Structured Electronic Design

EE3C11
Estimation of poles and zeros
in networks without feedback

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Time-constant matrix

Basis for intuitive determination of poles in networks without feedback

MNA matrix in first-order differential form: $\mathbf{M} = \mathbf{G} + s\mathbf{C}$

Characteristic equation: $\det\left(\mathbf{G} + s\mathbf{C}\right) = 0$

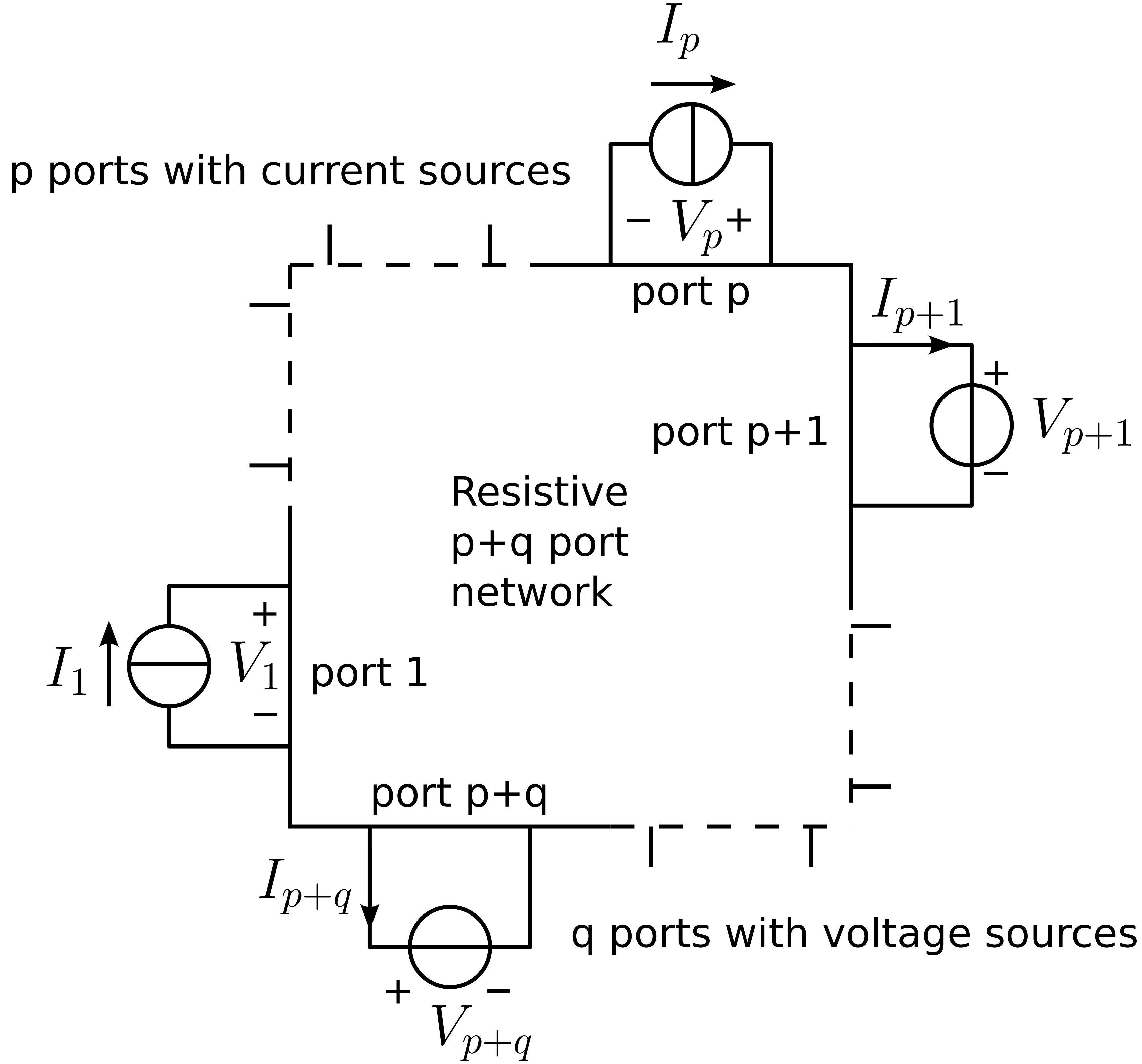
Time-constant matrix: $\mathbf{T} = \mathbf{G}^{-1} \cdot \mathbf{C} = \mathbf{R} \cdot \mathbf{C}$

Characteristic equation: $\det\left(\mathbf{I} + s\mathbf{T}\right) = 0$

Generalized eigenvalue problem: $\det\left(\mathbf{A} - \lambda\mathbf{I}\right) = 0$

If τ_i is an eigenvalue of $\mathbf{T}$ then $p_i = -\frac{1}{\tau_i}$ is a pole of the network.

