Structured Electronic Design
Physics and modeling of
Linear(ized) time-invariant dynamic systems

Systems with energy storage

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A system:

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In which energy is stored

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- Which is not in its quescent state

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Linear(ized) time-invariant dynamic systems

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Modeling with linear differential equations with constant coefficients

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Sum of a number of derivatives of the response = Sum of a number of derivatives of the excitation

Linear(ized) time-invariant dynamic systems

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$$\sum_{i=0}^{i=n} a_i \frac{d^i y(t)}{dt^i} = \sum_{k=0}^{k=m} b_k \frac{d^k x(t)}{dt^k}$$

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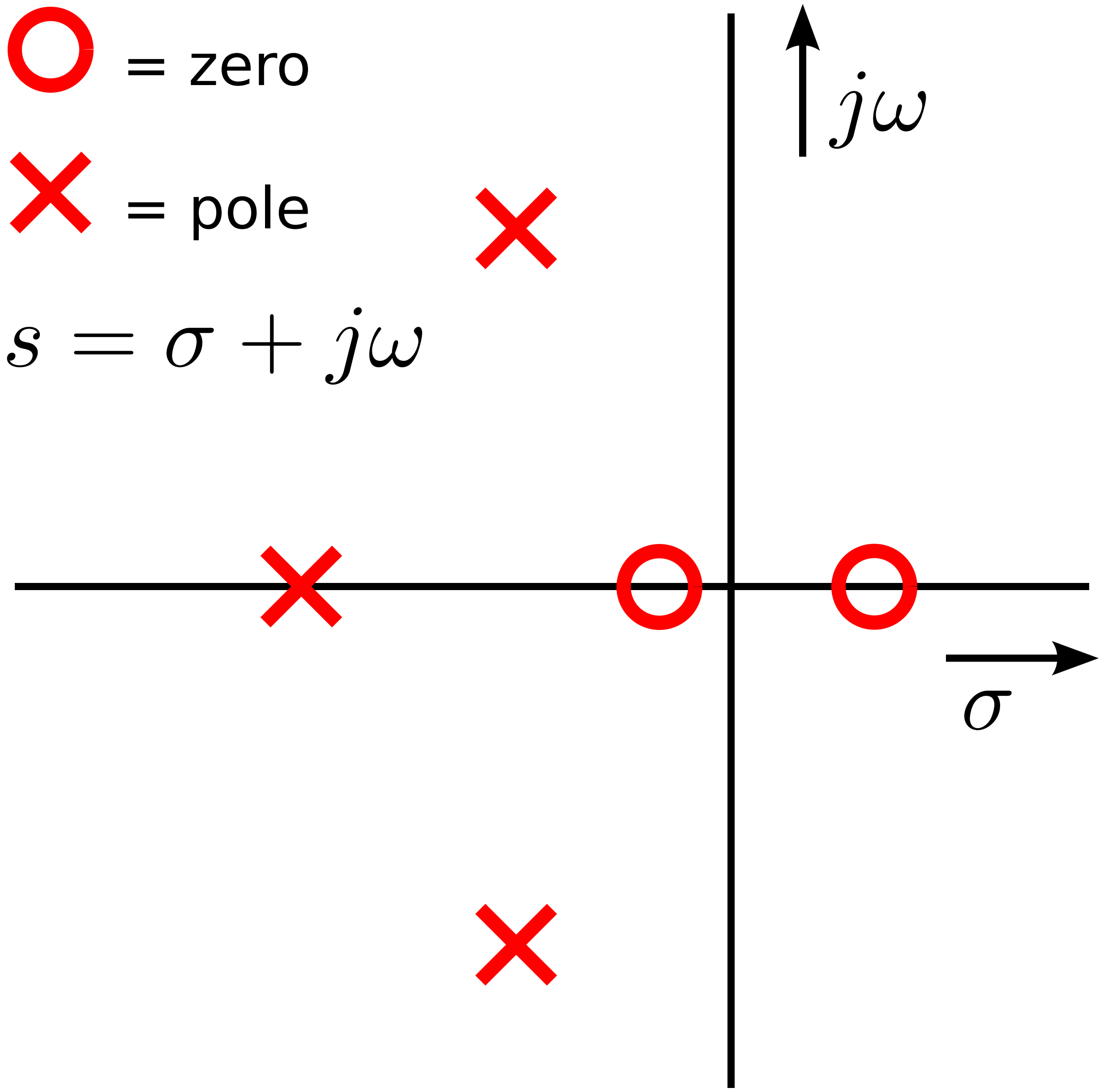

