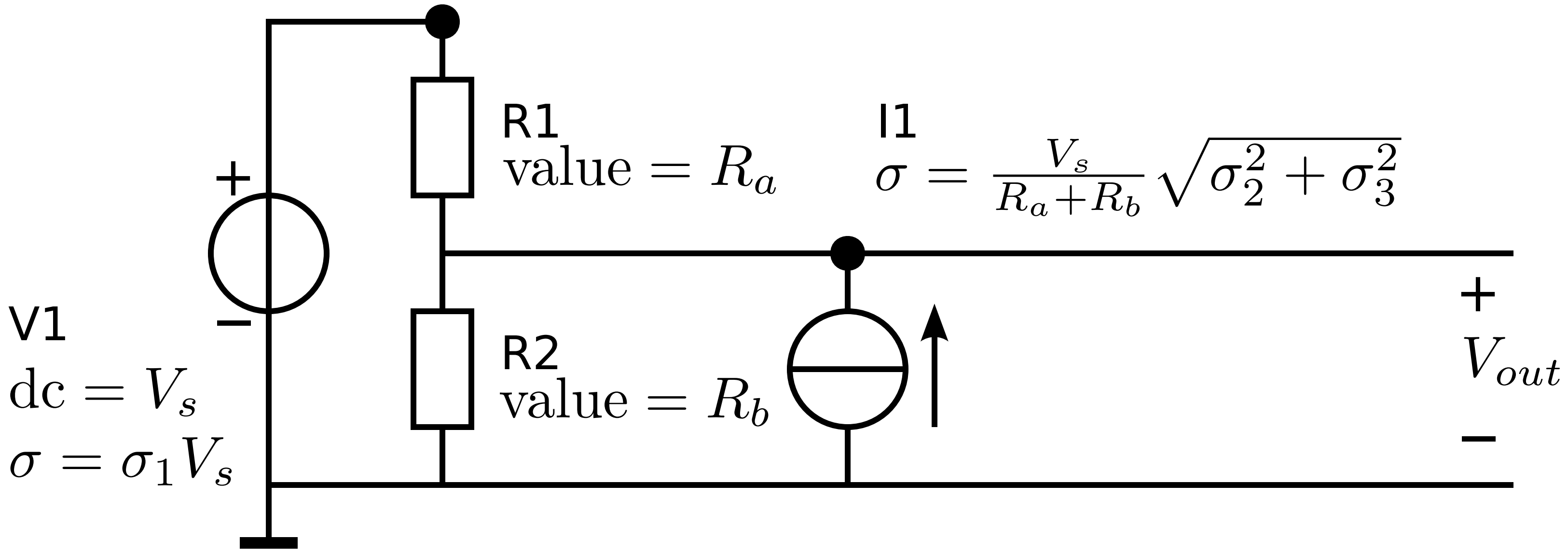


$$\overline{V_{out}} = V_s \frac{R_b}{R_a+R_b}$$

$$\frac{\sigma_{V_{out}}}{\overline{V_{out}}} = \sqrt{\left(\frac{R_a}{R_a+R_b}\right)^2 (\sigma_2^2+\sigma_3^2)+\sigma_1^2}$$

Influence of supply and resistor tolerances





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Influence of controller biasing errors