Resistance matrix



$$\begin{pmatrix} V_1 \\ \vdots \\ V_p \\ I_{p+1} \\ \vdots \\ I_{p+q} \end{pmatrix} = \begin{pmatrix} r_{11} & \cdots & r_{1p} & \mu_{11} & \cdots & \mu_{1q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ r_{p1} & \cdots & r_{pp} & \mu_{p1} & \cdots & \mu_{pq} \\ \alpha_{11} & \cdots & \alpha_{1p} & g_{11} & \cdots & g_{q1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \alpha_{q1} & \cdots & \alpha_{qp} & g_{1q} & \cdots & g_{qq} \end{pmatrix} \begin{pmatrix} I_1 \\ \vdots \\ I_p \\ V_{p+1} \\ \vdots \\ V_{p+q} \end{pmatrix}$$

Diagonal elements: $\mathcal{R}_{i,i}$

Resistance, conductance at port i

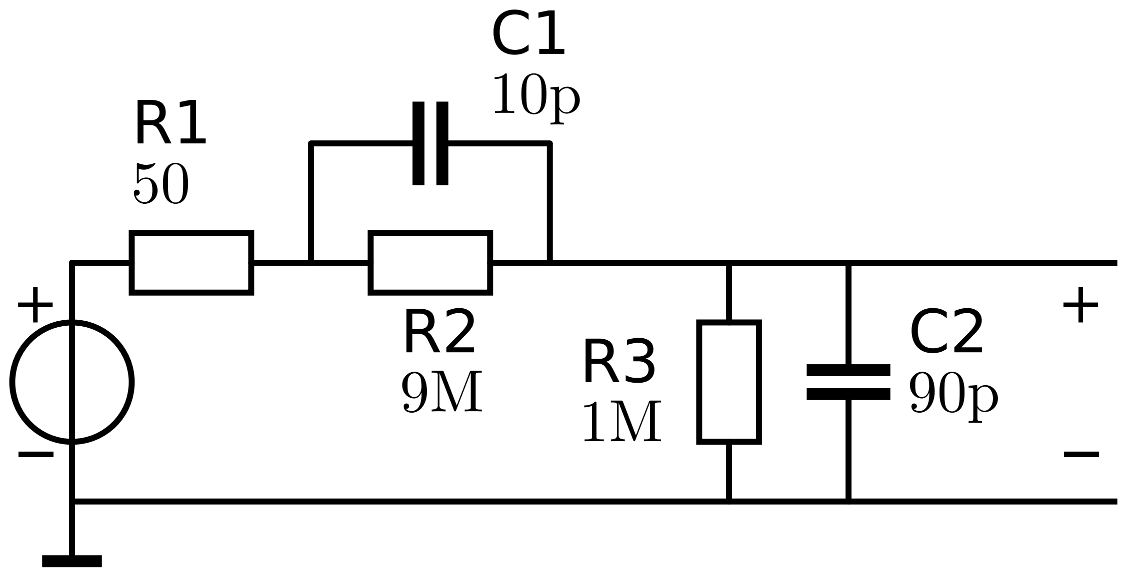
Off-diagonal elements: $\mathcal{R}_{i,j}$

transresistance
transconductance
voltage gain factor
current gain factor

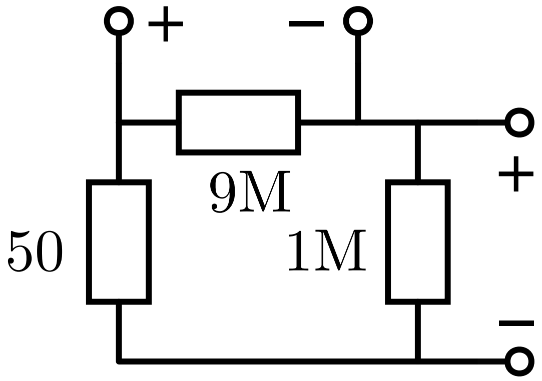
from port i to port j

RC matrix example

Oscilloscope probe circuit



R-port



R-matrix

$$\begin{pmatrix} \frac{9\cdot10^6\left(50+10^6\right)}{50+10\cdot10^6} & -\frac{9\cdot10^6\times10^6}{50+10\cdot10^6} \\ -\frac{9\cdot10^6\times10^6}{50+10\cdot10^6} & \frac{10^6\left(50+9\cdot10^6\right)}{50+10\cdot10^6} \end{pmatrix}$$