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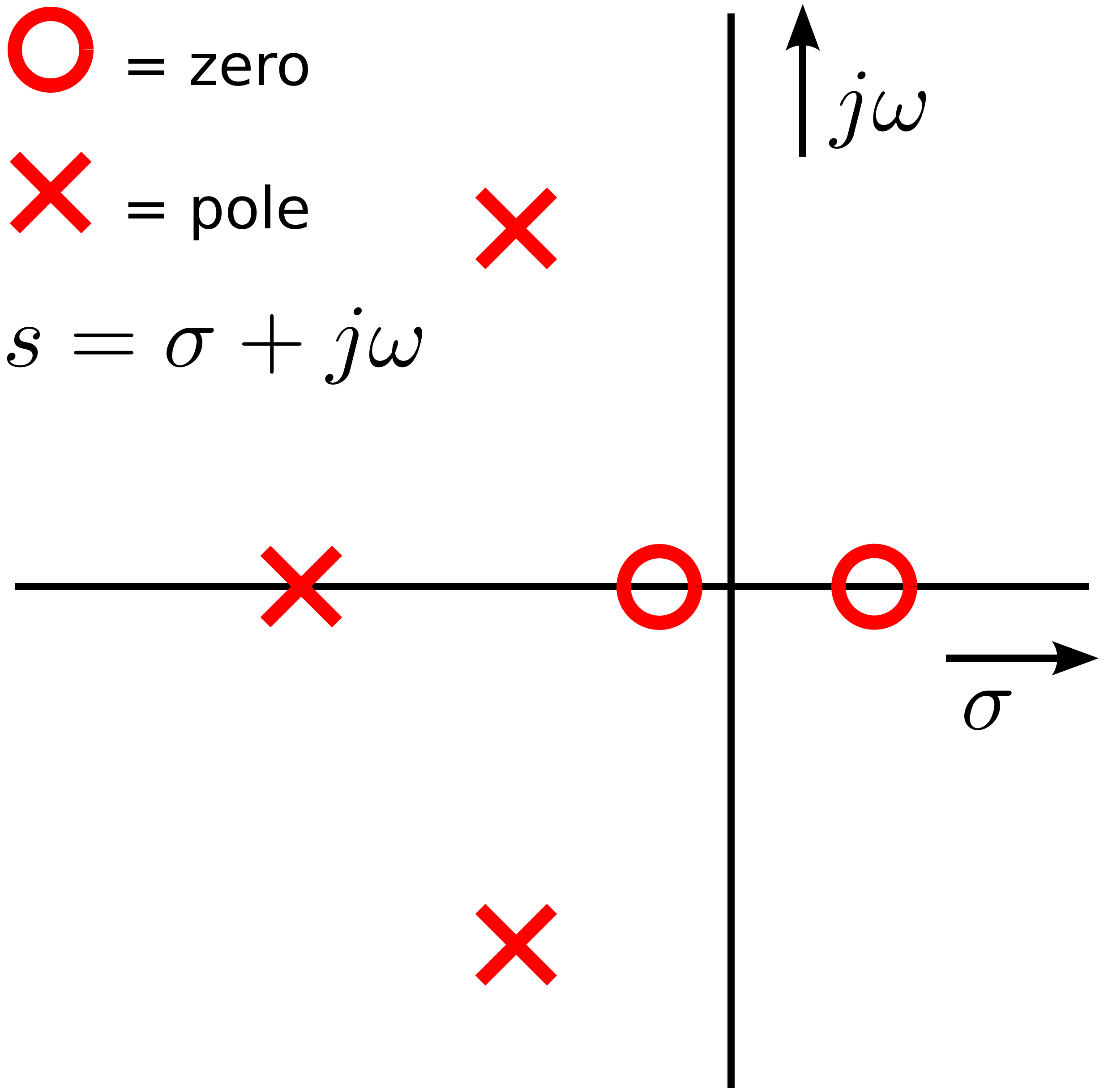

