Structured Electronic Design

EE3C11
Topics from Network Theory
Nodal Analysis

Anton J.M. Montagne

Nodal Analysis

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

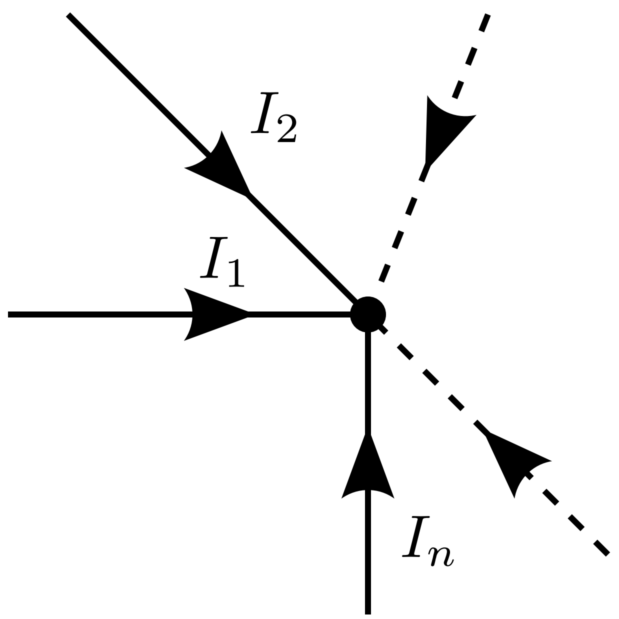
Branch: element between two nodes

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

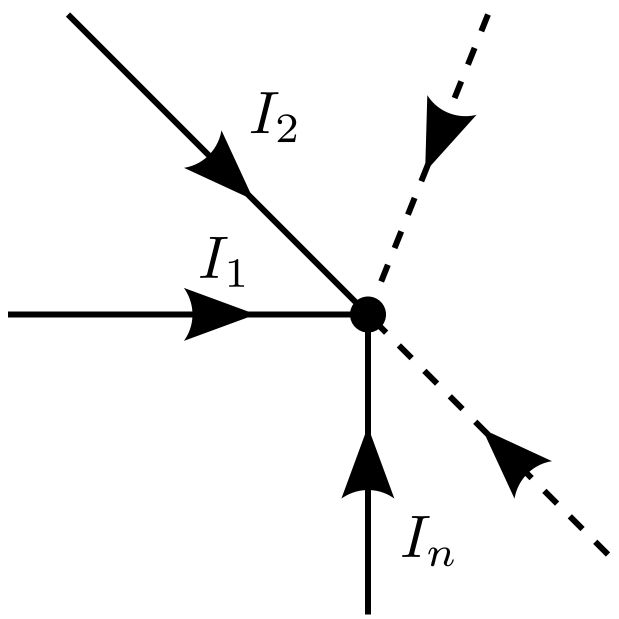
The sum of the branch currents that flow into (or from) a node, equals zero:

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

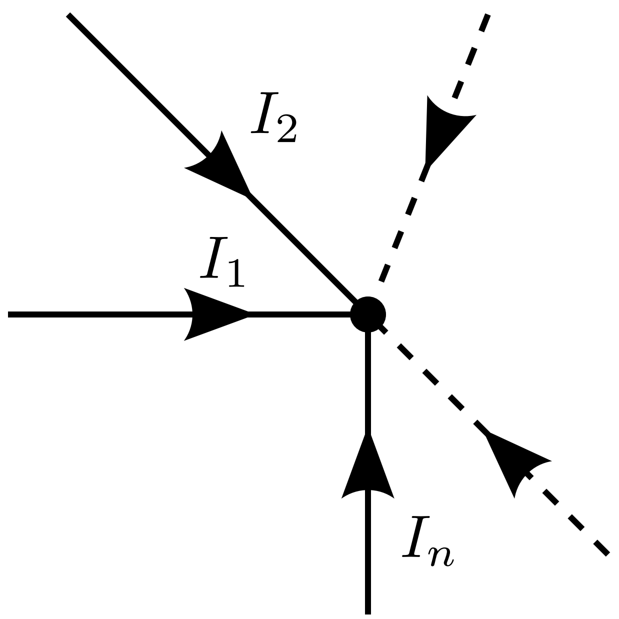
$$\sum_{i=1}^{i=n} I_i = 0$$

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

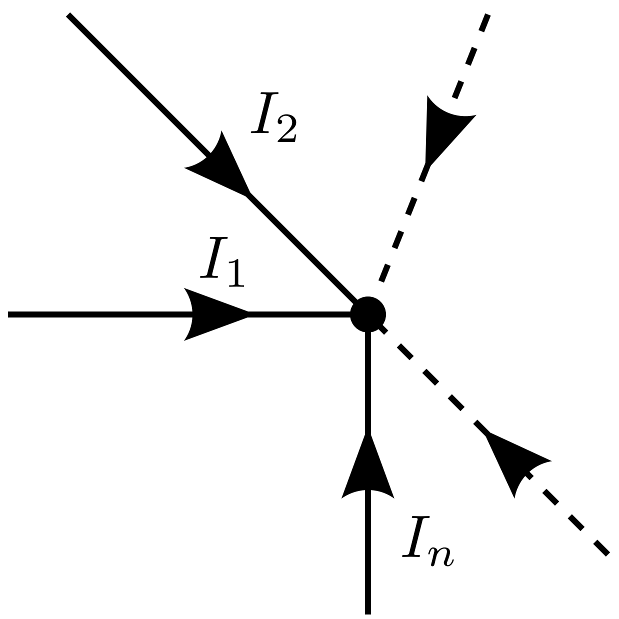
1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