C-matrix

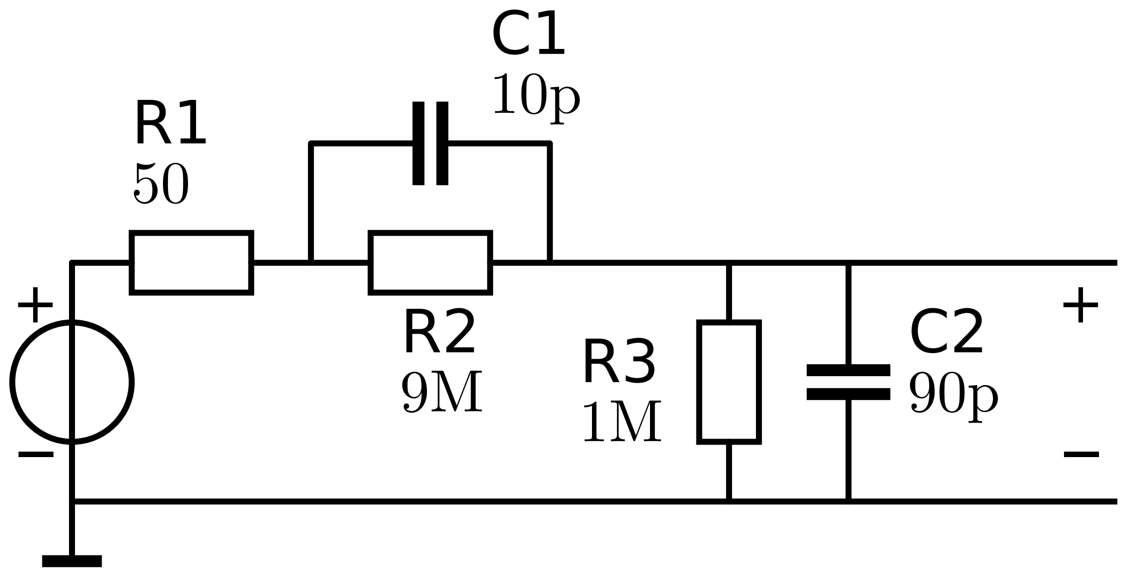
$$\begin{pmatrix} 10^{-11} & 0 \\ 0 & 90\cdot10^{-12} \end{pmatrix}$$

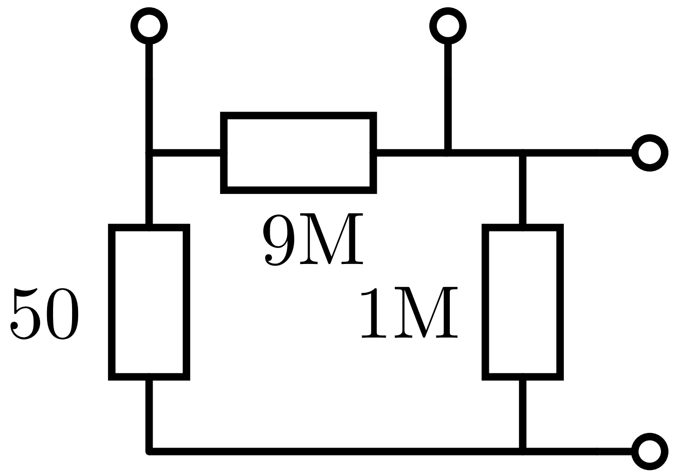
Poles from eigenvalues of RC:

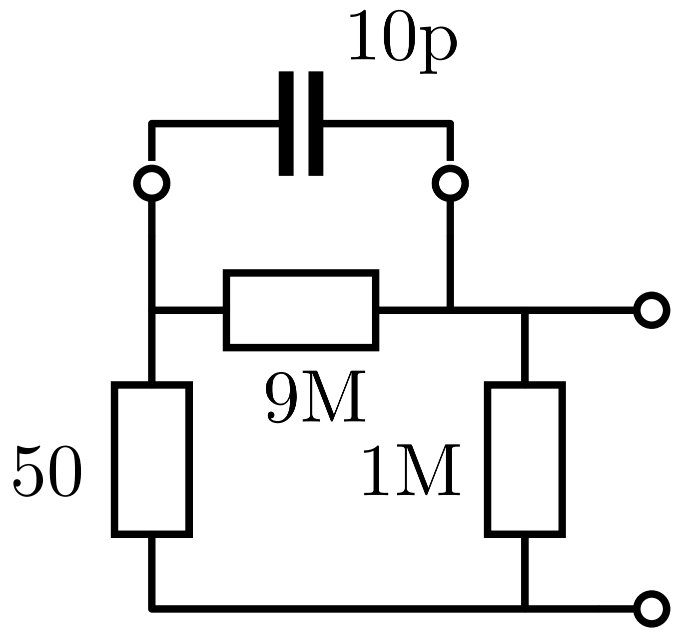
$p_1 = -1768\text{Hz}$ $\quad$ $p_2 = -0.3537\text{GHz}$