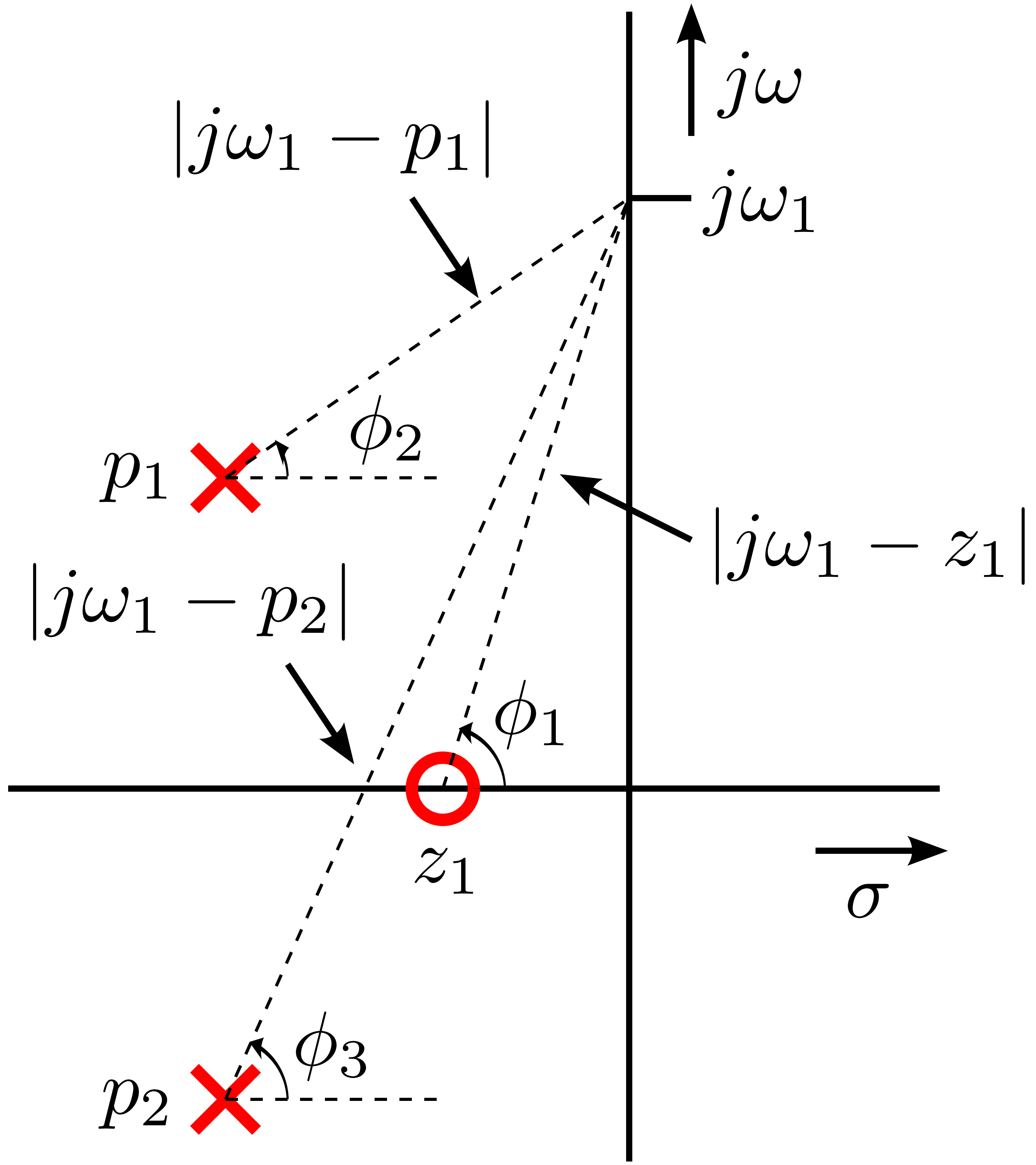

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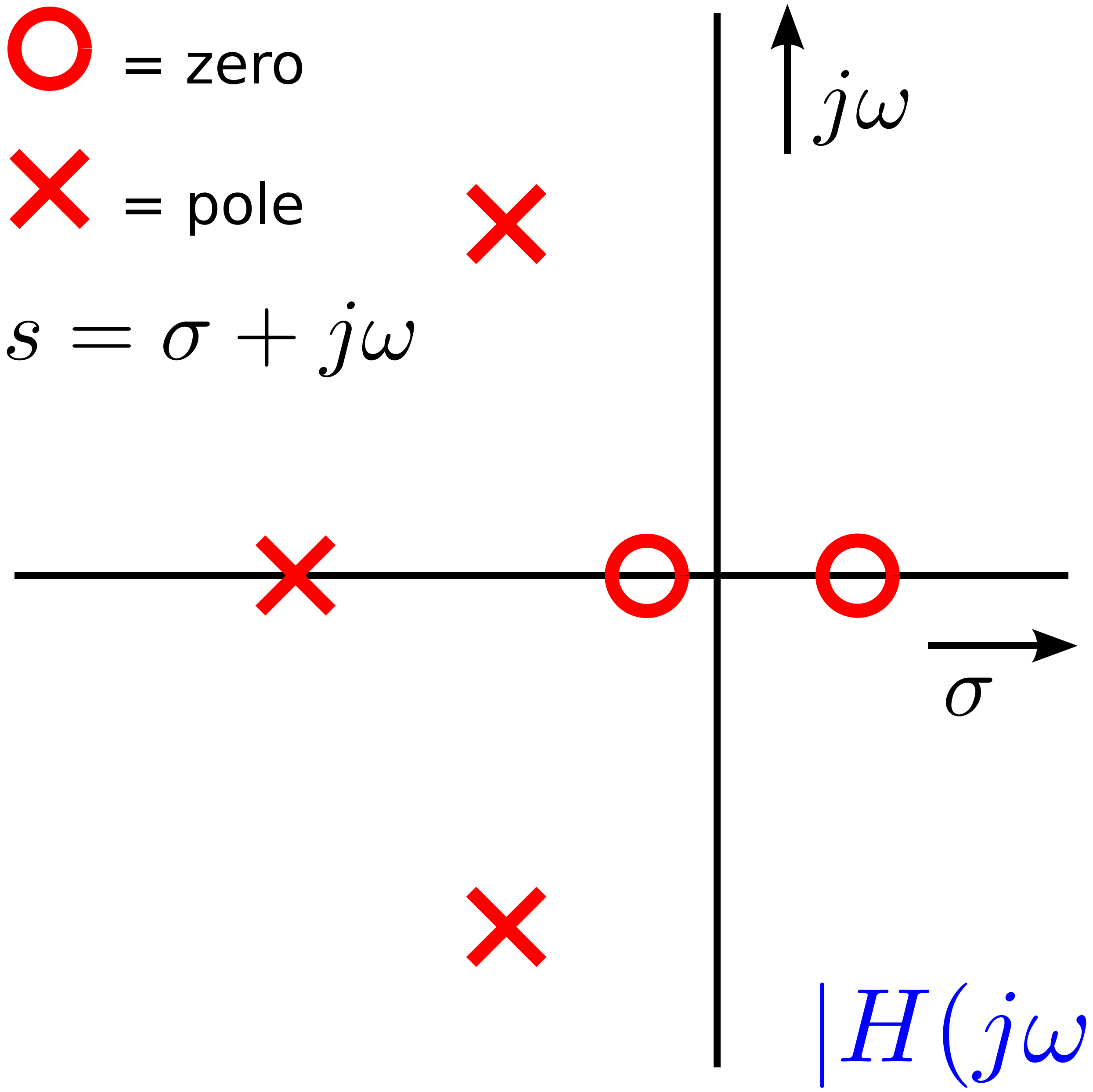