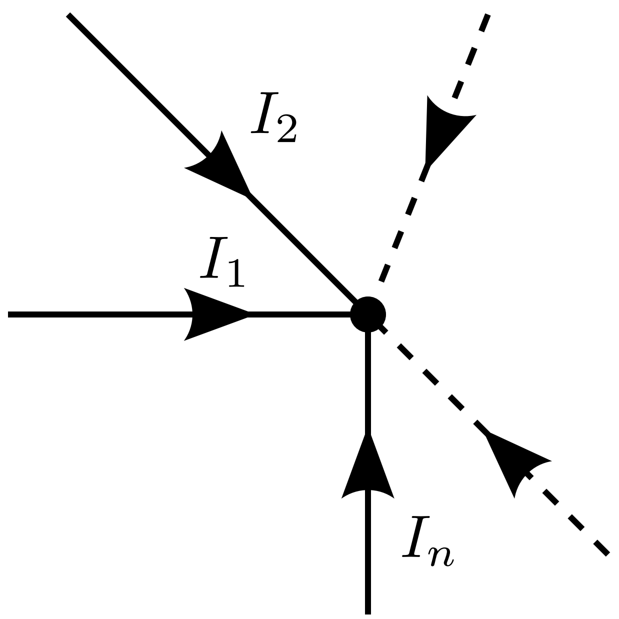
Only voltage-controlled elements:
Branch current can be written as
a function of the branch voltage

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

Only voltage-controlled elements:
Branch current can be written as a function of the branch voltage

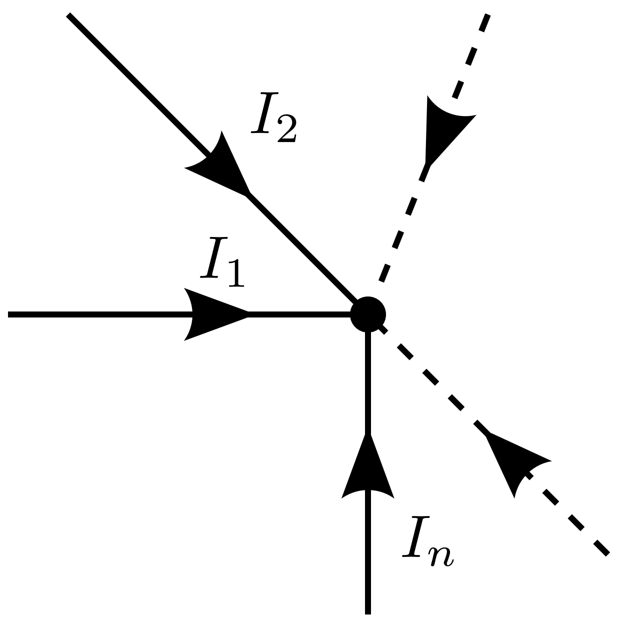
$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

Only voltage-controlled elements:
Branch current can be written as a function of the branch voltage

$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$

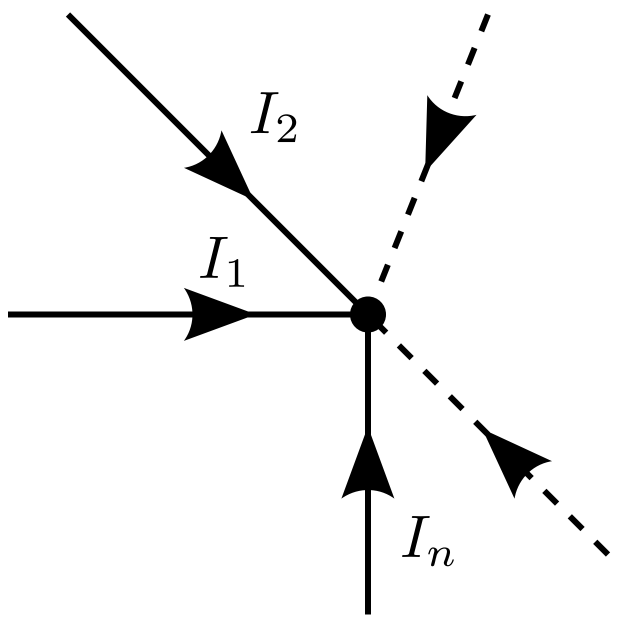
Vector with sum of independent currents flowing into a node

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

Only voltage-controlled elements:
Branch current can be written as a function of the branch voltage