Estimation of the poles from the diagonal elements of the RC matrix

Oscilloscope probe circuit

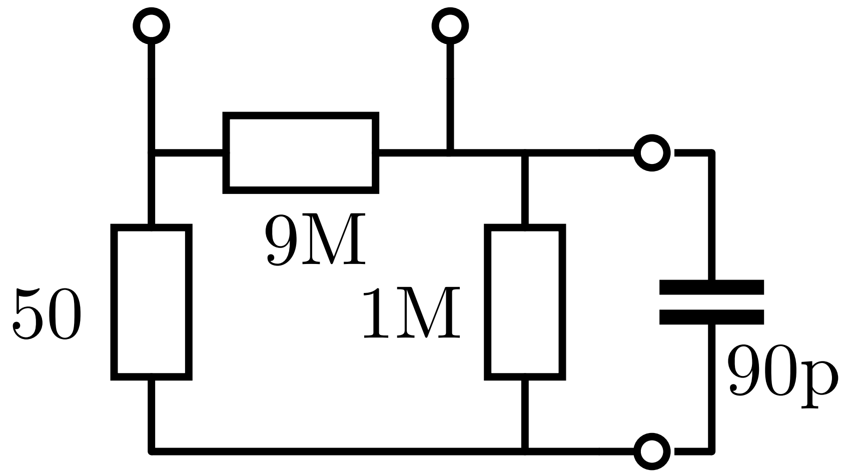






$\tau = 9\mu s$

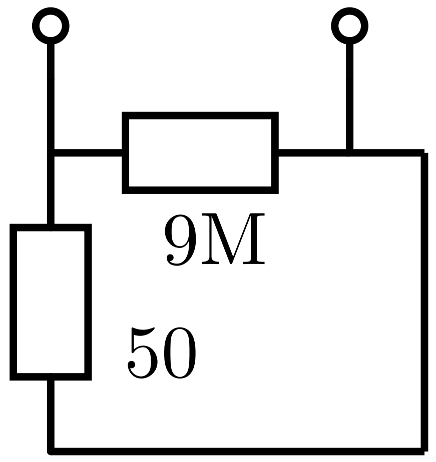
dominant pole

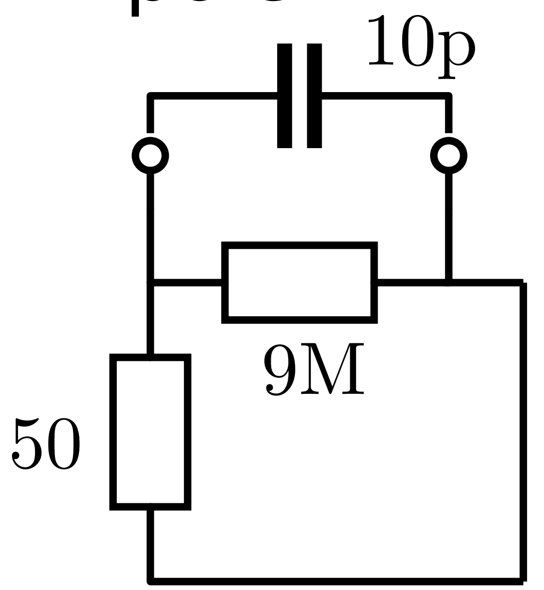


$\tau = 81\mu s$

$p_1 = -1965\text{Hz}$

non-dominant pole





$\tau = 500\text{ps}$

$p_2 = -0.3183\text{GHz}$

Rules

The number of poles equals the sum of the number of independent capacitor voltages plus the number of independent inductor currents.

The number of independent capacitor voltages equals the number of capacitors minus the number of independent loops of capacitors or capacitors and voltage sources.