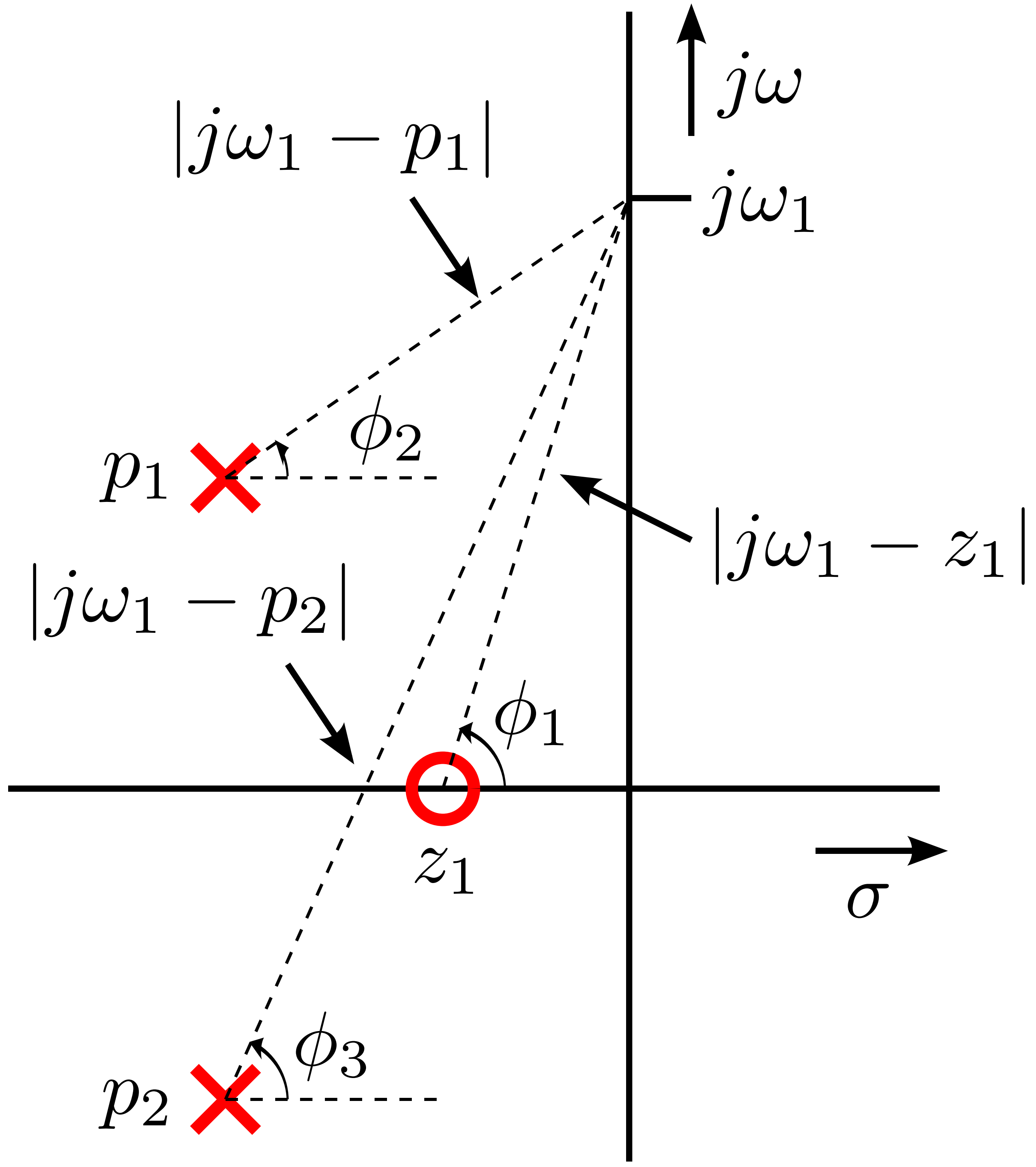