Influence of controller biasing errors

SLiCAP model of ideal
OpAmp with bias errors

Influence of controller biasing errors

SLiCAP model of ideal
OpAmp with bias errors

- Correlated bias currents

Influence of controller biasing errors

SLiCAP model of ideal
OpAmp with bias errors

- Correlated bias currents
- Uncorrelated offsets

Influence of controller biasing errors

SLiCAP model of ideal OpAmp with bias errors

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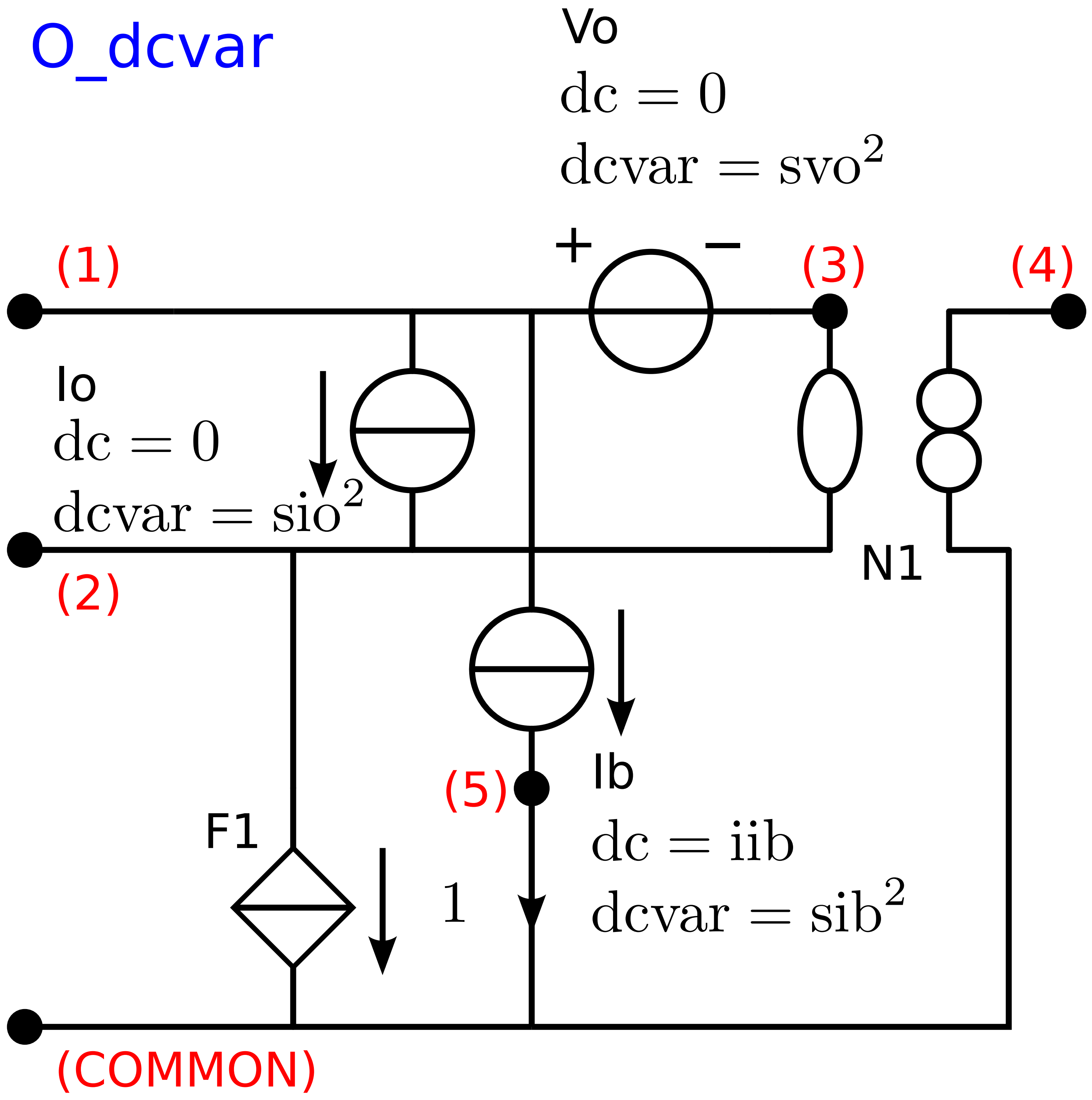
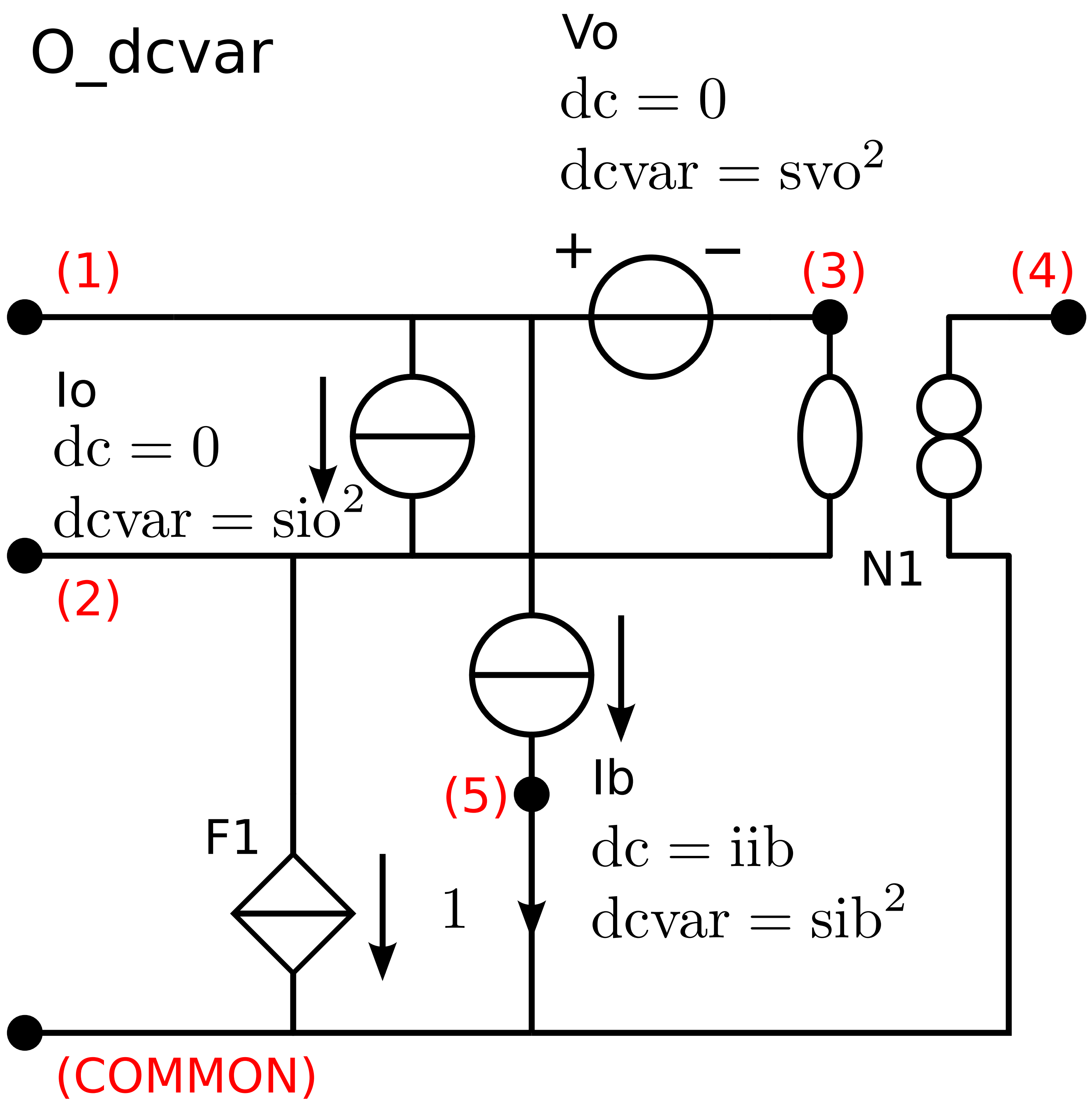