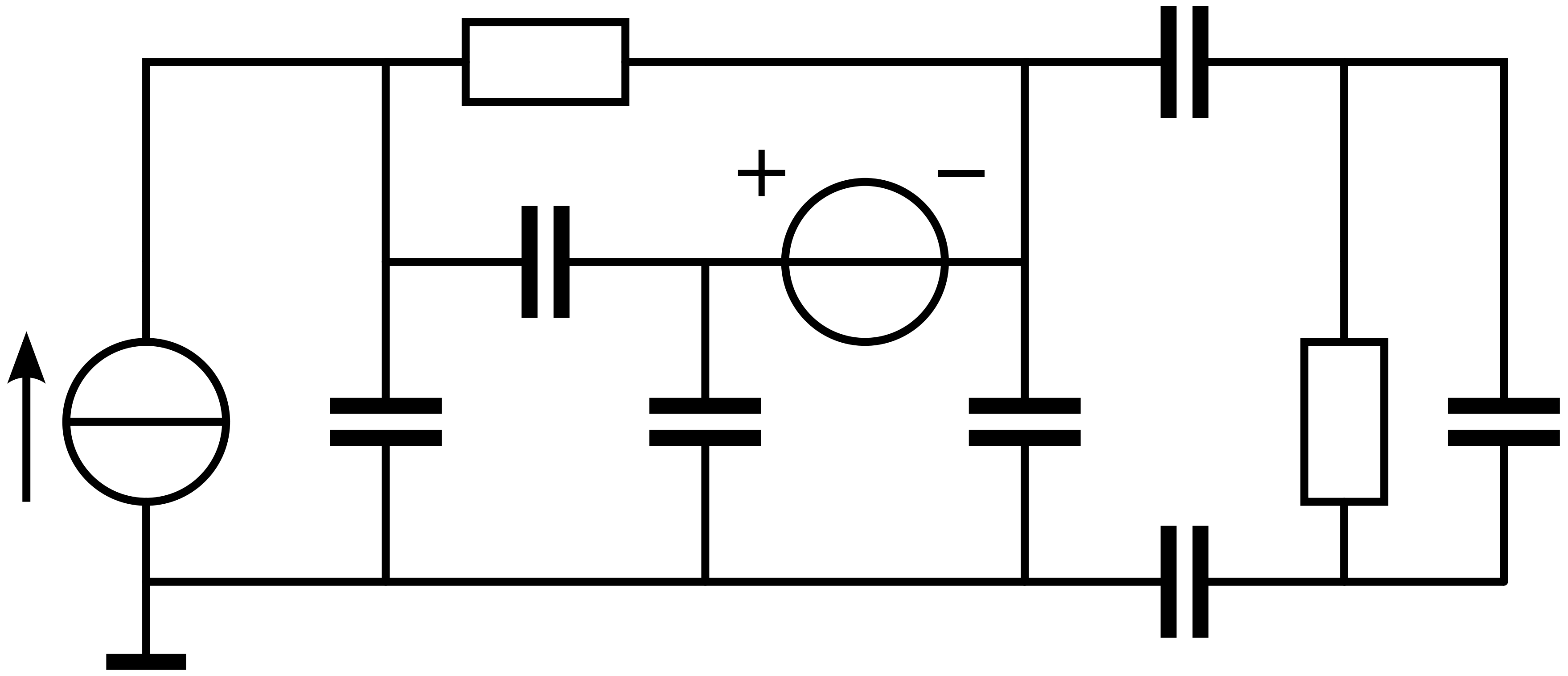
The number of independent loops of capacitors or capacitors and voltage sources equals the number of capacitors and/or voltage sources that must be removed from the network to break all the loops; this yields a tree of a network.

The number of independent inductor currents equals the number of inductors minus the number of independent cut sets of inductors and inductors and current sources.

The number of independent cut sets of inductors and inductors and current sources equals the number of inductors and/or current sources that need to be replaced in the network to obtain a connected graph, after all inductors and current sources that are part of such a cut set have been removed; this yields a tree of the network.