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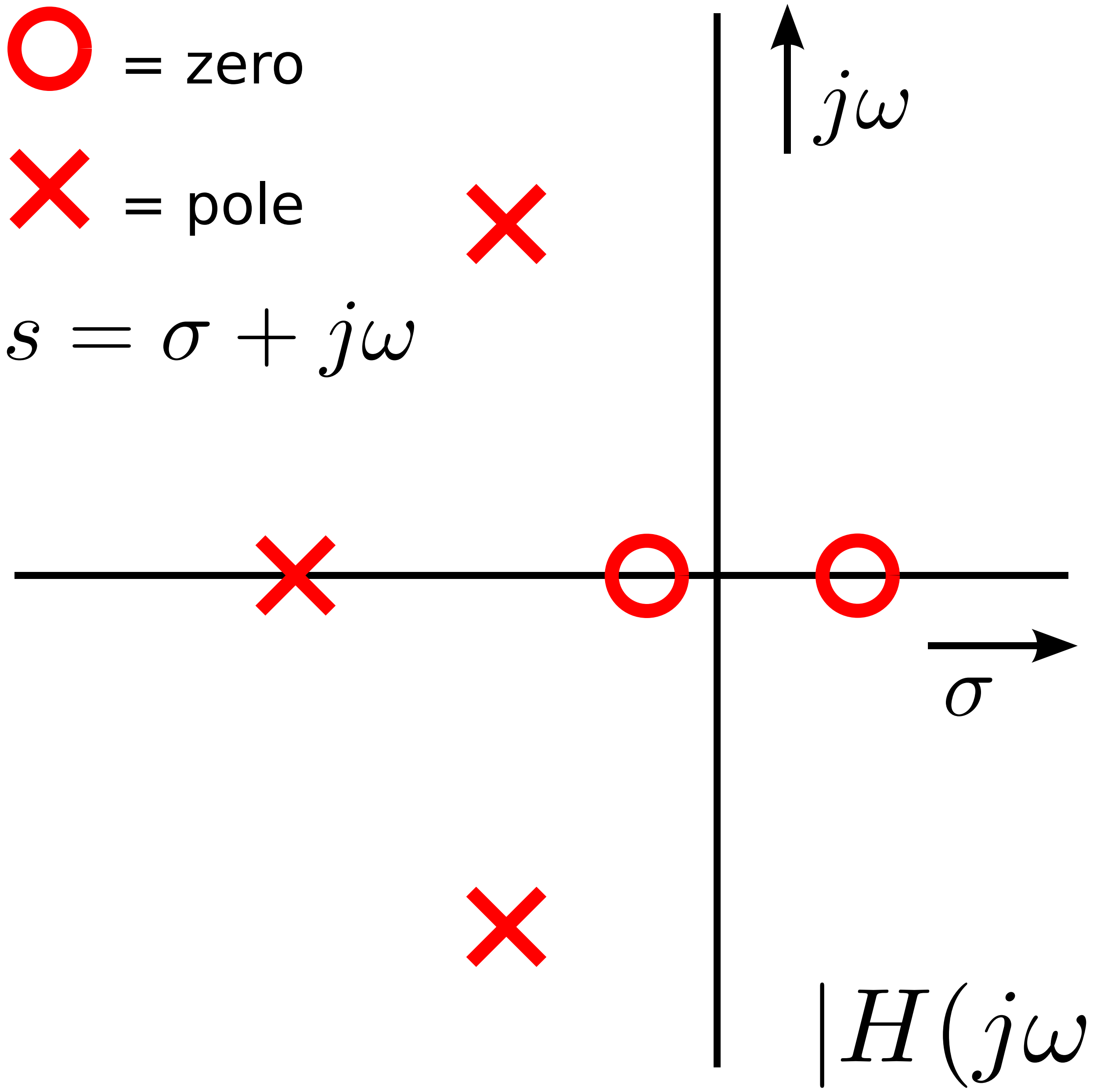