Influence of controller biasing errors

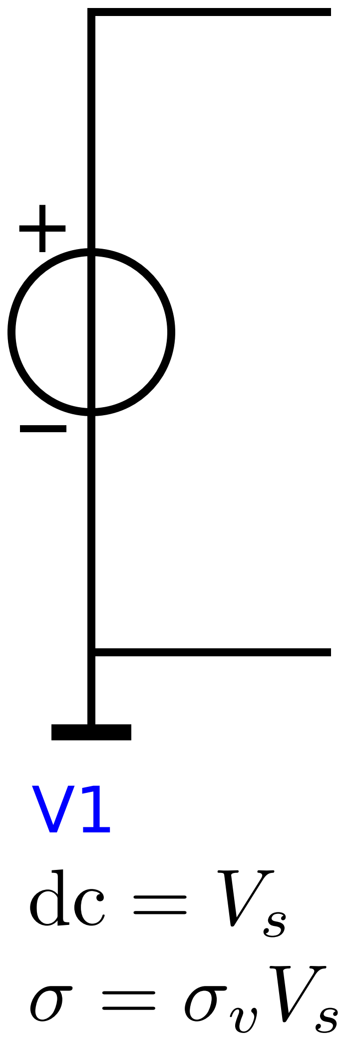
SLiCAP model of ideal OpAmp with bias errors