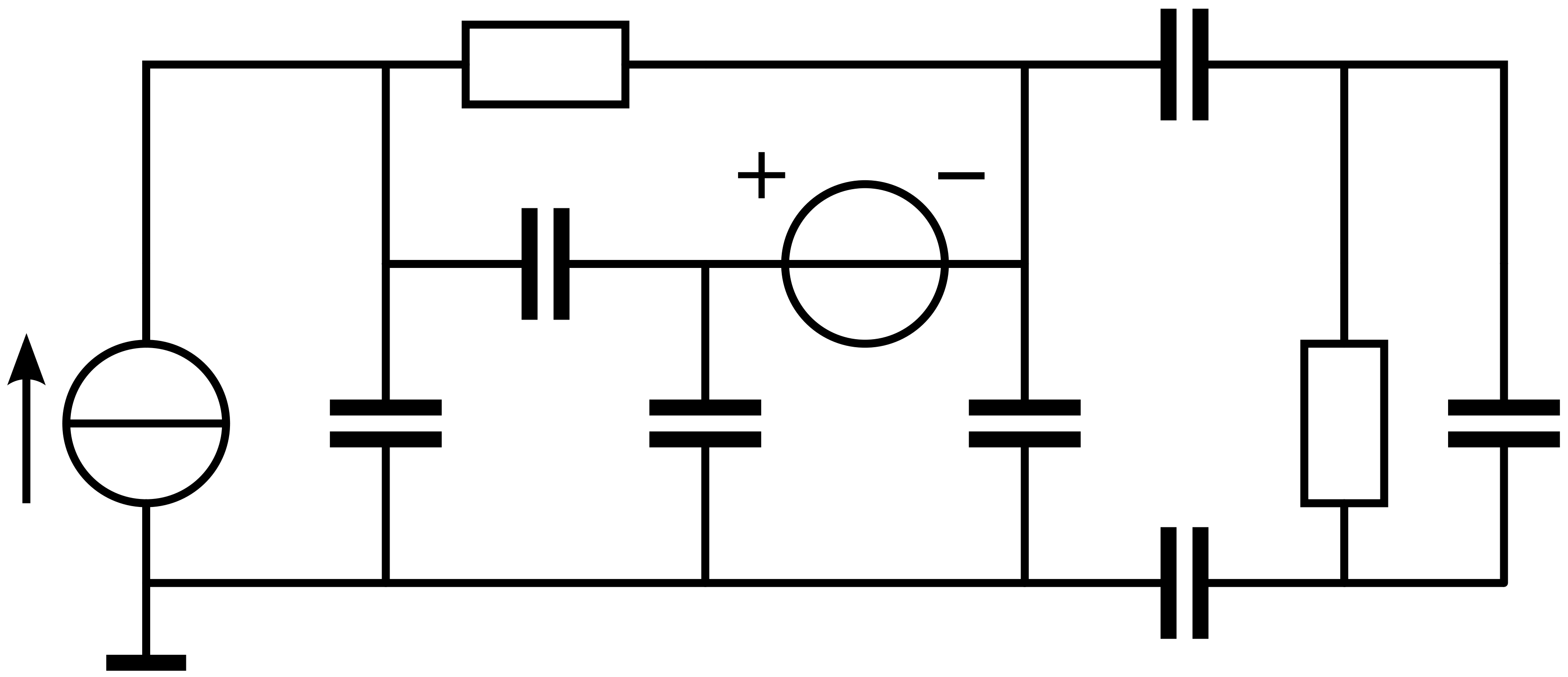
The number of poles at zero frequency equals the number of independent cut sets of capacitors and capacitors and current sources plus the number of independent loops of inductors and inductors and voltage sources.

Example



1. How many capacitors?
2. How many independent loops of capacitors and voltage sources?
3. How many poles?
4. How many independent cut sets of capacitors or of capacitors and current sources?
5. How many poles at s=0?