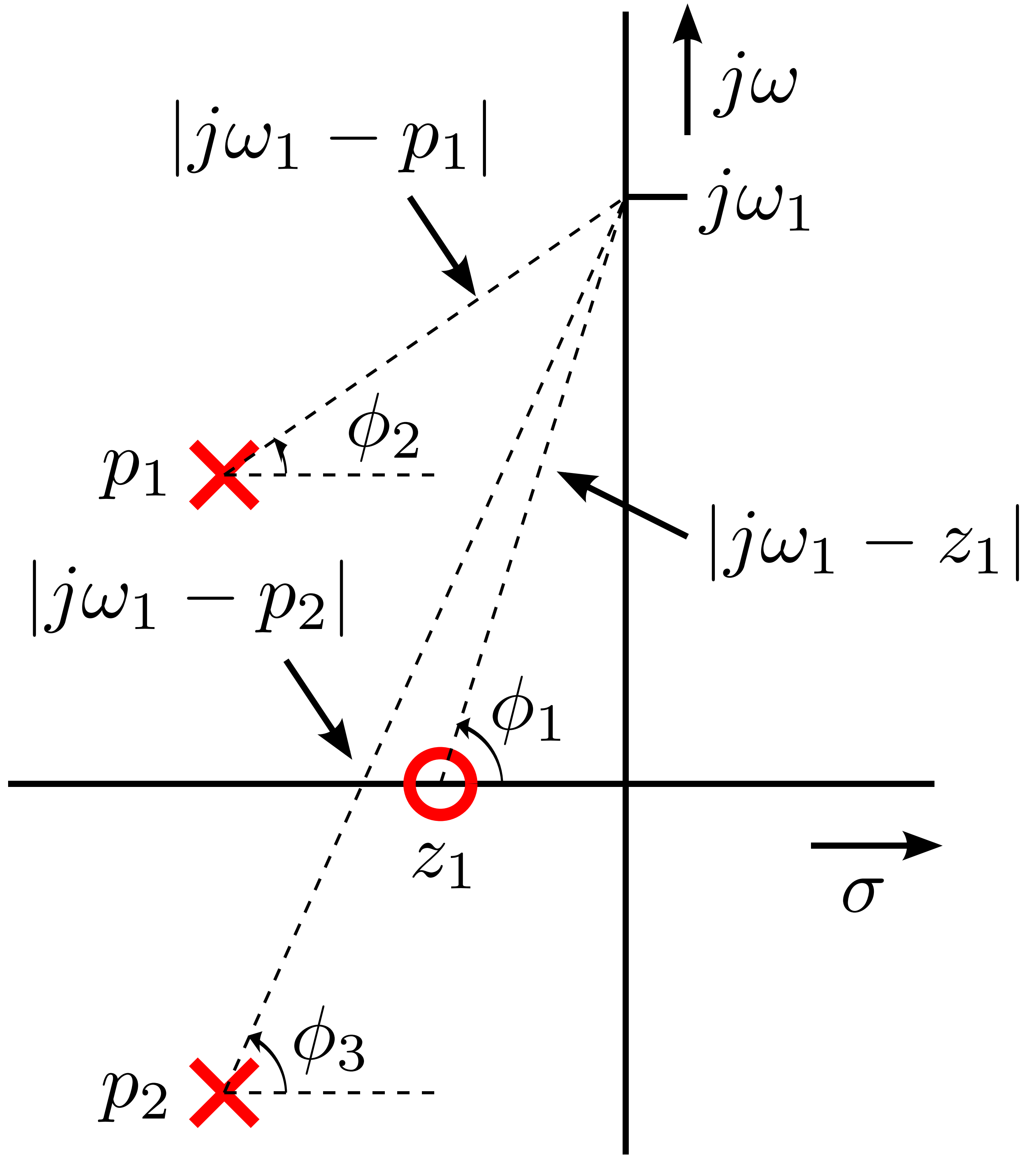