$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$

Vector with sum of independent currents flowing into a node

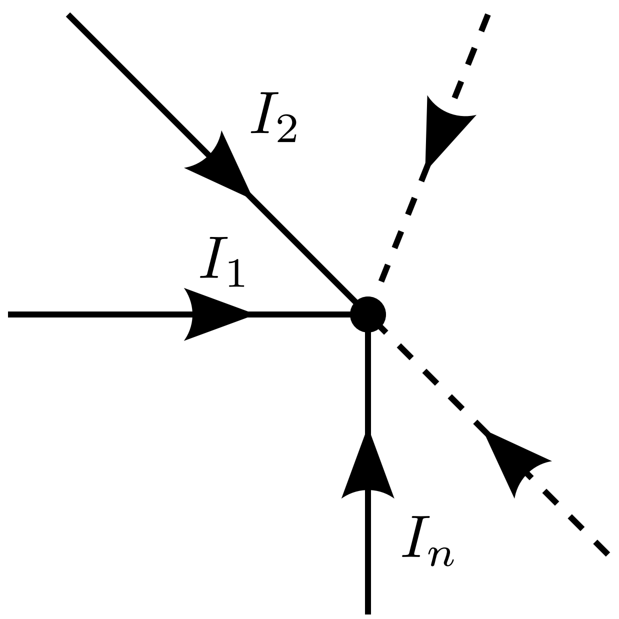
Admittance matrix

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

Only voltage-controlled elements:
Branch current can be written as a function of the branch voltage

$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$

Vector with sum of independent currents flowing into a node

Admittance matrix

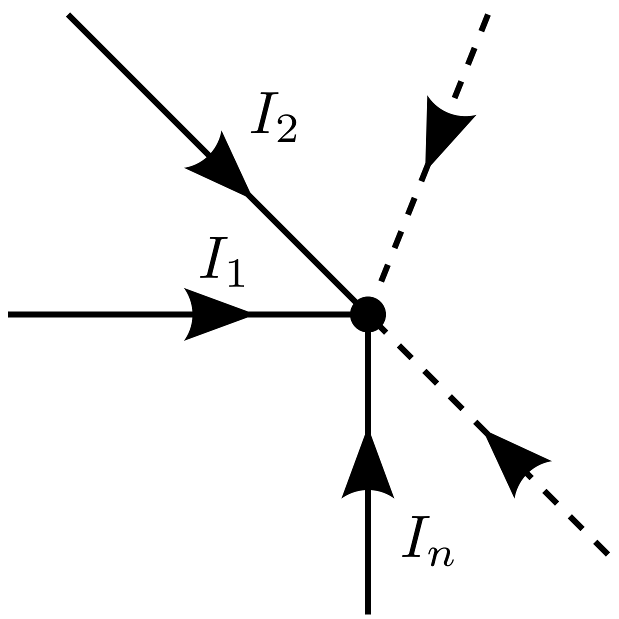
Vector with nodal voltages

Nodal Analysis

An electrical network consists of interconnected network elements

Node: interconnection point

Branch: element between two nodes



Kirchhoff's current law

The sum of the branch currents that flow into (or from) a node, equals zero:

$$\sum_{i=1}^{i=n} I_i = 0$$

Network with n nodes

1 node selected as reference node
n-1 nodal voltages w.r.t. voltage of ref. node
n-1 independent nodal equations

Only voltage-controlled elements:
Branch current can be written as a function of the branch voltage

$$\mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$$

Vector with sum of independent currents flowing into a node

Admittance matrix

Vector with nodal voltages

Admittance Matrix

Admittance Matrix

General form of nodal equation:

Admittance Matrix

General form of nodal equation:

$$\sum i_k = -\sum \mathbf{Y}_{k,1} v_1 - \sum \mathbf{Y}_{k,2} v_2 \quad \ldots + \sum \mathbf{Y}_{k,k} v_k \quad \ldots - \sum \mathbf{Y}_{k,n-1} v_{n-1}$$

Admittance Matrix

General form of nodal equation:

Sum of independent currents flowing into node k

$$\sum i_k = -\sum \mathbf{Y}_{k,1} v_1 - \sum \mathbf{Y}_{k,2} v_2 \quad ... + \sum \mathbf{Y}_{k,k} v_k \quad ... - \sum \mathbf{Y}_{k,n-1} v_{n-1}$$

Admittance Matrix

General form of nodal equation:

Sum of independent currents flowing into node k

Diagonal element: sum of admittances connected to node k

$$\sum i_k = -\sum \mathbf{Y}_{k,1} v_1 - \sum \mathbf{Y}_{k,2} v_2 \quad \ldots + \sum \mathbf{Y}_{k,k} v_k \quad \ldots - \sum \mathbf{Y}_{k,n-1} v_{n-1}$$

Admittance Matrix

General form of nodal equation:

Sum of independent currents flowing into node k

Diagonal element: sum of admittances connected to node k

$$\sum i_k = -\sum \mathbf{Y}_{k,1} v_1 - \sum \mathbf{Y}_{k,2} v_2 \quad ... + \sum \mathbf{Y}_{k,k} v_k \quad ... - \sum \mathbf{Y}_{k,n-1} v_{n-1}$$

Off-diagonal element: $\sum \mathbf{Y}_{k,j} =$

Admittance Matrix

General form of nodal equation:

Sum of independent currents flowing into node k

Diagonal element: sum of admittances connected to node k

$$\sum i_k = -\sum \mathbf{Y}_{k,1} v_1 - \sum \mathbf{Y}_{k,2} v_2 \quad \ldots + \sum \mathbf{Y}_{k,k} v_k \quad \ldots - \sum \mathbf{Y}_{k,n-1} v_{n-1}$$

Off-diagonal element: $\sum \mathbf{Y}_{k,j} =$ Sum of admittances connected between node k and node j

Admittance Matrix

General form of nodal equation:

Sum of independent currents flowing into node k

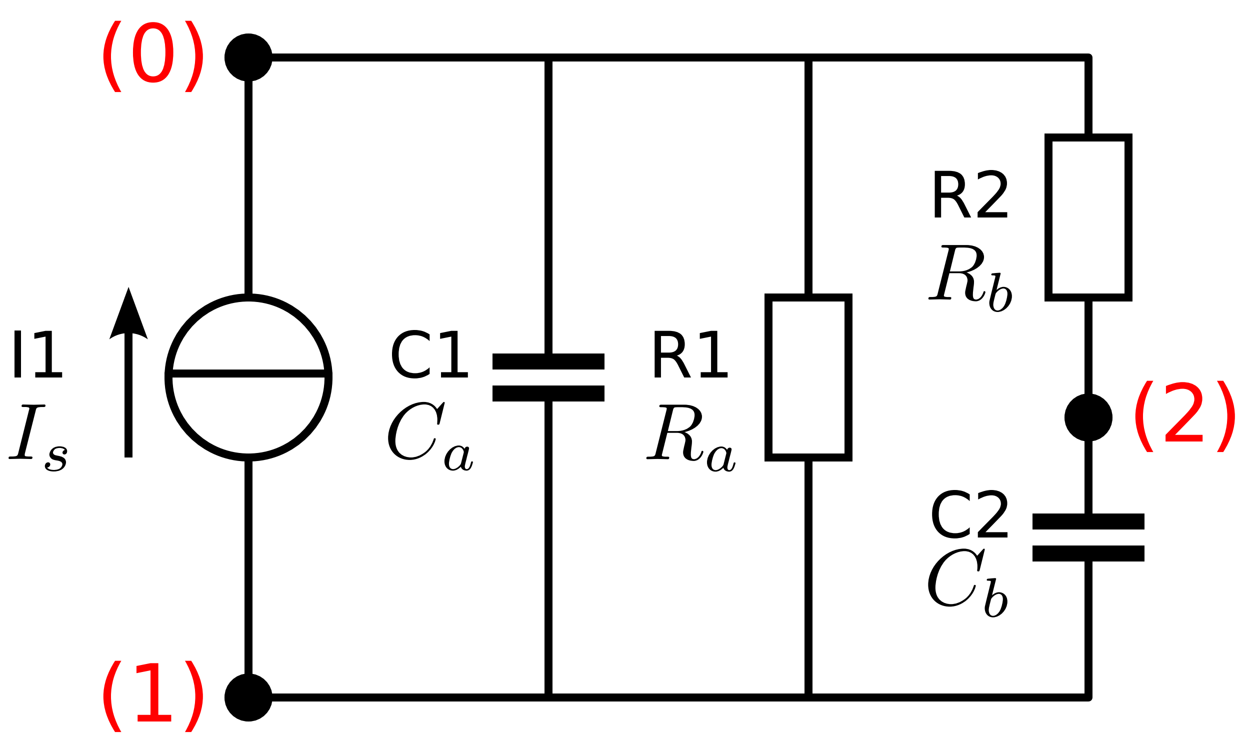
Diagonal element: sum of admittances connected to node k

$$\sum i_k = -\sum \mathbf{Y}_{k,1} v_1 - \sum \mathbf{Y}_{k,2} v_2 \quad ... + \sum \mathbf{Y}_{k,k} v_k \quad ... - \sum \mathbf{Y}_{k,n-1} v_{n-1}$$

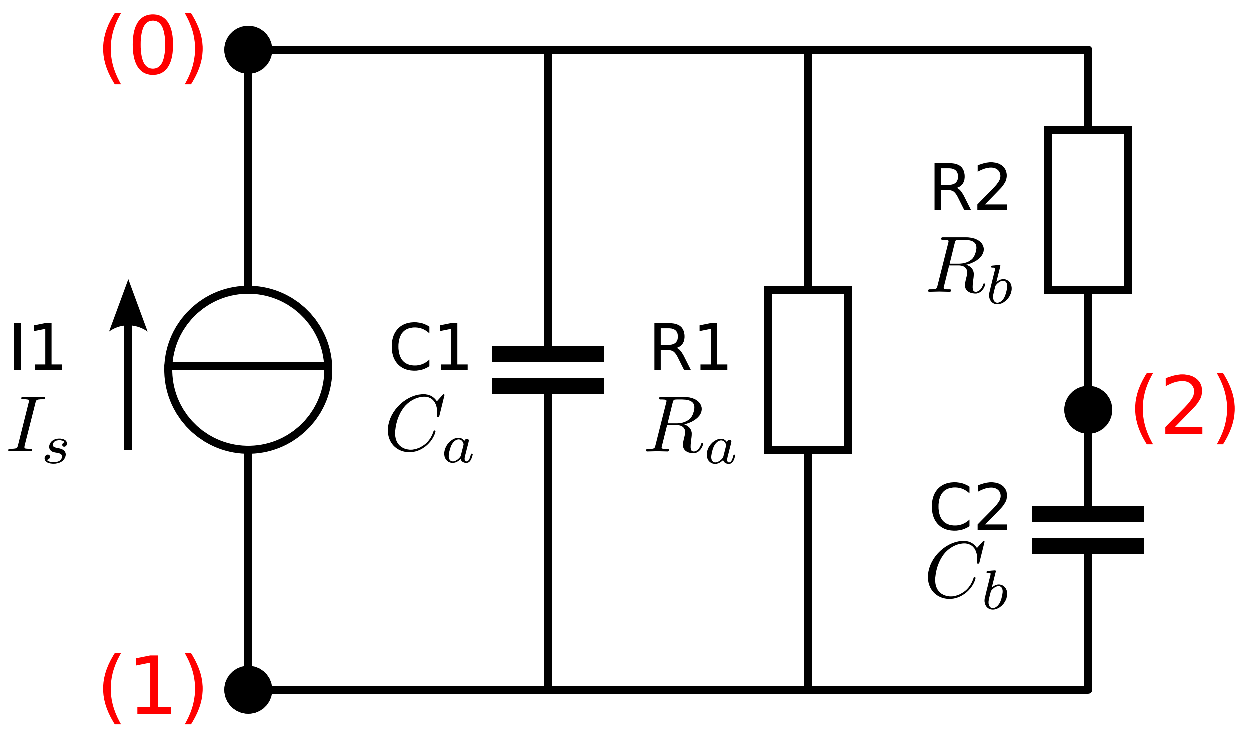
Off-diagonal element: $\sum \mathbf{Y}_{k,j} =$ Sum of admittances connected between node k and node j