- Correlated bias currents
- Uncorrelated offsets

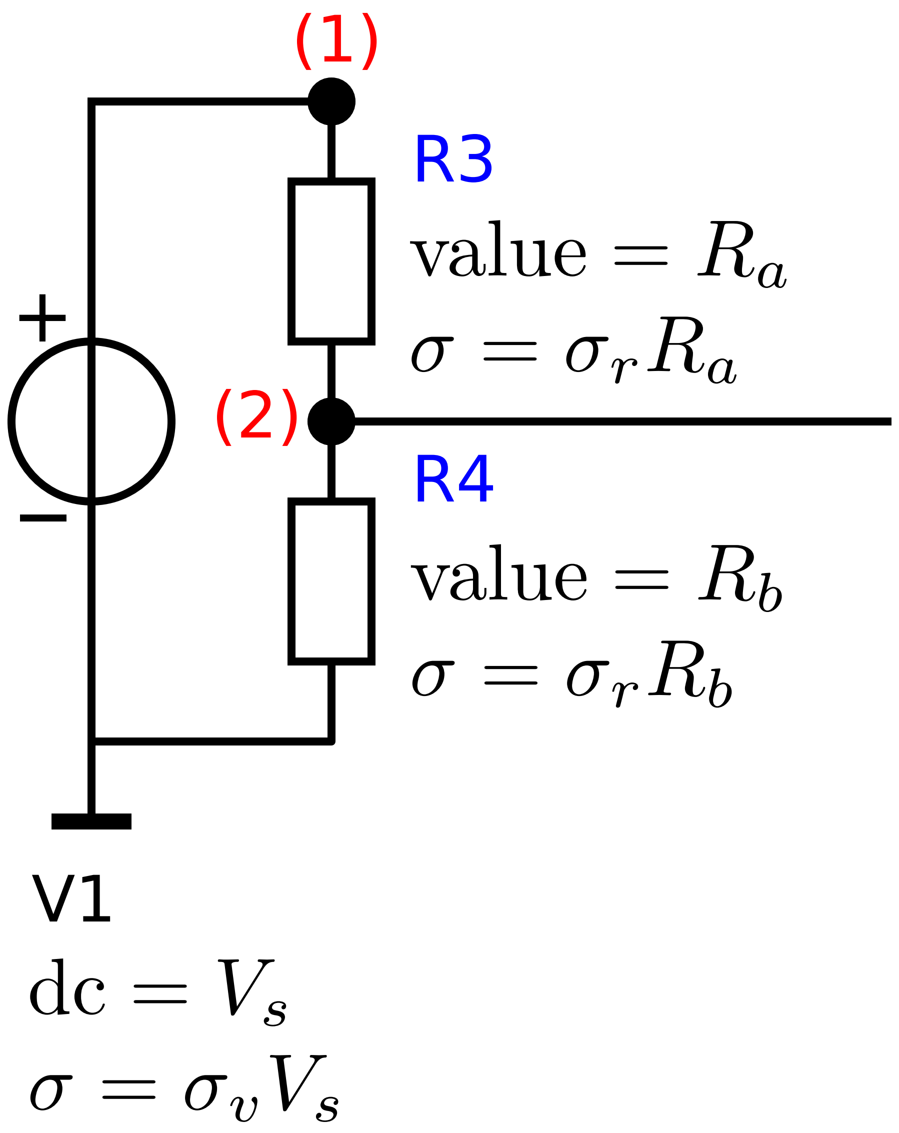


Total bias errors

Total bias errors



Total bias errors



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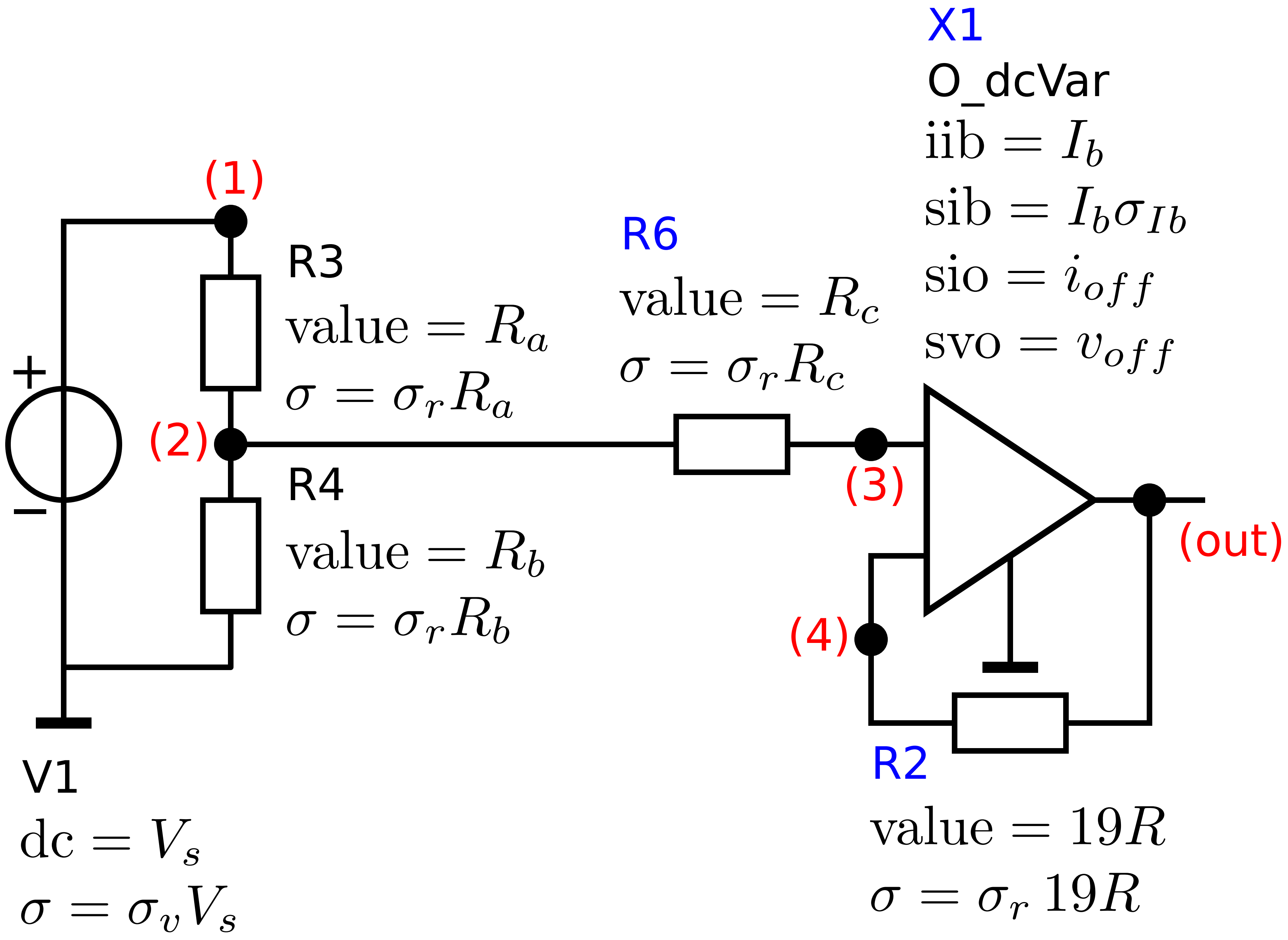