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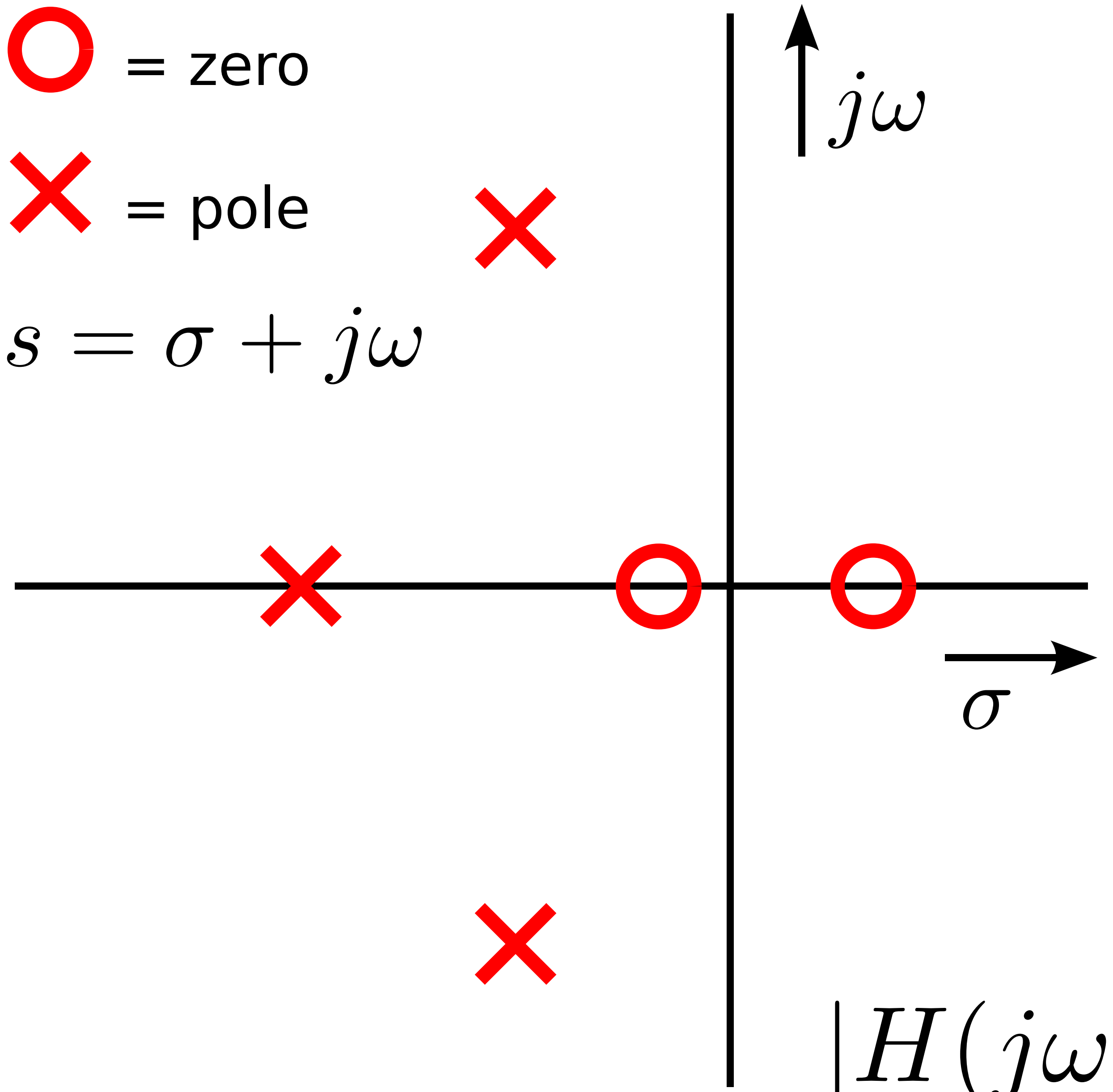