Nodal Analysis

Nodal Analysis

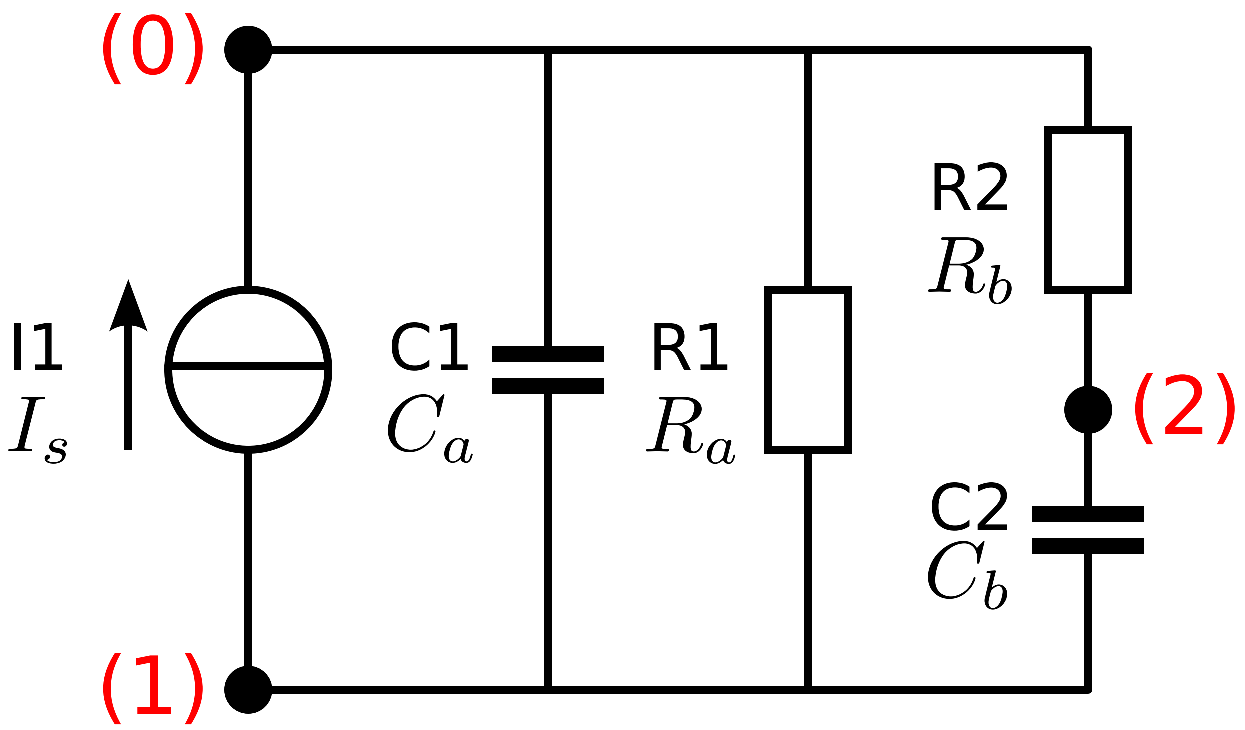


Nodal Analysis



Node (0) is reference node

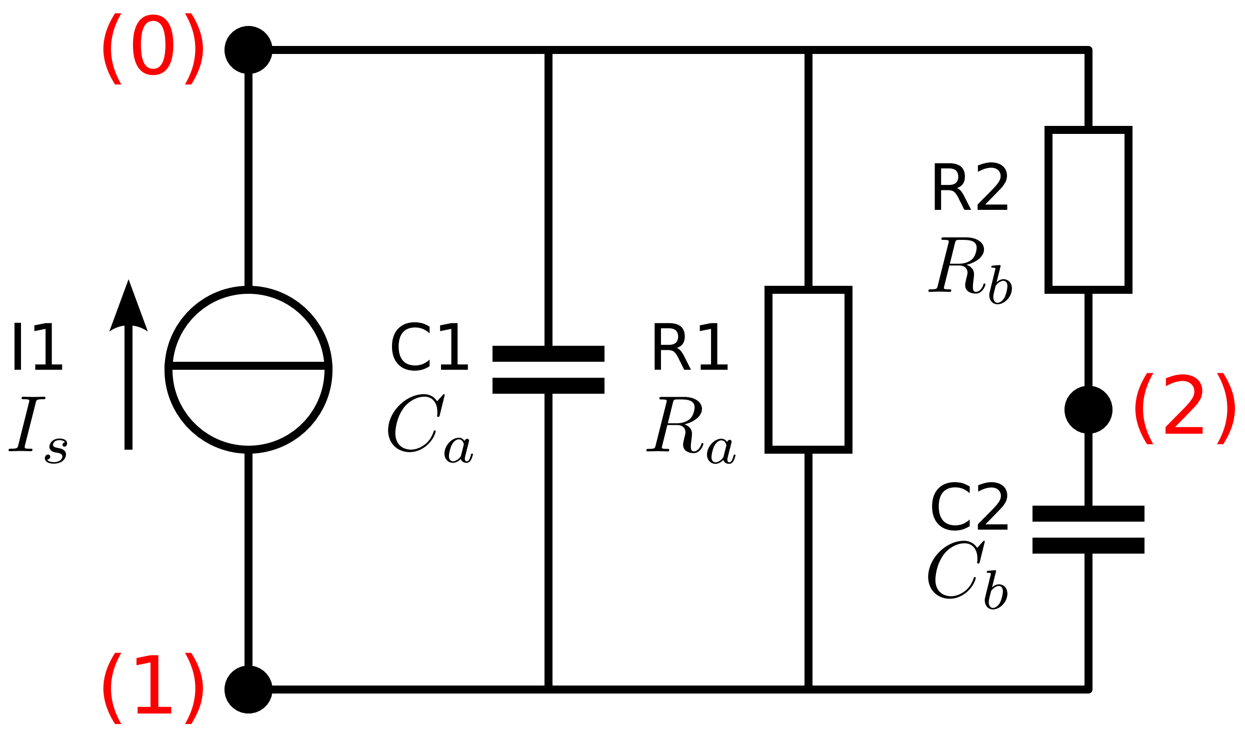
Nodal Analysis