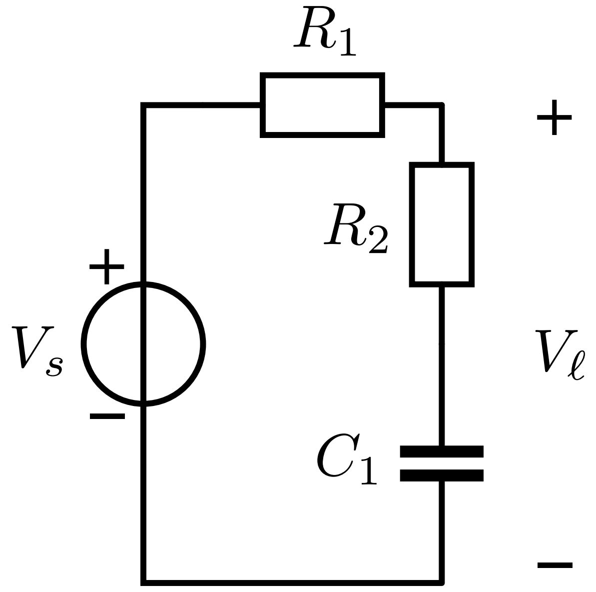
Example



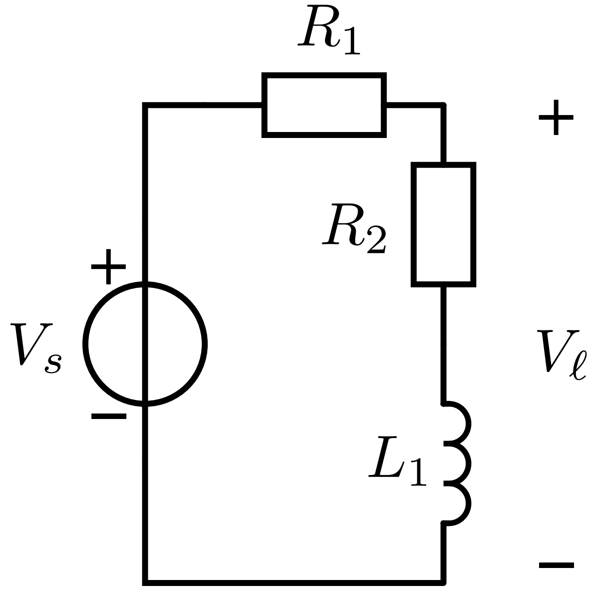
1. How many capacitors? 7
2. How many independent loops of capacitors and voltage sources? 3
3. How many poles? 4
4. How many independent cut sets of capacitors or of capacitors and current sources? 2
5. How many poles at s=0? 2

Zeros

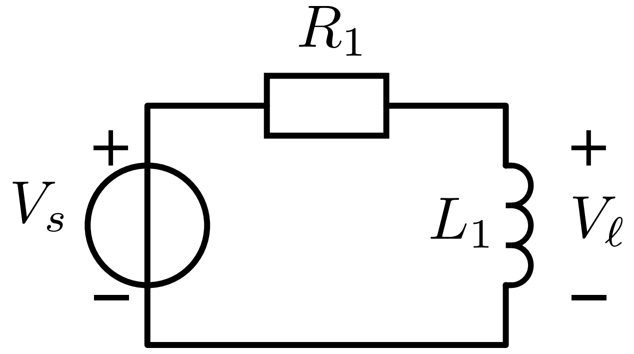
Short circuit in parallel with the signal path at complex frequency