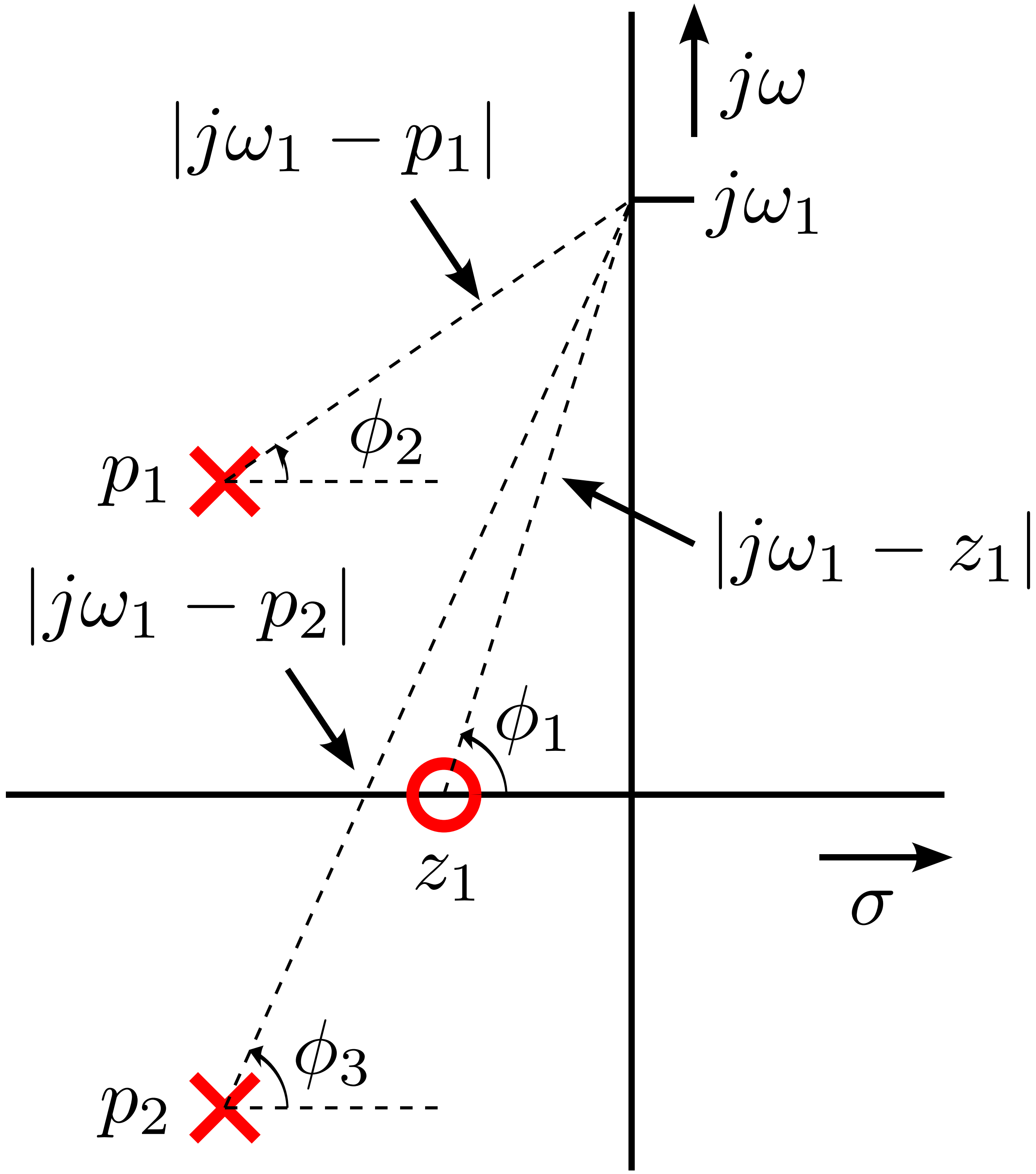

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Unit impulse response

$$h(t) = \mathcal{L}^{-1}\left\{H(s)\right\}$$

$$\mathcal{L}^{-1}\left\{H(s)\right\} = \sum_{i=1}^{n} \mathrm{Res}_i\left[e^{st}F\left(s\right)\right]$$

$$\mathrm{Res}_i = \frac{1}{(\ell-1)!}\lim_{s\to p_i}\frac{d^{\ell-1}}{ds^{\ell-1}}\left[\left(s-p_i\right)^{\ell}F\left(s\right)e^{st}\right]$$

$$\text{stable: } \mathrm{Re}(p_i) < 0\,\forall\, i$$

Unit step response

$$a(t) = \int h(t)dt = \mathcal{L}^{-1}\left\{\tfrac{1}{s}H(s)\right\}$$