Node (0) is reference node

Two independent nodal equations:

Nodal Analysis

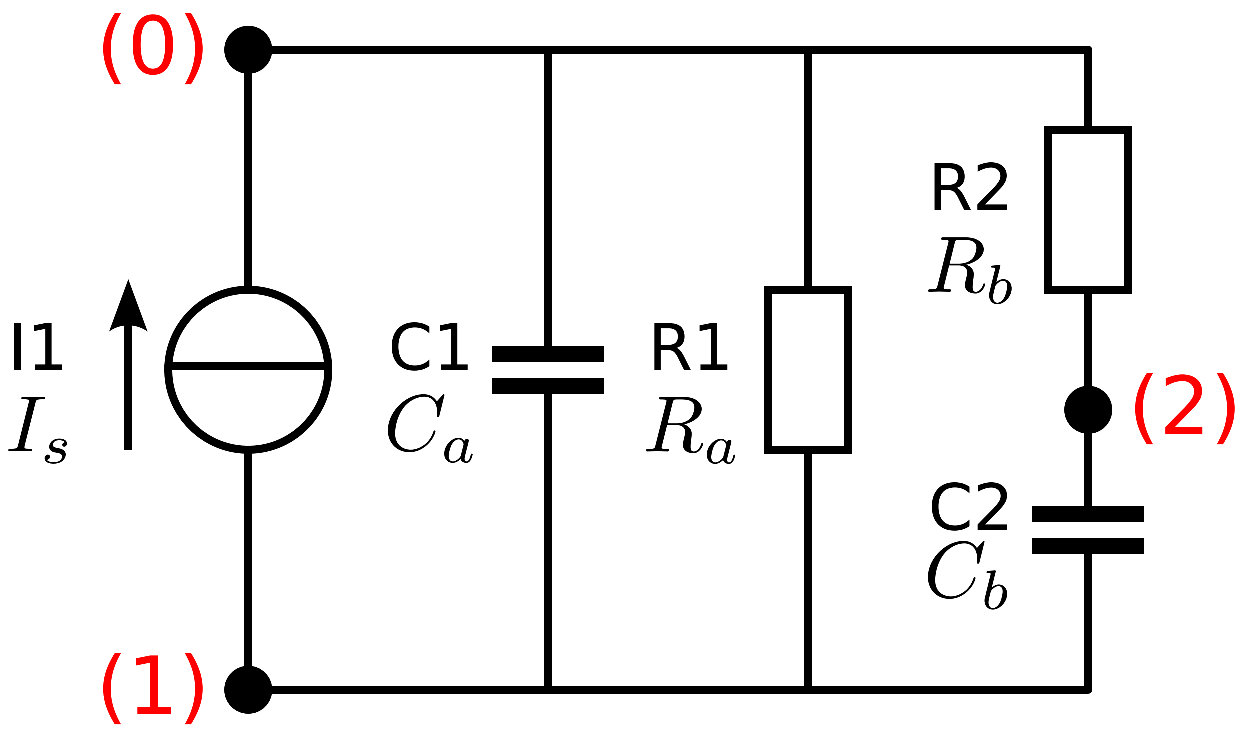


Node (0) is reference node

Two independent nodal equations:

(1) $$0 = I_s + V_1 s C_a + V_1 \frac{1}{R_a} + (V_1 - V_2) s C_b$$

Nodal Analysis



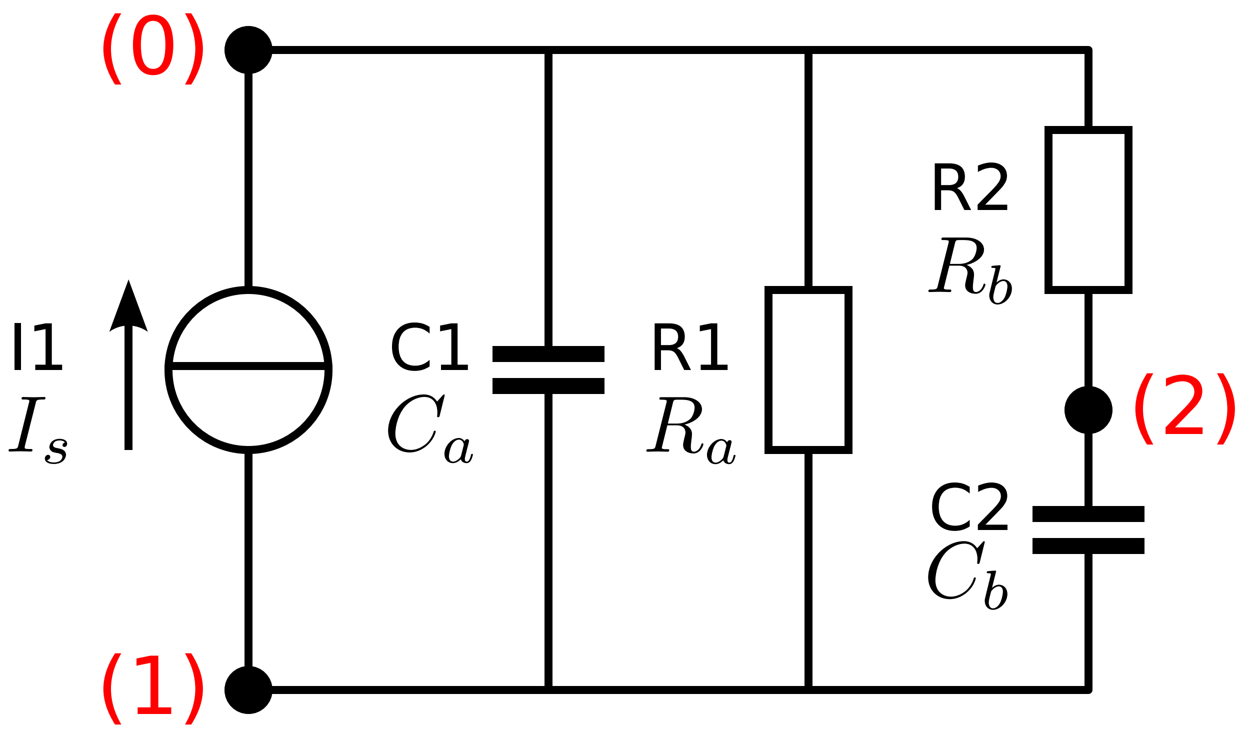
Node (0) is reference node

Two independent nodal equations:

(1) $$0 = I_s + V_1 sC_a + V_1 \frac{1}{R_a} + (V_1 - V_2)sC_b$$

(2) $$0 = V_2 \frac{1}{R_b} + (V_2 - V_1)sC_b$$

Nodal Analysis



Node (0) is reference node

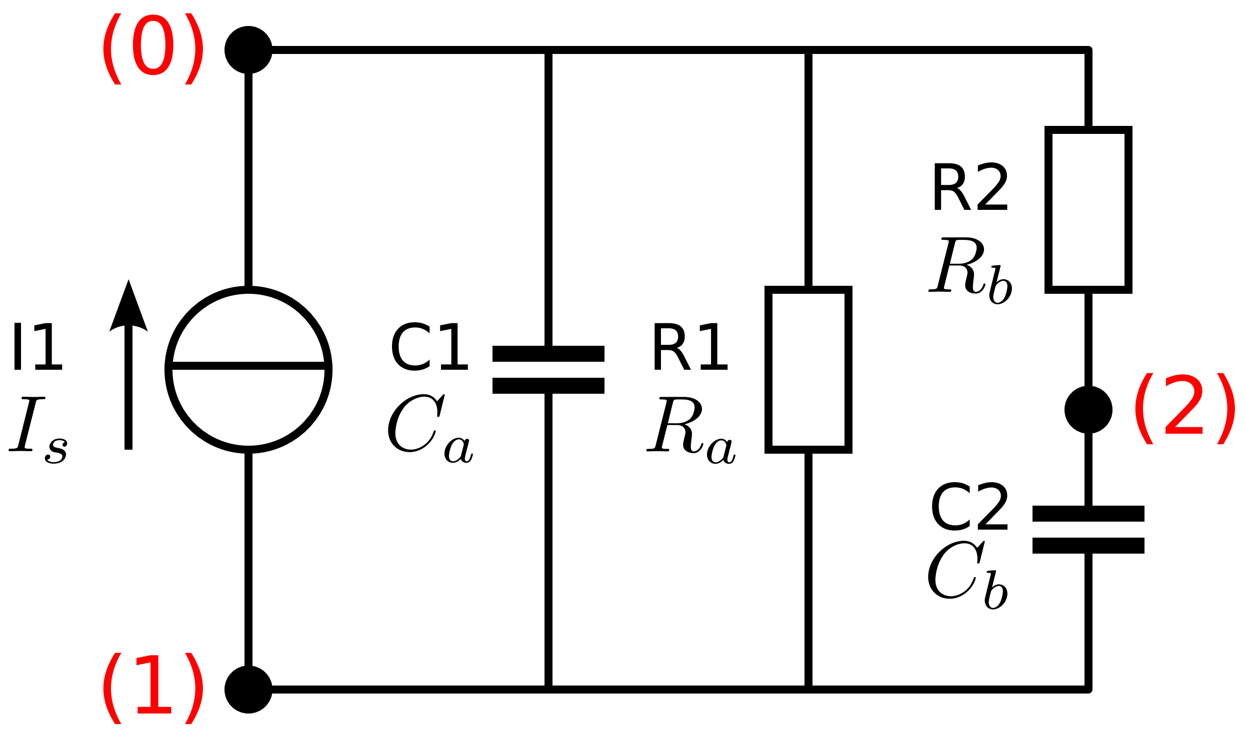
Two independent nodal equations:

(1) $$0 = I_s + V_1 sC_a + V_1 \frac{1}{R_a} + (V_1 - V_2) sC_b$$

(2) $$0 = V_2 \frac{1}{R_b} + (V_2 - V_1) sC_b$$

In matrix form:

Nodal Analysis



Node (0) is reference node

Two independent nodal equations:

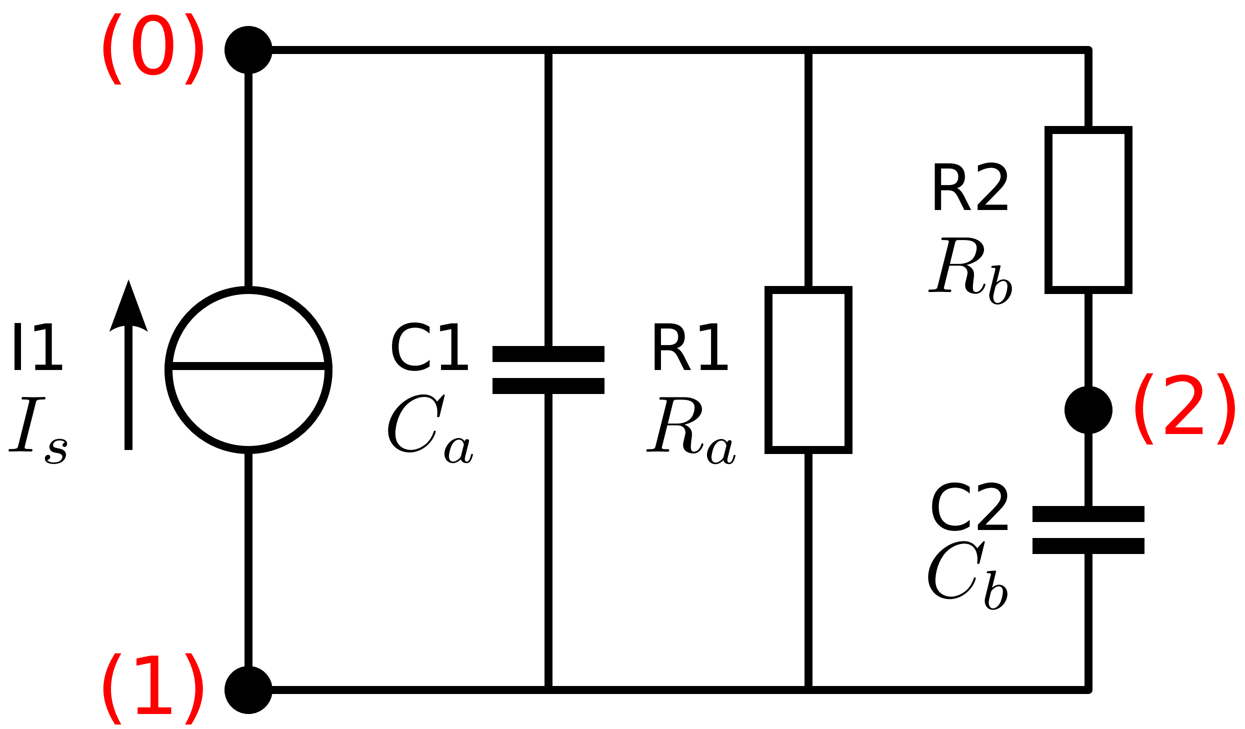
(1) $$0 = I_s + V_1 sC_a + V_1 \frac{1}{R_a} + (V_1 - V_2) sC_b$$

(2) $$0 = V_2 \frac{1}{R_b} + (V_2 - V_1) sC_b$$

In matrix form:

$$\begin{pmatrix} -I_s \\ 0 \end{pmatrix} = \begin{pmatrix} s\,(C_a + C_b) + \frac{1}{R_a} & -sC_b \\ -sC_b & sC_b + \frac{1}{R_b} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Nodal Analysis



Node (0) is reference node

Two independent nodal equations:

(1) $$0 = I_s + V_1 sC_a + V_1 \frac{1}{R_a} + (V_1 - V_2)sC_b$$

(2) $$0 = V_2 \frac{1}{R_b} + (V_2 - V_1)sC_b$$

In matrix form:

$$\begin{pmatrix} -I_s \\ 0 \end{pmatrix} = \begin{pmatrix} s\left(C_a + C_b\right) + \frac{1}{R_a} & -sC_b \\ -sC_b & sC_b + \frac{1}{R_b} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$