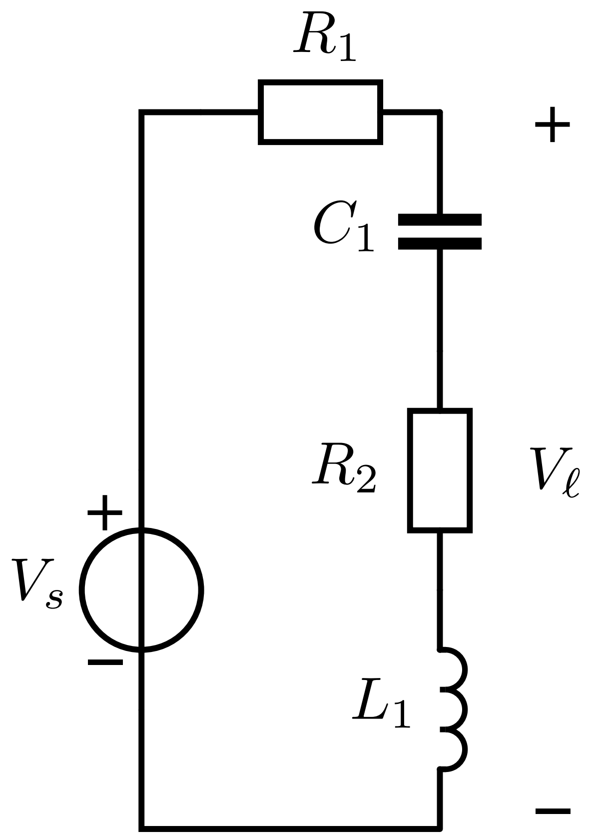
$$R_2 + \frac{1}{sC_1} = 0$$



$$R_2 + sL_1 = 0$$



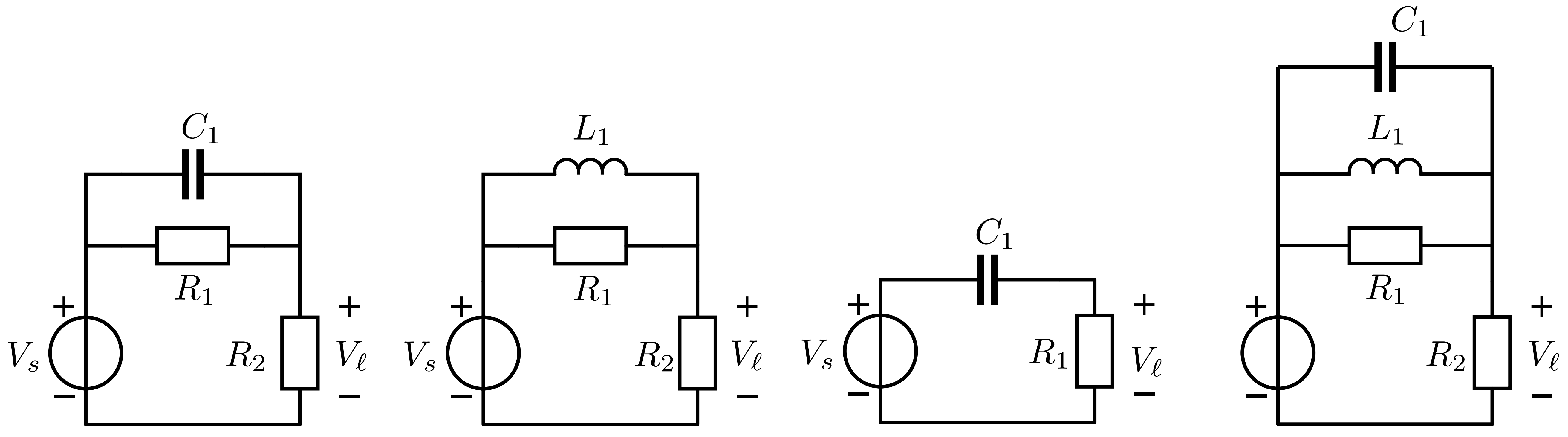
$$sL_1 = 0$$



$$R_2 + \frac{1}{sC_1} + sL_1 = 0$$

Zeros

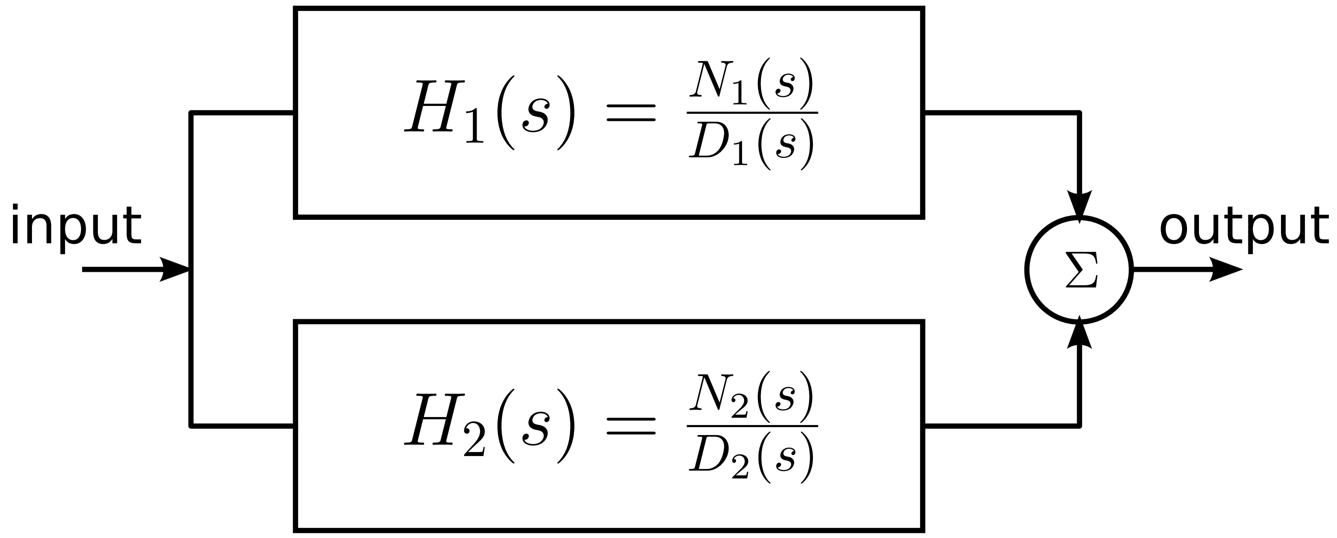
Open circuit in series with the signal path at complex frequency



$$sC_1 + \frac{1}{R_1} = 0 \qquad R_1 + \frac{1}{sL_1} = 0 \qquad sC_1 = 0 \qquad sC_1 + \frac{1}{R_1} + \frac{1}{sL_1} = 0$$

Zeros

Transfer through multiple paths that cancel each other at complex frequency



$$H_1(s) + H_2(s) = \frac{N_1(s)D_2(s)+N_2(s)D_1(s)}{D_1(s)D_2(s)}$$