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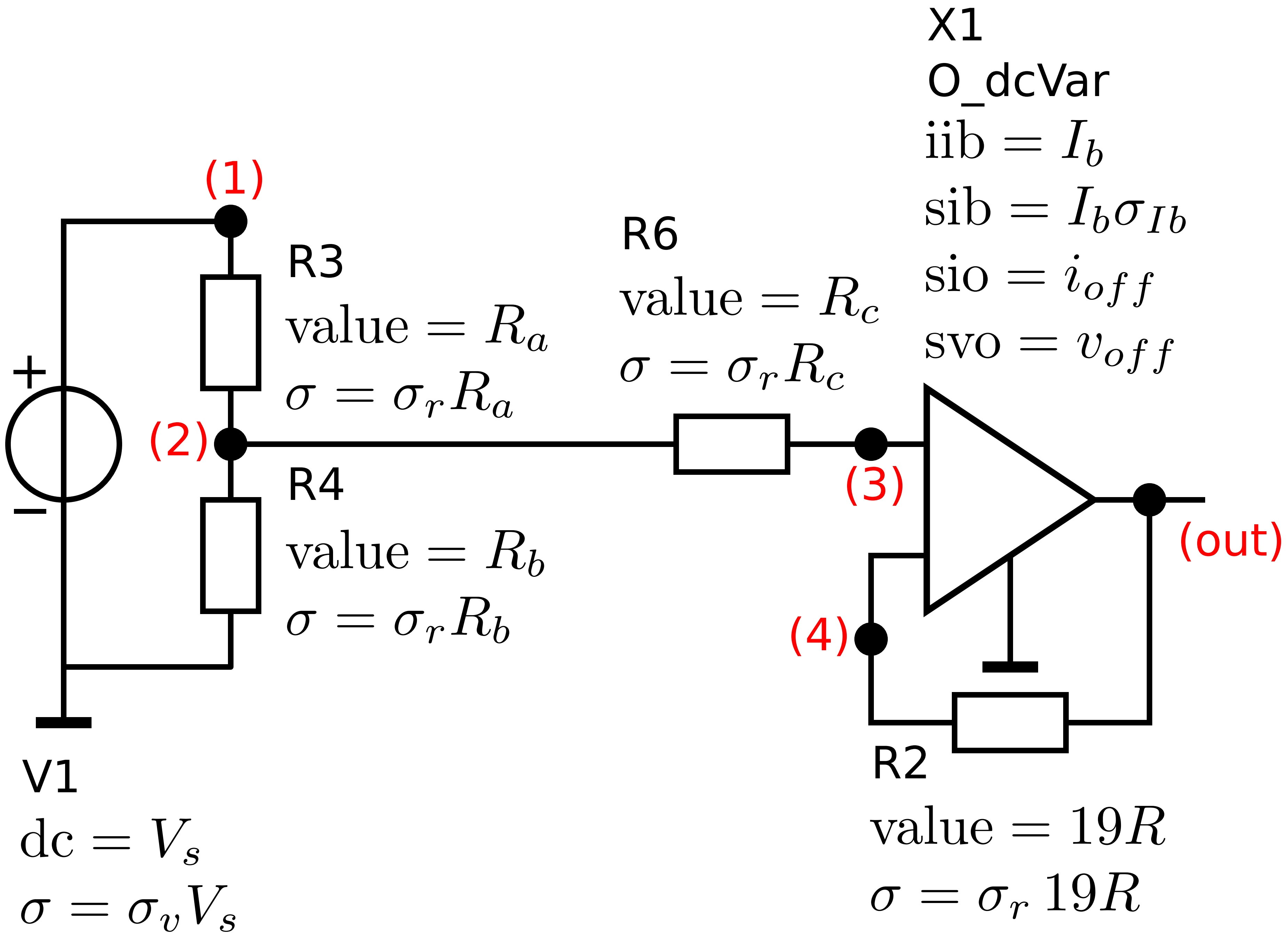
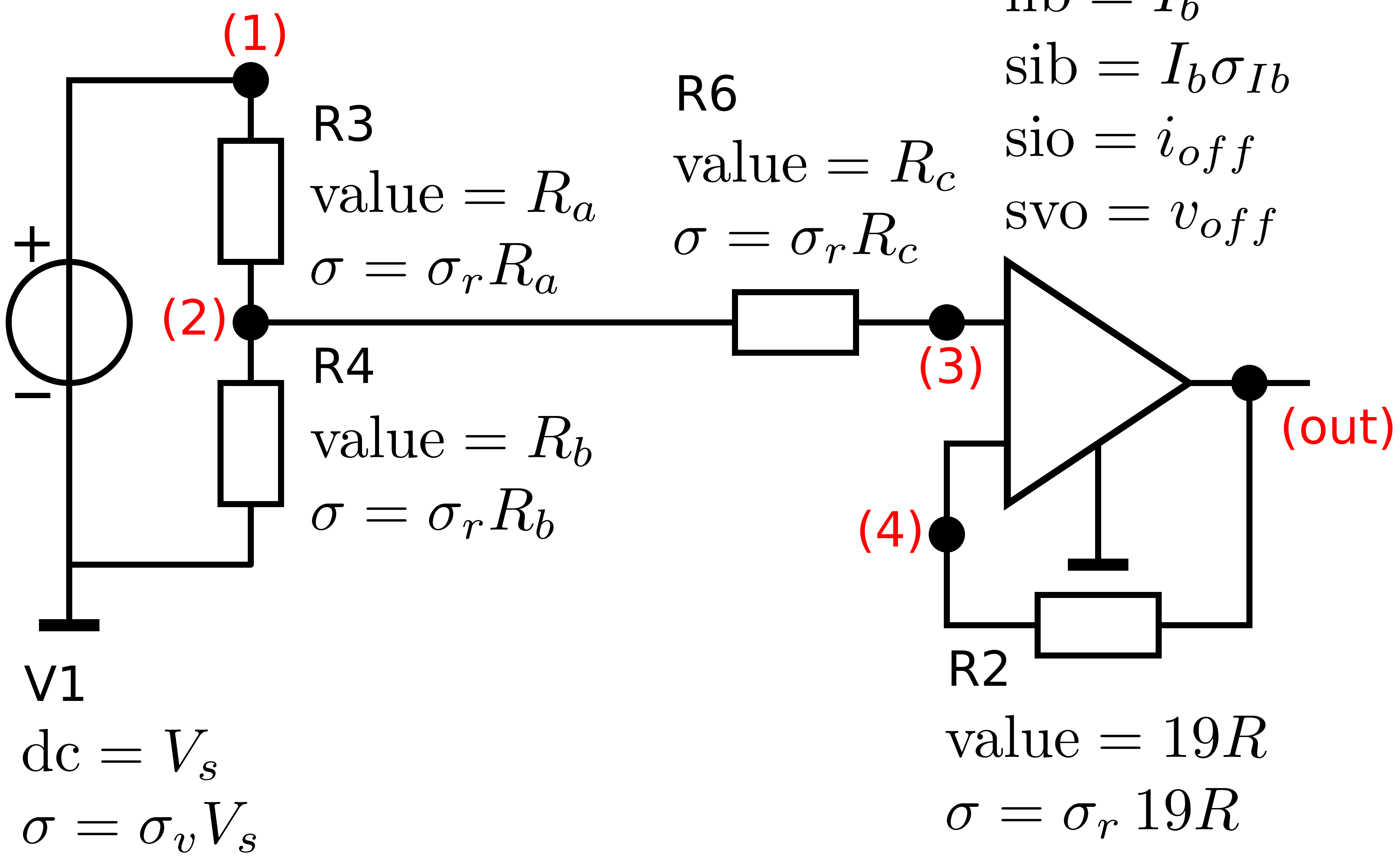


Simplified result: $R_c \gg \frac{R_a R_b}{R_a + R_b}$

Total bias errors



Simplified result: $R_c \gg \frac{R_a R_b}{R_a + R_b}$

$$
\begin{aligned}
\sigma^2_{V\,out} = {} & 2\sigma_r^2 \left(\frac{V_s}{R_a + R_b}\right)^2 \left(\frac{R_a R_b}{R_a + R_b}\right)^2 \\
& + \sigma_v^2 V_s^2 \left(\frac{R_b}{R_a + R_b}\right)^2 \\
& + v_{off}^2 \\
& + i_{off}^2 \left(R_c + 19R\right)^2 \\
& + \sigma_{Ib}^2 I_b^2 \left(R_c - 19R\right)^2 \\
& + \sigma_r^2 I_b^2 \left(R_c^2 + (19R)^2\right)
\end{aligned}
$$

